A Study of Channel Estimation Techniques Based on Pilot Arrangement in OFDM Systems

Shriram gupta, Praveen sharma
Institute of Technology and Management, Gwalior, MP, India

Abstract- The channel estimation techniques for OFDM systems based on pilot arrangement are investigated. The channel estimation based on comb type pilot arrangement is studied through different algorithms for both estimating channel at pilot frequencies and interpolating the channel. The estimation of channel at pilot frequencies is based on LS and LMS while the channel interpolation is linear interpolation, second order interpolation, low-pass interpolation, spline cubic interpolation, and time domain interpolation. Time-domain interpolation is obtained by passing to time domain through IDFT, zero padding and going back to frequency domain through DFT. In addition, the channel estimation based on block type pilot arrangement is performed by sending pilots at every sub-channel and using this estimation for a specific number of following symbols. We have also implemented decision feedback equalizer for all sub-channels followed by periodic block-type pilots. We have compared the performances of all schemes by measuring bit error rate with 16QAM, QPSK, DQPSK and BPSK as modulation schemes, and multipath rayleigh fading and AR based fading channels as channel models.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has recently been applied widely in wireless communication systems due to its high data rate transmission capability with high bandwidth efficiency and its robustness to multipath delay. It has been used in wireless LAN standards such as American IEEE802.11a and the European equivalent HIPERLAN/2 and in multimedia wireless services such as Japanese Multimedia Mobile Access Communications.

A dynamic estimation of channel is necessary before the demodulation of OFDM signals since the radio channel is frequency selective and time-variant for wideband mobile communication systems. The channel estimation can be performed by either inserting pilot tones into all of the subcarriers of OFDM symbols with a specific period or inserting pilot tones into each OFDM symbol. The first one, block type pilot channel estimation, has been developed under the assumption of slow fading channel. Even with decision feedback equalizer, this assumes that the channel transfer function is not changing very rapidly. The estimation of the channel for this block-type pilot arrangement can be based on LS or MMSE. The MMSE estimate has been shown to give 10-15 dB gain in SNR for the same mean square error of channel estimation over LS estimate [1]. In [2], a low-rank approximation is applied to linear MMSE by using the frequency correlation of the channel to eliminate the major drawback of MMSE, which is complexity. The later, the comb-type pilot channel estimation, has been introduced to satisfy the need for equalizing the significant changes even in one OFDM block. The comb-type pilot channel estimation consists of algorithms to estimate the channel at pilot frequencies and to interpolate the channel.

The estimation of the channel at the pilot frequencies for comb-type based channel estimation can be based on LS, MMSE or LMS. MMSE has been shown to perform much better than LS. In [3], the complexity of MMSE is reduced by deriving an optimal low-rank estimator with singular-value decomposition.

The interpolation of the channel for comb-type based channel estimation can depend on linear interpolation, second order interpolation, low-pass interpolation, spline cubic interpolation, and time domain interpolation. In [3], second-order interpolation has been shown to perform better than the linear interpolation. In [4], time-domain interpolation has been proven to give lower BER compared to linear interpolation.

In this paper, our aim is to compare the performance of all of the above schemes by applying 16QAM, QPSK, DQPSK and BPSK as modulation schemes, and multipath rayleigh fading and AR based fading channels as channel models. In section II, the description of the OFDM system based on pilot channel estimation is given. In section III, the estimation of the channel based on block-type pilot arrangement is discussed. In section IV, the estimation of the channel at pilot frequencies is presented. In section V, the different interpolation techniques are introduced. In section VI, the simulation environment and results are described. Section VII concludes the paper.

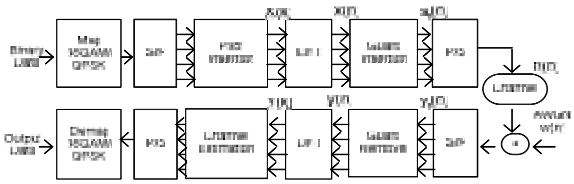

Fig. 1 Baseband OFDM system

II.SYSTEM DESCRIPTION

The OFDM system based on pilot channel estimation is given in Figure 1. The binary information is first grouped and mapped according to the modulation in “signal mapper”. After inserting pilots either to all sub-carriers with a specific period or uniformly between the information data sequence, IDFT block is used to transform the data sequence of length N {X(k)} into time domain signal {x(n)} with the following equation:

$$x(n) = IDFT\{X(k)\} = \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi kn}{N}} \quad n = 0,1,2,\ldots N-1 \qquad (1)$$

where N is the DFT length.

Following IDFT block, guard time, which is chosen to be larger than the expected delay spread, is inserted to prevent inter-symbol interference. This guard time includes the cyclically extended part of OFDM symbol in order to eliminate inter-carrier interference (ICI). The resultant OFDM symbol is given as follows:

$$x_f(n) = \begin{cases} x(N+n), & n = -N_g, -N_g+1, \cdots, -1 \\ x(n), & n = 0,1,\cdots N-1 \end{cases} \qquad (2)$$

where N_g is the length of the guard interval.

After following D/A converter, this signal will be sent from the transmitter with the assumption of the baseband system model. The transmitted signal will pass through the frequency selective time varying fading channel with additive noise. The received signal is given by:

$$y_f = x_f(n) \otimes h(n) + w(n) \qquad (3)$$

where w(n) is additive white gaussian noise and h(n) is the channel impulse response, which can be represented by: [4]

$$h(n) = \sum_{i=0}^{r-1} h_i e^{j\frac{2\pi}{N} f_{Di} Tn} \delta(\lambda - \tau_i) \quad 0 \le n \le N-1 \qquad (4)$$

where r is the total number of propagation paths, h_i is the complex impulse response of the i^{th} path, f_{Di} is the i^{th} path Doppler frequency shift, λ is delay spread index, T is the sample period and τ_i is the i^{th} path delay normalized by the sampling time.

At the receiver, after passing to discrete domain through A/D and low pass filter, guard time is removed:

$$y(n) = y_f(n + N_g) \quad n = 0,1,\cdots N-1 \qquad (5)$$

Then y(n) is sent to DFT block for the following operation:

$$Y(k) = DFT\{y(n)\} = \frac{1}{N}\sum_{n=0}^{N-1} y(n) e^{-j\frac{2\pi kn}{N}} \quad k = 0,1,2,\ldots N-1 \qquad (6)$$

Assuming there is no ISI, [7] shows the relation of the resulting Y(k) to H(k)=DFT{h(n)}, I(k) that is ICI because of Doppler frequency and W(k)=DFT{w(n)}, with the following equation:

$$Y(k) = X(k)H(k) + I(k) + W(k) \quad k = 0,1\cdots N-1 \qquad (7)$$

Following DFT block, the pilot signals are extracted and the estimated channel $H_e(k)$ for the data sub-channels is obtained in channel estimation block. Then the transmitted data is estimated by:

$$X_e(k) = \frac{Y(k)}{H_e(k)} \quad k = 0,1,\cdots N-1 \qquad (8)$$

Then the binary information data is obtained back in “signal demapper” block.

III. CHANNEL ESTIMATION BASED ON BLOCK-TYPE PILOT ARRANGEMENT

In block-type pilot based channel estimation, the pilot is sent in all sub-carriers with a specific period. Assuming the channel is constant during the block, it is insensitive to frequency selectivity. Since the pilots are sent at all carriers, there is no interpolation error. The estimation can be performed by using either LS or MMSE [1] [2]. The LS estimate is represented by:

$$h_{LS} = X^{-1} y$$
$$\text{where} \quad X = diag\{x_0, x_1, \cdots x_{N-1}\}$$
$$y = \begin{bmatrix} y_0 \\ \vdots \\ y_{N-1} \end{bmatrix} \qquad (9)$$

where x_i is the pilot value sent at the i^{th} subcarrier and y_i is the value received at the i^{th} sub-carrier.

If the time domain channel vector g is Gaussian and uncorrelated with the channel noise, the frequency-domain MMSE estimate of g is given by:

$$h_{MMSE} = FR_{gy}R_{yy}^{-1}y \quad where$$

$$F = \begin{bmatrix} W_N^{00} & \cdots & W_N^{0(N-1)} \\ \vdots & \ddots & \vdots \\ W_N^{(N-1)0} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix} and$$

$$W_N^{nk} = \frac{1}{N}e^{-j2® \frac{n}{N}k} \qquad (10)$$

where R_{gy} and R_{yy} is cross covariance matrix between g and y and the auto-covariance matrix of y respectively. When the channel is slow fading, the channel estimation inside the block can be updated using the decision feedback equalizer at each sub-carrier. Decision feedback equalizer for the k^{th} sub-carrier can be described as follows:

- The channel response at the k^{th} sub-carrier estimated from the previous symbol $\{H_e(k)\}$ is used to find the estimated transmitted signal $\{X_e(k)\}$.

$$X_e(k) = \frac{Y(k)}{H_e(k)} \quad k = 0,1,\cdots N-1 \qquad (11)$$

- $\{X_e(k)\}$is mapped to the binary data through "signal demapper" and then obtained back through "signal mapper" as $\{X_{pure}(k)\}$.
- The estimated channel $\{H_e(k)\}$ is updated by:

$$H_e(k) = \frac{Y(k)}{X_{pure}(k)} \quad k = 0,\cdots N-1 \qquad (12)$$

Since the decision feedback equalizer has to assume that the decisions are correct, the fast fading channel will cause the complete loss of estimated channel parameters. Therefore, as the channel fading becomes faster, there happens to be a compromise between the estimation error due to the interpolation and the error due to loss of channel tracking. For fast fading channels, as will be shown in simulations, the comb-type based channel estimation performs much better.

IV. CHANNEL ESTIMATION AT PILOT FREQUENCIES IN COMB-TYPE PILOT ARRANGEMENT

In comb-type pilot based channel estimation, the N_p pilot signals are uniformly inserted into X(k) according to the following equation:

$$X(k) = X(mL + l) = \begin{cases} x_p(m), & l = 0 \\ \inf.data, & l = 1,\ldots L-1 \end{cases} \qquad (13)$$

where L=number of carriers/N_p and $x_p(m)$ is the m^{th} pilot carrier value.

We define $\{H_p(k)\ k=0,1,\ldots N_p\}$as the frequency response of the channel at pilot sub-carriers. The estimate of the channel at pilot sub-carriers based on LS estimation is given by:

$$H_e(k) = \frac{Y_p(k)}{X_p(k)} \qquad k = 0,1,\cdots N_p - 1 \qquad (14)$$

where $Y_p(k)$ and $X_p(k)$ are output and input at the k^{th} pilot sub-carrier respectively.

Since LS estimate is susceptible to noise and inter-carrier interference (ICI), MMSE is proposed while compromising complexity. Since MMSE includes the matrix inversion at each iteration, the simplified linear MMSE estimator is suggested in [5]. In this simplified version, the inverse is only need to be calculated once. In [3], the complexity is further reduced with a low-rank approximation by using singular value decomposition.

V. INTERPOLATION TECHNIQUES IN COMB-TYPE PILOT ARRANGEMENT

In comb-type pilot based channel estimation, an efficient interpolation technique is necessary in order to estimate channel at data sub-carriers by using the channel information at pilot sub-carriers.

The linear interpolation method is shown to perform better than the piecewise-constant interpolation in [6]. The channel estimation at the data-carrier k, mL<k<(m+1)L, using linear interpolation is given by:

$$\begin{aligned} H_e(k) &= H_e(mL + l) \\ &= \left(H_p(m+1) - H_p(m)\right)\frac{l}{L} + H_p(m) \\ & 0 \le l < L \end{aligned} \qquad (15)$$

The second-order interpolation is shown to fit better than linear interpolation[3]. The channel estimated by second-order interpolation is given by:

$$\begin{aligned} H_e(k) &= H_e(mL + l) \\ &= c_1 H_p(m-1) + c_0 H_p(m) + c_{-1} H_p(m+1) \end{aligned}$$

$$where \begin{cases} c_1 = \frac{\sim(\sim - 1)}{2}, \\ c_0 = -(\sim - 1)(\sim + 1), \qquad \sim = l/N \\ c_{-1} = \frac{\sim(\sim + 1)}{2}, \end{cases} \qquad (16)$$

The low-pass interpolation is performed by inserting zeros into the original sequence and then applying a special lowpass FIR filter that allows the original data to pass through unchanged and interpolates between such that the mean-square error between the interpolated points and their ideal values is minimized *(interp in MATLAB).*

The spline cubic interpolation produces a smooth and continuous polynomial fitted to given data points *(spline in MATLAB).*

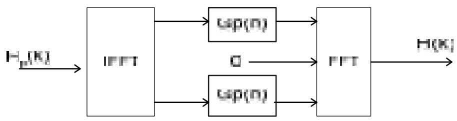

Fig. 2 Time Domain Interpolation

The time domain interpolation is a high-resolution interpolation based on zero-padding and DFT/IDFT [7]. After obtaining the estimated channel $\{H_p(k),\ k=0,1,\ldots N_p-1\}$, we first convert it to time domain by IDFT:

$$G_N(n) = \sum_{k=0}^{N_p-1} H_p(k) e^{j\frac{2\pi kn}{N_p}},\ n = 0,1,\cdots N_p - 1 \quad (17)$$

Then, by using the basic multi-rate signal processing properties [8], the signal is interpolated by transforming the N_p points into N points with the following method:

$$G_N = \begin{cases} G_p(n), & 0 \le n < \frac{N_p}{2} - 1 \\ 0, & \frac{N_p}{2} \le n < N - \frac{N_p}{2} - 1 \\ G_p(n - N + 1 + N_p), & N - \frac{N_p}{2} - 1 \le n < N - 1 \end{cases} \quad (18)$$

The estimate of the channel at all frequencies is obtained by:

$$H(k) = \sum_{n=0}^{N-1} G_N(n) e^{-j\frac{2\pi}{N}nk} \quad 0 \le k \le N - 1 \quad (19)$$

VI. SIMULATION

A) DESCRIPTION OF SIMULATION

I. System parameters

OFDM system parameters used in the simulation are as follows: the number of sub-carriers is 1024, pilot ratio is 1/8, guard length is 256 and carrier modulation is QPSK, DQPSK, BPSK or 16QAM. We assume to have perfect synchronization since the aim is to observe channel estimation performance. Moreover, we have chosen the guard interval to be greater than the maximum delay spread in order to avoid inter-symbol interference. Simulations are carried out for different signal-to-noise (SNR) ratios and for different Doppler spreads.

II. Channel model

Two multipath fading channel models are used in the simulations. The 1st channel model is the ATTC (Advanced Television Technology Center) and the Grande Alliance DTV laboratory's ensemble E model, whose static case impulse response is given by:

$$h(n) = \delta(n) + 0.3162\,\delta(n-2) + 0.1995\,\delta(n-17) + 0.1296\,\delta(n-36) + 0.1\delta(n-75) + 0.1\delta(n-137) \quad (20)$$

The 2nd channel model is the simplified version of DVB-T channel model, whose static impulse response is given in Table I.

In the simulation, we have used Rayleigh fading channel. In order to see the effect of fading on block type based and LMS based channel estimation, we have also modeled channel that is time-varying according to the following autoregressive (AR) model:

$$h(n+1) = ah(n) + w(n) \quad (21)$$

where a is the fading factor and w(k) is AWGN noise vector. "a" is chosen to be close to 1 in order to satisfy the assumption that channel impulse response does not change within one OFDM symbol duration. In the simulations, "a" changes from 0.90 to 1.

Table 1
Channel Impulse Response For Channel 2

Delay (OFDM samples)	Gain	Phase(rad)
0	0.2478	-2.5649
1	0.1287	-2.1208
3	0.3088	0.3548
4	0.4252	0.4187
5	0.49	2.7201
7	0.0365	-1.4375
8	0.1197	1.1302
12	0.1948	-0.8092
17	0.4187	-0.1545
24	0.317	-2.2159
29	0.2055	2.8372
49	0.1846	2.8641

III. Channel estimation based on block-type pilot arrangement

We have modeled two types of block-type pilot based channel estimation. Each block consists of a fixed number of symbols, which is 30 in the simulation. Pilots are sent in all the sub-carriers of the first symbol of each block and channel estimation is performed by using LS estimation. According to the first model, the channel estimated at the beginning of the block is used for all the following symbols of the block and according to the second method, the decision feedback equalizer, which is described in section III, is used for the following symbols in order to track the channel.

IV. Channel estimation based on comb-type pilot arrangement

We have used both LS and LMS to estimate the channel at pilot frequencies. The LS estimator description is given in section IV. The LMS estimator uses one tap LMS adaptive filter at each pilot frequency. The first value is found directly through LS and the following values are calculated based on the previous estimation and the current channel output as shown in Figure 3.

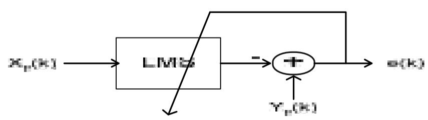

Fig. 3. LMS Scheme

After estimating the channel at pilot frequencies by using either LS or LMS, all of the possible interpolation techniques (linear interpolation, second order interpolation, low-pass interpolation, spline cubic interpolation, and time domain interpolation) are applied to investigate the effects.

B) SIMULATION RESULTS

Figures 4,5,6 and 7 give the bit error rate performance of channel estimation algorithms for different modulations and for rayleigh fading channel, whose static channel response is given in (20) with Doppler frequency 70Hz. In these simulations, Block-type estimation showed 10-15dB higher BER than that of comb-type estimation.

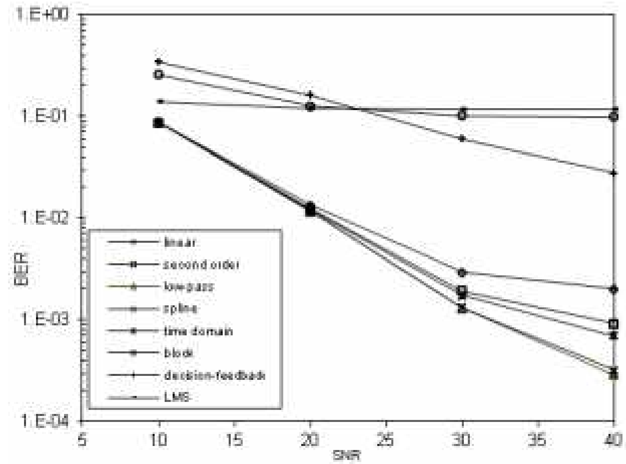

Fig. 4. BPSK (Channel 1) Rayleigh Fading

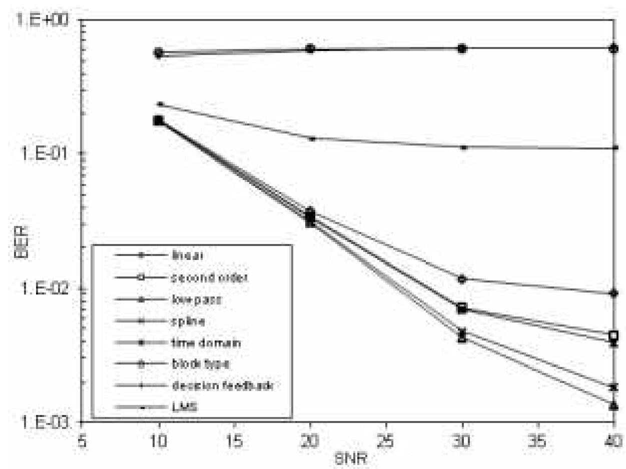

Fig. 5. QPSK (Channel 1) Rayleigh Fading

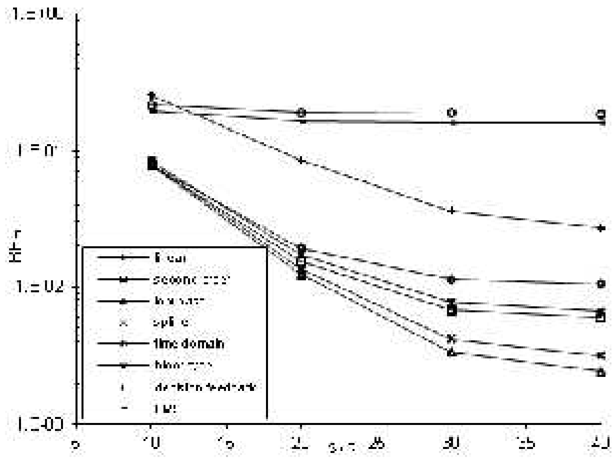

Fig. 6 16QAM (Channel 1) Rayleigh Fading

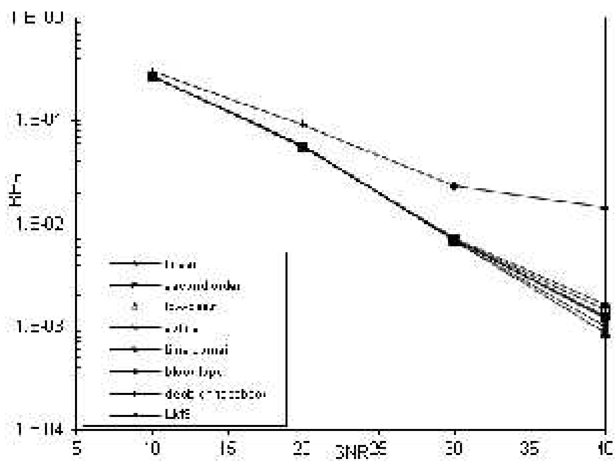

Fig. 7 DQPSK (Channel 1) Rayleigh Fading

The comb-type channel estimation with low pass interpolation achieves the best performance among all the estimation techniques for BPSK, QPSK and 16QAM modulation. The performance among comb-type channel estimation techniques usually ranges from the best to the worst as follows: low-pass, spline, time-domain, second-order and linear. The result was expected since the low-pass interpolation used in simulation does the interpolation such that the mean-square error between the interpolated points and their ideal values is minimized. These results are also consistent with those obtained in [3] and [4].

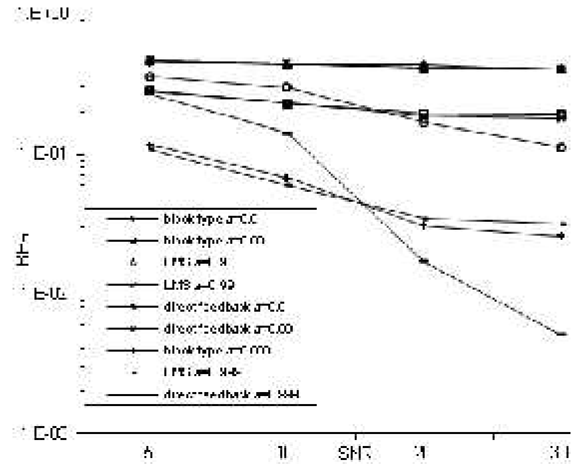

Fig. 8 16QAM (Channel 1) AR fading

DQPSK modulation based channel estimation shows almost the same performance for all channel estimation techniques except the comb-type channel estimation with spline interpolation method. The BER performance for all estimation types is much better than channel estimation techniques with other modulations for high SNR whereas it is worse for low SNR.

The effect of fading on the block type and LMS estimation can be observed from Fig.8 for autoregressive channel model with different fading parameters. As the fading factor "*a*" in equation (21) increases from 0.9 to 0.999, the performance of both block based methods and LMS improves. When fading is fast, this means higher fading parameter, the estimation does not improve as SNR increases. The reason for this is that the tracking error in fast fading channel avoids improving the performance. On the other hand, for slow fading channel, the BER of the decision feedback block-type channel estimation tracks the channel much better compared to the other two schemes as SNR increases.

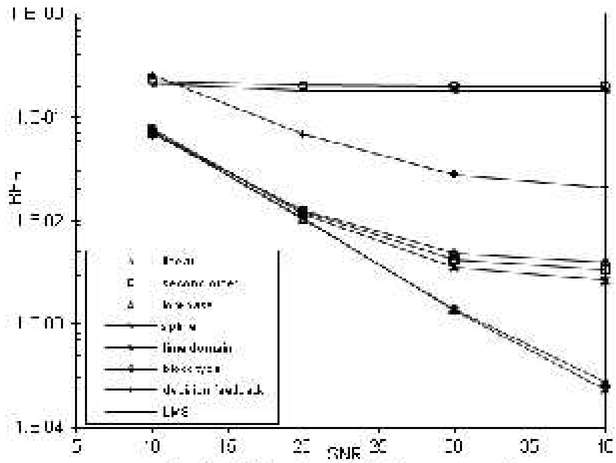

Fig. 9 16QAM (Channel 2) Rayleigh fading

The general characteristics of the channel estimation techniques perform the same for Rayleigh fading channel, whose static impulse response is given in table 1 for 16QAM as can be seen in Fig.9.

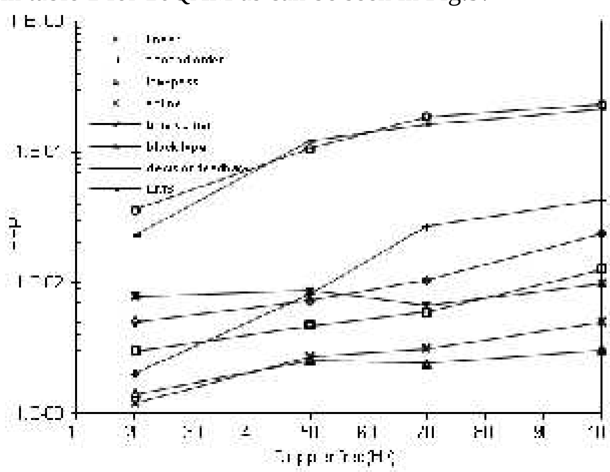

Fig. 10 16QAM (Channel 1) Rayleigh Fading

Figure 10 shows the performance of channel estimation methods for 16QAM modulation, Rayleigh fading channel whose static response is given in (20) and 40dB SNR for different Doppler frequencies. The general behavior of the plots is that BER increases as the Doppler spread increases. The reason is the existence of severe ICI caused by Doppler shifts. Another observation from this plot is that decision feedback block type channel estimation performs better than comb-type based channel estimation for low Doppler frequencies as suggested in [10] except low-pass and spline interpolation. We also observe that time-domain interpolation performance improves compared to other interpolation techniques as Doppler frequency increases.

VII.CONCLUSION

In this paper, a full review of block-type and comb-type pilot based channel estimation is given. Channel estimation based on block-type pilot arrangement with or without decision feedback equalizer is described. Channel estimation based on comb-type pilot arrangement is presented by giving the channel estimation methods at the pilot frequencies and the interpolation of the channel at data frequencies. The simulation results show that comb-type pilot based channel estimation with low-pass interpolation performs the best among all channel estimation algorithms. This was expected since the comb-type pilot arrangement allows the tracking of fast fading channel and low-pass interpolation does the interpolation such that the mean-square error between the interpolated points and their ideal values is minimized. In addition, for low Doppler frequencies, the performance of decision feedback estimation is observed to be slightly worse than that of the best estimation. Therefore, some performance degradation can be tolerated for higher data bit rate for low Doppler spread channels although low-pass interpolation comb-type channel estimation is more robust for Doppler frequency increase.

REFERENCES

[1] J.-J van de Beek, O. Edfors, M. Sandell, S.K. Wilson and P.O. Borjesson, " On channel estimation in OFDM systems" in *Proc. IEEE 45th Vehicular Technology Conference,* Chicago, IL, Jul. 1995, pp. 815-819

[2] O. Edfors, M. Sandell, J.-J van de Beek, S.K. Wilson and P.O. Borjesson, "OFDM channel estimation by singular value decomposition" in *Proc. IEEE 46th Vehicular Technology Conference,* Atlanta, GA, USA, Apr. 1996, pp. 923-927

[3] M. Hsieh and C. Wei, "Channel estimation for OFDM systems based on comb-type pilot arrangement in frequency selective fading channels" in *IEEE Transactions on Consumer Electronics,* vol. 44, no.1, February 1998

[4] R. Steele, Mobile Radio Communications, London, England, Pentech Press Limited, 1992

[5] U. Reimers, "Digital video broadcasting," *IEEE Communications Magazine,* vol. 36, no. 6, pp. 104-110, June 1998.

[6] L. J. Cimini, Jr., "Analysis and simulation of a digital mobile channel using orthogonal frequency division multiplexing," *IEEE Trans. Commun.,* vol.33, no. 7, pp. 665-675, July 1985.

[7] Y. Zhao and A. Huang, "A novel channel estimation method for OFDM Mobile Communications Systems based on pilot signals and transform domain processing", in *Proc. IEEE 47th Vehicular Technology Conference,* Phoenix, USA, May 1997, pp. 2089-2093

[8] A. V. Oppenheim and R. W. Schafer, *Discrete-Time Signal Processing,* New Jersey, Prentice-Hall Inc., 1999

[9] *Digital video broadcasting (DVB): Framing, channel coding and modulation for digital terrestrial television,* Draft ETSI EN300 744 V1.3.1 (2000-08).

[10] Y. Li, "Pilot-Symbol-Aided Channel Estimation for OFDM in Wireless Systems", in *IEEE Transactions on Vehicular Technology,* vol. 49, no.4, July 2000.

