# OPTIMIZATION AND SENSITIVITY ANALYSIS OF FLUTTER OF AN AIRCRAFT WING

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### Abstract

The main objective of this research is to obtain a preliminary design of an aircraft wing with an underwing store for improved flutter performance in the transonic regime. Since the transonic flow regime is already highly non-linear, the presence of an underwing store creates store-induced non-linearities in addition to those non-linearites associated with the wing. All these non-linearities are captured using a computational tool called Computational Aeroelasticity **P**rogram Transonic Small Disturbance (CAP-TSD). In this work, a methodology is developed to incorporate the non-linearities into a multidisciplinary optimization algorithm. Generally, combining a non-linear analysis with optimization is a computationally expensive and difficult task. Therefore, the parameters that are insignificant in the analyses are identified and excluded. A wing with different store configurations is modeled using finite elements. Using CAP-TSD, several parametric studies on the flutter of various wing-underwing store configurations are conducted in the transonic regime.

# 1.0 Introduction

Computational fluid dynamics methods have been developed extensively and used in blade design on a daily basis [1]. Methodology development for turbomachinery blade shape optimization typically follows that for the external flows, e.g. those around aircraft wings. A starting point of the departure of the former from the latter is the interactive nature among components in turbomachines. An 'optimized' bladerow in isolation may more than often behave differently when put in a multi-row environment. From a basic blade row/stage matching viewpoint, interactive blade rows should be optimized concurrently. An iterative alternative would typically be very lengthy.

Ever increased aerodynamic loading in turn increases the blade row interactions, hence by itself underlines the need for concurrent design and optimization. Furthermore, the enhanced aerodynamic loads give rise to concerns over the blade structure integrity. A simple pursuit of an aerodynamic performance gain often results in structurally unacceptable configurations even simply from a static stress limit consideration, although this may be overcome relatively easily and simply by imposing some simple structure constraints. It is more difficult to evaluate and control the dynamic stresses associated with aerodynamics induced blade vibrations, flutter and forced response. Flutter typically limits the performance for low pressure components, while forced response limits both low pressure and high pressure components.

A similar case can be also made for high pressure turbine blades in terms of the interaction between aerodynamics and heat transfer. An aerodynamically 'optimized' blading might lead to high heat transfer. The associated extra cooling required to keep to an acceptable blade metal temperature might give a very different trade-off both in terms of the extra cooling air consumption and the corresponding coolant-main stream mixing losses.

# 2.0 Concurrent Multi-Bladerow Design Optimization

Here, we take a gradient based approach to blade shape optimization. A typical issue one would face when dealing with a multi-component concurrent design optimization is how to get the gradient sensitivities for a large number of variables. In general, the number of design variables for detailed shaping of each blade could be in the range of  $10^2$ . For a multi-stage compressor, the total number of design variables could easily be in the order of  $10^3$ . The challenge is, how to get the gradient information for this large number of variables, simultaneously as required for a concurrent design optimization. An adjoint method is the one which can meet the challenge.

# 2.1 Adjoint Principle

For a typical design situation of practical interest, there are only few objective functions (e.g. isentropic efficiency, pressure ratio of a compressor)[2]. Consider an objective function I (scalar) in an aerodynamic design optimization as a function of the flow variable vector U and a design variable  $\alpha$ , expressed as,

$$I = I(U, \alpha) \tag{1}$$

The relation between the flow variable and the design variable is determined through the solution to the nonlinear flow equation (a vector equation),

$$\mathbf{R}(\mathbf{U},\,\boldsymbol{\alpha}) = \mathbf{0} \tag{2}$$

The gradient of the objective function to a design variable can be given by,

$$\frac{dI}{d\alpha} = \frac{\partial I}{\partial \alpha} + \frac{\partial I}{\partial U} \frac{\partial U}{\partial \alpha}$$
(3)

Of the three terms in the gradient expression,  $\partial I/\partial \alpha$  and  $\partial I/\partial U$  can be calculated analytically. The key term is the flow variable sensitivity  $\partial U/\partial \alpha$ . In a direct sensitivity calculation, the flow variable sensitivity to each design variable is obtained by solving the flow fields respectively, as the flow variables of the whole flow filed are coupled together. Thus a direct approach would mean that for *N* design variables, *N* flow field solutions are needed for each design cycle. This apparently would be unpractical (even if not impossible) for situations with a large number of design variables.

Having identified the problem of a direct sensitivity calculation, the central point is whether we can find a way to decouple the influence of different design variables on an objective function through the flow variable sensitivity  $\partial U/\partial \alpha$ . Put it more specifically, we would like to find a way to eliminate the explicit dependency of the objective function sensitivity dI/d $\alpha$  on the flow variable sensitivity  $\partial U/\partial \alpha$  in (3).

The adjoin formulation can now be introduced to accomplish the task of eliminating  $\partial U/\partial \alpha$  in Eq 3. The differentiation of the flow equation with respect to the design variable  $\Box$  is:

$$\frac{\partial R}{\partial \alpha} + \frac{\partial R}{\partial U} \frac{\partial U}{\partial \alpha} = 0 \tag{4}$$

Multiplying the right hand side of the linearized flow equation (4) with the adjoint variable vector (also called Lagrange multiplier)  $\lambda$  (noting  $\lambda$  is in fact a transpose of a vector of the

same dimension as the flow variable U), and subtracting the product from the gradient expression yields

$$\frac{dI}{d\alpha} = \frac{\partial I}{\partial \alpha} + \frac{\partial I}{\partial U} \frac{\partial U}{\partial \alpha} - \lambda \left[\frac{\partial R}{\partial \alpha} + \frac{\partial R}{\partial U} \frac{\partial U}{\partial \alpha}\right]$$
(5a)

Given the task of eliminating  $\partial U/\partial \alpha$ , we regroup Eq (5a),

$$\frac{\mathrm{dI}}{\mathrm{d}\alpha} = \frac{\partial \mathrm{I}}{\partial \alpha} - \lambda \frac{\partial \mathrm{R}}{\partial \alpha} + \left[\frac{\partial \mathrm{I}}{\partial \mathrm{U}} - \lambda \frac{\partial \mathrm{R}}{\partial \mathrm{U}}\right] \frac{\partial \mathrm{U}}{\partial \alpha}$$
(5b)

It is clear then that our task can be achieved by choosing the adjoint variables to satisfy the following to nullify the influence of  $\partial U/\partial \alpha$ ,

$$\frac{\partial I}{\partial U} - \lambda \frac{\partial R}{\partial U} = 0$$
(6)

Eq (6) is called the adjoint equation. This is a field equation in the same dimension as the flow equations.

Upon satisfying the adjoint equation (Eq.6), the gradient of the objective function is reduced to:

$$\frac{\mathrm{dI}}{\mathrm{d}\alpha} = \frac{\partial \mathrm{I}}{\partial \alpha} - \lambda \frac{\partial \mathrm{R}}{\partial \alpha}$$
(7)

which is not longer dependent on the flow variable sensitivity. Furthermore, the adjoint equation (Eq.6) does not depend on any design variable. This implies that the gradient of a scalar objective function to <u>ALL</u> the design variables can be obtained by solving only two sets of equations for the computational domain:

- 1) the standard RANS flow equation (Eq.2);
- *2) the adjoint equation (Eq.6).*

For each design cycle, only the above two field solutions are required (Fig.1). The calculation of the gradient expression (Eq.7) is effectively equivalent to a post-processing and hence can be done very efficiently.



# Figure 1: Flow Chart of Aerodynamic Design Optimization

## 2.2 Adjoint Mixing-Plane Treatment for Bladerow Interface

It is recognized that the conventional multi-bladerow analysis methods in most design systems are based on the mixing-plane treatment for the rotor-stator interfaces (Denton [2]). A multibladerow adjoint optimization would point to a need for an equivalent adjoint mixing-plane method.

The strategy for such a development is that an adjoint mixing-plane should "reflect" the physical domain mixing-plane treatment as the adjoint equations reflect the corresponding flow equations in the physical domain. To follow this through, we need to start with a recognition of the difference in the information/disturbance propagation between a direct flow problem in a physical domain and that of an adjoint one[6]. In a physical domain, a perturbation to a design variable would propagate through flow characteristics (acoustic, entropic and vortical disturbances). This is how an objective function will be influenced in a direct analysis. The purpose of introducing of the adjoint variable (as shown in Eq.5), however, is exactly to 'block' the direct information propagation.

| Table 1:   | Number | of | boundary | conditions | for | inlet | and | exit |
|------------|--------|----|----------|------------|-----|-------|-----|------|
| boundaries |        |    |          |            |     |       |     |      |

|              | Physical                      | Adjoint                       |
|--------------|-------------------------------|-------------------------------|
| Domain Inlet | Inlet BC:                     | Adjoint "Exit" BC:            |
|              | Specified : 4                 | Specified: 1                  |
|              | Extrapolated from interior: 1 | Extrapolated from interior: 4 |
| Domain Exit  | Exit BC:                      | Adjoint "Inlet" BC:           |
|              | Specified : 1                 | Specified : 4                 |
|              | Extrapolated from interior: 4 | Extrapolated from interior: 1 |

The consistent understanding of the "anti-physics" adjoint characteristics propagation is very helpful in formulating an adjoint mixing plane. For a rotor-stator interface in the physical domain, the number of physical flow characteristics across the interface and their directions are known. The corresponding number of the adjoint characteristics across the interface and their directions can be worked out exactly based on the 'anti-physics" path. A set of adjoint mixing-plane interface conditions can thus be consistently formulated and implemented as demonstrated by Wang and He [6].

An example of illustrating the anti-physics adjoint characteristics is taken from [3] for a 2D section of the compressor stage. The relative Mach number contours are shown in Fig 2. The upstream rotor is choked with a passage shock, whilst the downstream stator is of a typical subsonic flow pattern. The corresponding field for an adjoint variable is shown in Fig.3. Clearly the adjoint solution looks to behave in a complete opposite way: the physical upstream domain becomes the adjoint downstream domain. The 'adjoint wakes' flowing reversely are clearly visible in both blade rows (Fig.3).



Figure 2: Relative Mach Number Contours for a Transonic Compressor Stage



Figure 3: Contours of an Adjoint Variable for a Transonic Compressor Stage



Figure 4: Meridional view and blade to blade view of Siemens 3-stage

compressor computational domain

The demonstration sample example is taken from Wang et al [4]. In this case, the computational domain consists of 7 rows and its blade to blade view of a mid-span section and meridional view are shown in Fig.4. In the design optimization, the IGV remains unchanged as for the three-row design optimization, whilst the other 6 rows are allowed to be changed (a total number of design variables of 1023). For each blade row, the design variables are distributed on 11 spanwise sections with the same number of design variables for each section. A single-point design optimization is carried out at the original design point of the compressor. Twenty nine design cycles are completed over 11 days (single processor)[10].

The performance comparison between the original compressor and the optimised one at the chosen operating point is presented in Table 3. The optimised design has an efficiency that is 2.47% point higher than the original one with 0.34% increase in mass flow rate and 0.08% decrease in pressure ratio.

#### Table 3: Performance comparison between original and optimised blades

(Siemens 3-stage compressor redesign. 1023 design variables)

|           | mass flow rate<br>(kg/s) | pressure<br>ratio | isentropic<br>efficiency(%) |
|-----------|--------------------------|-------------------|-----------------------------|
| original  | 26.46                    | 2.9885            | 86.81                       |
| optimised | 26.55                    | 2.9860            | 89.28                       |
| change    | +0.34%                   | -0.08%            | +2.47                       |

#### Conclusions

Development of modern high performance gas turbine balding calls for more concurrent multicomponent & disciplinary approaches toward design & optimization. In this lecture, some recent efforts are reviewed with emphasis on the blade shape optimization under influence of blade row-to-row interactions, and aerodynamic-aeroelastic interactions. The efforts are largely based on the gradient based approach, and an adjoin approach is adopted for efficient evaluations of gradient sensitivities. The results have demonstrated the effectiveness of the adjoin approach, but more importantly the benefit or/need for a concurrent approach.

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