ON ECCENTRIC CONNECTIVITY INDEX OF SUBDIVISION-RELATED GRAPHS

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***Abstract*: The eccentric connectivity index based on degree and eccentricity of the vertices of a graph is a widely used graph invariant in mathematics. In this paper we present the explicit generalized expression for the eccentric connectivity index of the subdivision-related graph of some special graphs.**

***Keywords* - Eccentricity, Eccentric connectivity index, Subdivision graph, Subdivision-related graph.**

I.INTRODUCTION

A topological index, based on degree and eccentricity of a vertex of a graph, known as eccentric connectivity index, first appeared for structure-property and structure activity studies of molecular graphs [6] and shown to give high degree of predictability of pharmaceutical properties. Now for any simple connected graph $G=(V(G),E(G))$ with n vertices and m edges, the distance between the vertices v_i and v_j of $V(G)$, is equal to the length that is the number of edges of the shortest path connecting v_i and v_j [1]. Also for a given vertex v_i of $V(G)$ its eccentricity $\varepsilon_G(v_i)$ is the largest distance from v_i to any other vertices of G. The radius and diameter of the graph are respectively smallest and largest eccentricity among all the vertices of G where as the average eccentricity of a graph is denoted by $ece(G)$ and is defined as

$$\xi^c(G)=\frac{1}{n}\sum_{i=1}^{n}\varepsilon_G(v_i).$$

The eccentric connectivity index $\xi^C(G)$ of a graph G [6] is defined as $\xi^c(G)=\sum_{i=1}^{n} d_G(v_i).\varepsilon_G(v_i)$ where $d_G(v_i)$ is the degree i.e. number of first neighbour of v_i of $V(G)$. Compare to other topological indices as the eccentric connectivity index has been found to have a low degeneracy [5], it subject to a large number of chemical and mathematical studies.

A number of graph operations have been defined and studied that have led to several results dealing with the Hamiltonian and Eulerian properties. One of the simplest operations is that of the subdivision graph of a graph [7]. The subdivision-related graph $R(G)$ [2] is the graph obtained from G by adding a new vertex corresponding to each edge of G, then joining each new vertex to the end vertices of the corresponding edge. Another way to describe $R(G)$ is to replace each edge of G by a triangle.

The barbell graph B_n is defined as the simple graph obtained by connecting two copies of a complete graph K_n by a bridge. The Gear graph G_n is also known as bipartite wheel graph. It is a wheel graph with a graph vertex added between each pair of adjacent graph vertices of the outer cycle. The Double-Wheel graph DW_{2n+1} is defined as the graph $2C_n + K_1$, where K_1 is the singleton graph and C_n is the cycle graph. The corona graph operation [4] $G_1 * G_2$ of two graphs G_1 and G_2 is a new graph obtained by taking n_1 copies of the graph G_2 and then joining i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2 respectively. Eccentric connectivity index [3] gives the explicit formulae for the values of eccentric connectivity index of the above graphs. In this paper we determine the eccentric connectivity index for its corresponding subdivision - related graphs.

II. RESULTS

In this section we derived an expression for the *eccentric connectivity index* of the *subdivision-related* graphs of the *Barbell* graphs, the *Double - Wheel* graphs, *Gear* graphs and *Corona* graph.

Theorem 2.1: The eccentric connectivity index of subdivision-related graph of Barbell graph B_n is

$$\xi^{(c)} R(B_n) = 26n^2 - 34n + 26, \quad \textit{for all } n \geq 3.$$

Proof:

The $R(B_n)$ contain the subgraphs, two copies of $R(K_n)$ and a bridge connecting copies of $R(K_n)$ that is known as $R(P_1)$. The subdivision-related graph of Barbell graph B_n contains $n(n + 1)+1$ vertices. Among this, 2n vertices are actual vertices and $[n(n - 1) + 1]$ vertices are the subdivision vertices of the graph $R(B_n)$.

Among $2n$ vertices, $2(n - 1)$ vertices are of degree $2(n - 1)$ and the remaining 2 vertices are of degree $2n$. The subdivision vertices of $R(B_n)$ are of degree 2.

Let us assume u_s and u_t be the end vertices of bridge connecting copies of $S(K_n)$ and these vertices are of degree n. Among the subdivision vertices, 1 vertex is the subdivision vertex of the bridge that is labelled as e_l .

The vertices of degree $2(n - 1)$ are of eccentricity 4 and the end vertices of the bridge are of eccentricity 3. The vertex e_l is of eccentricity 3.

Irrespective e_l there are $n(n - 1)$ subdivision vertices. Among which $(n - 1)$ vertices are the neighbours of u_s and $(n - 1)$ vertices are the neighbours of u_t that are of eccentricity 4 and the remaining subdivision vertices are of eccentricity 5.

In general, for all $n \geq 3$

$$\begin{aligned}\xi^{(c)} R(B_n) &= 2(n-1).2(n-1).4 + 2.2n.3 + 1.2.3 + 2(n-1).2.4 + (n^2 - 3n + 2).2.5 \\ &= 26n^2 - 34n + 26.\end{aligned}$$

Hence, for all $n \geq 3$ $\xi^{(c)} R(B_n) = 26n^2 - 34n + 26$.

Theorem 2.2: The eccentric connectivity index of subdivision-related graph of the Double Wheel graph DW_{2n+1} is $\xi^{(c)} R(DW_{2n+1}) = 72n, \quad \textit{for all } n \geq 3.$

Proof:

The subdivision-related graph of Double Wheel graph DW_{2n+1} contains the subgraphs *outer cycle* $R(C_n)$, the *inner cycle* $R(C_n)$ and the *hub* of the wheel respectively. The graph $R(DW_{2n+1})$ contains $6n + 1$ vertices. Among which $2n$ vertices are actual vertices, one vertex is the hub of wheel and the remaining vertices are the subdivision vertices.

For all the values of n, $n \geq 3$

The $2n$ vertices are of degree 6. The hub of the wheel is of degree $4n$. The subdivision vertices are of degree 2. The actual $2n$ vertices of the inner cycle and outer cycle are of eccentricity 3. The hub of the wheel is of eccentricity 2. The subdivision vertices of the inner cycle and outer cycle are of eccentricity 4. The subdivision vertices in the spokes are of eccentricity 3.

The eccentric connectivity index of $R(DW_{2n+1})$ is calculated as follows.

$$\begin{aligned}\xi^{(c)}S(DW_{2n+1}) &= 2n.6.3+1.4n.2+2n.2.4+2n.2.3\\ &= 72n.\end{aligned}$$

Hence, for all values of n, $n \geq 3$ $\xi^{(c)}R(DW_{2n+1}) = 72n.$

Theorem 2.3: The eccentric connectivity index of subdivision-related graph of Gear graph G_n is

$$\xi^{(c)}R(G_n) = \begin{cases} 168, & \text{for } n = 3, \\ 60n, & \text{for } n \geq 4. \end{cases}$$

Proof:

The subdivision-related graph of the Gear graph contains the subgraph $R(C_{2n})$ and the hub of the wheel. The cardinality of the vertex set of the subgraph $R(C_{2n})$ is $5n$ +1vertices.

For all the values of n, $n \geq 3$

The hub of the wheel is of degree $2n$ and eccentricity 2. Among $4n$ vertices of the subgraph $R(C_{2n})$, the n vertices are of degree 6 and the graph vertex added between each pair of adjacent graph vertices in $R(C_{2n})$ are of degree 4. The subdivision vertices are of degree 2.

For n = 3, among the $4n$ vertices of the subgraph $R(C_{2n})$, the $2n$ vertices are of eccentricity 3 and the remaining vertices are of eccentricity 4. The subdivision vertices in the spokes are of eccentricity 3.

For all $n \geq 4$, among actual $2n$ vertices the n vertices are of eccentricity 3 and the remaining n vertices are of eccentricity 4. The subdivision vertices of the cycle are of eccentricity 4. The subdivision vertices in the spokes are of eccentricity 3 respectively.

In general, the eccentric connectivity index of the subdivision-related graph of Gear graph G_n is given by

$\xi^{(c)}R(G_n) = 168,$ *for the case* $n = 3,$

$\xi^{(c)}R(G_n) = 60n$, *for the case* $n \geq 4.$

Theorem 2.4: Let C_n and K_1 be two graphs, then the eccentric connectivity index of the subdivision-related graph of corona graph $C_n * K_1$ is

$$\xi^{(c)}R(C_n * K_1) = \begin{cases} 6n^2+12n, & \text{when } n \geq 3 \text{ and } n \text{ is odd}, \\ 6n^2+16n, & \text{when } n > 3 \text{ and } n \text{ is even}. \end{cases}$$

Proof:

The subdivision-related graph of the corona graph $R(C_n * K_1)$ contains $2n$ vertices in the cycle $R(C_n)$ and $2n$ vertices in the n copies of K_1 that joins i^{th} vertex of C_n to every vertex in the i^{th} copy of K_1 . Thus, the cardinality of the vertex set of $R(C_n * K_1)$ is $4n$ vertices.

For all the values of n, $n \geq 3$, the actual n vertices of the cycle $R(C_n)$ are of degree 6. The vertices of n copies of K_1 are of degree 2. All the subdivision vertices in the corona graph $R(C_n * K_1)$ are of degree 2.

For all $n \geq 3$ and n is odd, the actual n vertices of the cycle $R(C_n)$ are of eccentricity $\left(\frac{n+1}{2}\right)$ and the n vertices of K_1 and the subdivision vertices are of eccentricity $\left(\frac{n+3}{2}\right)$.

For all $n > 3$ and n is even, all the vertices of the subgraph cycle $R(C_n)$ are of eccentricity $\left(\frac{n}{2}+1\right)$. The n vertices of copies of K_1 and the subdivision vertices of copies of K_1 are of eccentricity $\left(\frac{n}{2}+2\right)$.

Therefore,

The eccentric connectivity index of the subdivision-related graph of corona graph $C_n * K_1$ is

$$\xi^{(c)} R(C_n * K_1) = 6n^2 + 12n, \quad \text{for the case } n \geq 3 \text{ and } n \text{ is odd}.$$
$$\xi^{(c)} R(C_n * K_1) = 6n^2 + 16n, \quad \text{for the case } n > 3 \text{ and } n \text{ is even}.$$

IV CONCLUSION

Using the generalized expression derived above one can easily obtain the eccentric connectivity index of subdivision - related graph of Barbell graph, Double-Wheel graph, Gear graph and corona graph $C_n * K_1$ respectively. Similarly, we can find the eccentric connectivity index for other family of graphs by incorporating same methodology.

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