

An Advanced Two Level Frequency Hopped CDMA Technique for Wireless Fading Channels

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Abstract: *Here, we propose a “two-level” frequency hopping code-division multiple-access (FH-CDMA) scheme for wireless communication systems. This new method gives us the flexibility to select various modulation codes and FH patterns. In this method, we have proposed a technique where available modulation codes can be divided so as to carry more possible users without increasing number of available FH patterns. The performance and spectral efficiency of the scheme are examined for additive white Gaussian noise (AWGN), Rayleigh and Rican fading channels are analyzed algebraically . Also, the simulation results show that the divided two level FH-CDMA scheme supports higher data rate and greater spectral efficiency than frequency-shift-keying FH-CDMA scheme. Therefore our two-level FH-CDMA scheme is more flexible in the selection of the modulation codes and FH patterns in order to meet different system operating requirements.*

Keywords: Code Division Multiple Access, Modulation Codes, Frequency Hopping, Spectral Efficiency.

I. INTRODUCTION

Frequency Hopping Code Division Multiple Access (FH-CDMA) provides frequency range and helps ease multi-path fading and vary intervention [1], [2]. The advantages of FH-CDMA over Direct-Sequence DS-CDMA [3], [4] include better resistance to multiple access interfering (MAI), less rigorous power control, and reduced “near-far” problem and multi-path interfering. By conveying a unique FH model to each user, a FH-CDMA system allows multiple users to share the same transmission channel concurrently [5], [6]. MAI occurs when more than one simultaneous user make use of the same carrier frequency in the same time slot. “One-hit” FH models have been designed in order to minimize MAI [7], [8].

M -ary frequency-shift-keying (MFSK) atop FH-CDMA scheme is used in order to enlarge data rate by transmitting symbols, instead of data bits. In addition, the uses of prime and Reed-Solomon (RS) sequences as

modulation codes atop FH-CDMA were proposed [9], [10], in which the symbols were represented by non-orthogonal sequences, rather than orthogonal MFSK. These prime FH-CDMA [9] and RS FH-CDMA [10] methods supported higher data rate than MFSK FH-CDMA scheme [5], at the expense of worse performance. However, the weights and lengths of the modulation codes and FH models needed to be the same in both methods, confining the choice of suitable modulation codes and FH models to use.

A new two level FH-CDMA scheme has been proposed in which both modulation codes and FH models do not need to have the same weight/length anymore. The only condition is that the weight of the FH models is at least equal to the length of the modulation codes, which is usually true in modulated FH-CDMA methods because each element of the modulation codes needs to be conveyed by an element of the FH models. Therefore, two level FH-CDMA method is more flexible in the selection of the modulation codes and FH models in order to meet different system operating requirements.

II.RELATED WORK

The usage of different groups of modulation codes as an additional level of address signature, the divided two level FH-CDMA method allows the assignment of the same FH model to multiple users, thus maximizing the number of possible users. The performance of two level FH-CDMA method over additive white Gaussian noise (AWGN), and fading channels are analyzed algebraically. While previous analyses [5], [9], [10] used a constant β_0 to approximate the “false-alarm” and removal probabilities caused by additive noise or fading, we include a more accurate model of β_0 better reflecting the actual effects of “false alarms” and deletions to the method performance. The new method with MFSK/FH-CDMA method in terms of performance and a more meaningful metric, spectral efficiency (SE) has been compared. Numerical examples show that two level FH-CDMA method provides a trade-off between performance and data rate. In the comparison

of SE, the divided two-level FH-CDMA method displays better system efficiency than MFSK/FH-CDMA method under some constraints.

III. PROPOSED METHODOLOGY

In two-level FH-CDMA scheme, the available transmission bandwidth is divided into M_h frequency bands with M_m carrier frequencies in each band giving a total of $M_h M_m$ carrier frequencies. In the first level a number of serial data bits is grouped together and represented by a symbol.

TABLE I
Prime Sequences

	Group0	Group1	Group2	Group3	Group4
i_2	$i_1=0$	$i_1=1$	$i_1=2$	$i_1=3$	$i_1=4$
0	0000x	0123x	02x13	031x2	0x321
1	1111x	123x0	1302x	1x203	10x32
2	2222x	23x01	2x130	203x1	210x3
3	3333x	3x012	302x1	31x20	3210x
4	xxxxx	x0123	x1302	x2031	x3210

Each symbol is in turns represented by a modulation code of dimension $M_m \times L_m$ and weight W_m , where M_m the number of data bits that can be represented by a symbol depends on the number of available modulation codes.

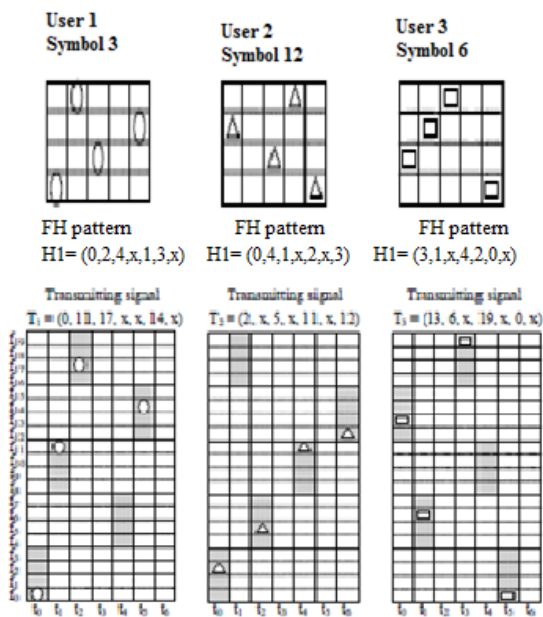


Figure 1 : Encoding Process

If there is Φ_m available modulation codes, each symbol can be represent up to $\lfloor \log_2 \Phi_m \rfloor$ data bits, where $\lfloor . \rfloor$ is the floor function. In the second (FH) level, each user is assigned a

unique FH pattern of dimension $M_h \times L_h$ and weight W_h , where M_h is the number of frequencies and L_h is the number of time slots.

The elements in the modulation codes and FH models conclude the carrier frequencies of the final FH-CDMA signals. While an element of modulation code define the carrier frequency used in a frequency band in a given time slot, an element of the FH pattern determines which frequency band to use. In proposed method families of $(M_m \times L_m, W_h, \lambda_{a,m}, \lambda_{c,m})$ modulation codes are chosen and $(M_h \times L_h, W_h, \lambda_{a,h}, \lambda_{c,h})$ FH models as long as $W_h \geq L_m$ prime sequences in Table I as the modulation codes.

Fig.1 shows the encoding process of three simultaneous users. The data symbols of these users are $S_1=S_3,0=(0,3,1,x,2)$, $S_2=(2,x,1,3,0)$, $S_3=S_1,1(1,2,3,x,0)$. Elements of $S_{i1,i2}$ determines which carrier frequency of a frequency band in a given time slot to use. $S_{i1,i2,i} = i_2 \oplus p(i_1 \odot p \ 1)$ [\oplus - modulo p addition, \odot - modulo p multiplication]. If the number of available carrier frequencies is restricted or the sequence weight needs to be varied in order to achieve certain method performance, the sequence weights are adjusted to be $W_m < P$ by dropping the largest $[p-W_m]$ elements in $S_{i1,i2}$. As a result the construction algorithm gives $\lfloor \Phi_m = P^2 - P + W_m \rfloor$ prime sequences of weight $W_m \leq P$ and length $L_m = P$ with, $\lambda_{c,m} = 1$. Using these prime sequences as the modulation codes, at most twenty four symbols are supported and each symbol represents $\lfloor \log_2 24 \rfloor = 4$ data bits. FH models can be chosen for the second level of two level FH-CDMA method as long as $W_h \geq L_m$. To illustrate this $(M_h \times L_h, W_h, \lambda_{a,h}, \lambda_{c,h}) = (5 \times 7, 5, 0, 1)$ prime sequences as the one-hit FH models are transmitting signal. Where $\lambda_{a,m}, \lambda_{a,h}$ and $\lambda_{c,m}, \lambda_{c,h}$ denotes the maximum autocorrelation and cross-correlation values of the modulation codes, respectively. To illustrate the main concept of two level FH-CDMA method, the prime sequences as the modulation codes are used, other codes, such as the RS sequences, quadratic congruence code (QCCs) and multilevel prime codes can also be used.

The carrier frequency used in each frequency band in a time slot is determined by superimposing all $W_m = 4$ elements of S_1 on top of this first W_m non-X elements of H_k and the X elements of S_i produce empty frequency bands in the final two level FH-CDMA signal, where $K=\{1,2,3\}$ the shaded columns in the transmitting signal, T_k of Figure 1 represent the frequency bands specified by the corresponding FH models, H_k for $K=\{1,2,3\}$. In review two level frequency hopping CDMA can be represented by $T_k = (T_{k,0}, T_{k,1}, \dots, T_{k,j}, \dots, T_{k,L,h-1} = S_k)$, where $T_{k,j}$ represents frequency used in i th time slot and Δ denotes super impose

operation for example two level FH CDMA signal of first user is found to be $T1=(0+, 0*4, 3+2*4, 1+4*x,x,x,2+3*4,x)=(0, 11, 17, x, x, 14, x)$ after super imposition similarly other two simultaneous users have $T2=(2,x,5,x,11,x,12)$ and $T3=(13,6,x,19,x,0,x)$. in receiver , received two level FH CDMA signal of all users and effects of MAI, fading and noise are hard limited, dehopped and finally decoded in order to recover transmitted data symbol. Fig 2 illustrates decoding and detection process of user 1. Received signal is first hard limited and then dehopped by user 1's FH pattern to give a dehopped signal R1 of dimesions 4X5. The role of the dehopping process simply brings the frequency bands in each time slot of back to the base band, according to the frequency bands specified by H1.

Although the prime sequences can only support up to $\lceil \log_2(p^2 - p + w_m) \rceil$ bit/symbol, it is important to point out that our two-level FH-CDMA scheme allows the use of other codes, such as the RS sequences ,QCCs ,and MPCs ,as the modulation codes. For example, the MPCs have p^{n-1} sequences of weight $w_m=p$ and length $L_m=p$ with $c, =$ (i.e., symbol interference), where n is a natural number. If the MPCs are used as the modulation codes, the date rate can be increased because the MPCs can support up to $\lceil \log_2 P_{n+1} \rceil$ bit/symbol at the expense of worsened symbol interference.

In general, the number of possible users in a FH-CDMA system is limited by the number of available FH patterns. However, our two-level FH-CDMA scheme can flexibly increase the number of possible users by trading for lower data rate through a reduction of symbol size. It is done by partitioning the modulation codes into several groups and each group contains reduced number of modulation codes with a lower $\lambda_{c,m}$ each user can now only use one group of modulation codes for symbol representation. In addition to the unique FH pattern assigned to a user, the group of modulation codes that the user can use adds another degree of user address signature. The same FH pattern can now be reused by multiple users as long as they have different groups of modulation codes. Let say there are ϕ_h FH patterns and ϕ_m modulation codes with $\lambda'_{c,m}$. If the modulation codes are partitioned into t groups of codes with $\lambda'_{c,m}$. The partition results in $\lambda'_{c,m} = \lambda'_{c,m} - 1$

We can then assign each user with one FH pattern and one of these t groups of modulation codes, thus supporting a total of $t \cdot \phi_h$ possible users. The tradeoff is that each group now has at most ϕ_m / t modulation codes and thus the number of bits per symbol is lowered from $\lceil \log_2 \phi_m \rceil$ to $\lceil \log_2 (\phi_m / t) \rceil$

For example, the twenty-four $\lambda_{c,m} = 1$ prime sequences in Table I can be partitioned into five groups of prime sequences of $\lambda'_{c,m} = 0$ and assigned to five different users with the same FH pattern. Although the number of bits represented by each symbol decreases from $\lceil \log_2 24 \rceil$ to $\lceil \log_2 5 \rceil$, the number of possible users is now increased from ϕ_h to $5\phi_h$. We can also choose the MPCs of length p and $\lambda_{c,m} = n$ as the modulation codes. The MPCs can be partitioned into $p^{n-n'}$ groups and each group has $\lambda'_{c,m} = n'$ and $\phi_m = p^{n+1}$, where $n > n'$. The number of possible users is increased to $\phi_h p^{n-n'}$, but the number of bits per symbol is reduced to $\lceil \log_2 P^{n+1} \rceil$

IV. PERFORMANCE ANALYSIS

In this FH-CDMA system MAI depends on the cross correlation values of FH patterns. For our two level FHCDMA scheme, the cross-correlation values of the modulation codes impose additional interference and need to be considered. Assume that one-hit FH patterns of dimension $M_h * L_h$ are used and the transmission band is divided into $M_m * M_h$ frequencies, in which M_m frequencies are used to carry the modulation codes of weight W_m the probability that a frequency of an interferer hits with one of the frequencies of the desired user is given by

$$q = \frac{W_m^2}{W_m M_h L_h} \tag{1}$$

Assume that there are K simultaneous users the probability row is given by

$$P(n) = \binom{W_m}{n} \sum_{i=0}^n (-1)^i \binom{n}{i} \left[1 - q + \frac{(n-1)q}{W_m} \right]^{K-1} \tag{2}$$

That the dehopped signal contains n entries in an undesired over AWGN, and Rayleigh and Rician fading channels, false alarms and deletions may introduce detection errors to the received FH-CDMA signal. A false alarm probability P_d is the probability that a receiver missed a transmission tone. For these three types of channels the false alarm probability is generally given by

$$P_f = \exp\left(\frac{-\beta_0^2}{2}\right) \tag{3}$$

For an AWGN channel, the deletion probability is given by

$$P_d = 1 - Q \left(\sqrt{2 \left(\frac{\bar{E}_b}{N_0} \right) \cdot \left(\frac{K_b}{W_m} \right)}, \beta_0 \right) \tag{4}$$

Where β_0 denotes the actual threshold divided by the root mean-squared receiver noise, is the number of bits per symbol, \bar{E}_b/N_0 is the average bit-to-noise density ratio

$Q(a, b) = \int_b^\infty x \exp[-(a^2 + x^2)/2] I_0(ax) dx$ is Marcum's Q function and $I_0(\cdot)$ is the modified Bessel function of the first kind and zeros order. To minimize the error probability the optimal β_0 of an AWGN channel should be a function of the signal to noise ratio (SNR) \bar{E}_b/N_0 and K_b/W_m can be more accurately written as

$$\beta_0 = \sqrt{2 + \frac{(\bar{E}_b/N_0) \cdot (K_b/W_m)}{2}} \tag{5}$$

Rather than an inaccurate constant value i.e. $\beta_0=3$, used in equations (5, 9, 10). For a Rayleigh fading channel the deletion probability is given in eq. (14)

$$P_d = 1 - \exp \left\{ \frac{-\beta_0^2}{2 + 2 \left(\frac{\bar{E}_b}{N_0} \right) \cdot \left(\frac{K_b}{W_m} \right)} \right\} \tag{6}$$

Similarly the optimal β_0 of a Rayleigh fading channel can be more accurately written as

$$\beta_0 = \sqrt{2 + \frac{2}{\left(\frac{\bar{E}_b}{N_0} \right) \cdot \left(\frac{K_b}{W_m} \right)}} \times \sqrt{\log[1 + \left(\frac{\bar{E}_b}{N_0} \right) \cdot \left(\frac{K_b}{W_m} \right)]} \tag{7}$$

Finally for a Rician fading channel the deletion probability is given in e.q (14)

$$P_d = \left[1 - Q \left(\sqrt{\frac{2\rho \left(\frac{\bar{E}_b}{N_0} \right) \cdot \left(\frac{K_b}{W_m} \right)}{1 + \rho + \left(\frac{\bar{E}_b}{N_0} \right) \cdot \left(\frac{K_b}{W_m} \right)}, \beta_1 \right) \right] \tag{8}$$

Where the Rician factor P is given as the ratio of the power in the seculars components to the power in multipath components. Similarly β_0 and β_1 can be more accurately written as

$$\beta_0 = \sqrt{2 + \frac{(\bar{E}_b/N_0) \cdot (K_b/W_m)}{2}} \tag{9}$$

$$\beta_1 = \frac{\beta_0}{\sqrt{1 + (\bar{E}_b/N_0) \cdot (K_b/W_m) / (1 + \rho)}} \tag{10}$$

Including the noise or fading effect, the probability that the dehopped signal contains entries in an undesired row .

$$P_i(n) = \sum_{j=0}^n \sum_{r=0}^{\min[n-j, W_m-n]} \left[P(n-j) \binom{n-j}{r} \times P_d^{r-j} (1-P_d)^{n-j-r} \binom{W_m-n+j}{r+j} \times P_f^{r+j} (1-P_f)^{W_m-n-r} \right] + \sum_{j=1}^{W_m-n} \sum_{r=j}^{\min[n+j, W_m-n]} \left[P(n+j) \binom{n+j}{r} \times P_d^{r-j} (1-P_d)^{n+j-r} \binom{W_m-n-j}{r-j} \times P_f^{r-j} (1-P_f)^{W_m-n-r} \right] \tag{11}$$

The FH-CDMA system, an error occurs when interference causes undesired rows in the dehopped signal to have equal or more entries than the desired rows. In addition an error may still occur in our two-level FH-CDMA scheme even when the undesired rows have fewer entries than the desired rows. Because the nonzero cross-correlation values of the modulation codes add extra undesired entries.

If there are $2 - 1$ incorrect rows, the probability that is the maximum number of entries and that exactly unwanted rows contain entries is given by

$$P_c(n) = \binom{W_m}{n} (1 - P_d)^n (P_d)^{W_m-n} \tag{12}$$

The desired symbol is detected wherever the maximum number of entries in the incorrect rows is less than . As the receiver decides which symbol (out of 2 symbols) is recovered by searching for the modulation code with the largest matching entries, the bit error probability (BEP) is finally given by

$$P_b(K) = \frac{2^{k_b}}{2(2^{k_b}-1)} \times \left\{ 1 - \sum_{n=1}^W \left[P_c(n) \sum_{t=0}^{2^{k_b}-1} \frac{1}{t+1} P_r(n, t) \right] \right\} \tag{13}$$

for No. Of simultaneous users.

V. EXPERIMENTAL RESULTS

In this section we compare the performances of the new two-level FH-CDMA and MFSK/FH-CDMA schemes under the condition of same transmission parameters: $M_g = M_m M_h$, $L_g = L_h$, and $W_g = W_m$, where M_g , L_g , and W_g are the number of frequencies, number of time slots, and weight of FH patterns respectively. MFSK/FH-CDMA scheme supports M_g modulation symbols (represented by the orthogonal frequencies), while the this FH-CDMA scheme supports $[P^2 - P + W_m]$ symbols with the symbol interference level $\lambda_{c,m} = 1$ if the prime sequences in section are used as the modulation codes.

The BEPs of both schemes are plotted against the number of simultaneous users K over a Rayleigh fading channel, based on the condition of same transmission parameters, where $M_g \times L_g = 44 \times 47$, $W_g = W_m = 4$, $M_m \times L_m = 4 \times 11$, $M_h \times L_h = 11 \times 47$, and $\bar{E}_b/N_0 = 25 \text{ dB}$. Using $p = 11$, our two-level FH-CDMA scheme supports $K_b = 6$ bits/symbol, while MFSK/FH-CDMA scheme supports $K = 5$ bits/symbol. Based on (7) and $K = \{5, 6\}$, we more accurately calculate $\beta_0 = \{3.4633, 3.5148\}$, respectively, instead of the constant $\beta_0 = 3$.

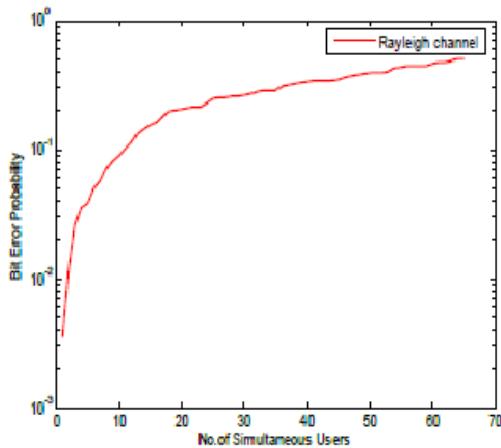


Figure 2: BEP of Rayleigh channel

The computer simulation of our two-level FH-CDMA scheme is performed as follows. The FH pattern assigned to each user is arbitrarily selected from all 47^2 possible $(11 \times 47, 11, 0, 1)$ prime sequences constructed from GF(47) and then all 11^2 possible $(4 \times 11, 4, 0, 1)$ prime sequences constructed from GF(11) are used as the modulation codes for each user. For each simulation point in the figure, the total number of data bits involved in the simulation ranges from 10^4 to 10^6 , depending on the targeted error probability.

In our partitioned two-level FH-CDMA scheme, the modulation codes (e.g., the prime sequences) are partitioned into p groups and the cross-correlation values of each group are zero (i.e. zero symbol interference).

In fig. 2 the BEPs of both schemes over a Rayleigh fading channel are plotted against the number of simultaneous users K , based on the conditions of same number of possible users and same transmission parameters, where $M_g \times L_g = 44 \times 47$, $W_g = W_m = 4$, $M_m \times L_m = 4 \times 11$, $M_h \times L_h = 11 \times 47$, and $\bar{E}_b/N_0 = 25 \text{ dB}$. Our partitioned scheme supports $K = 3$ bits/symbol, while scheme supports $K = 2$ bits/symbol.

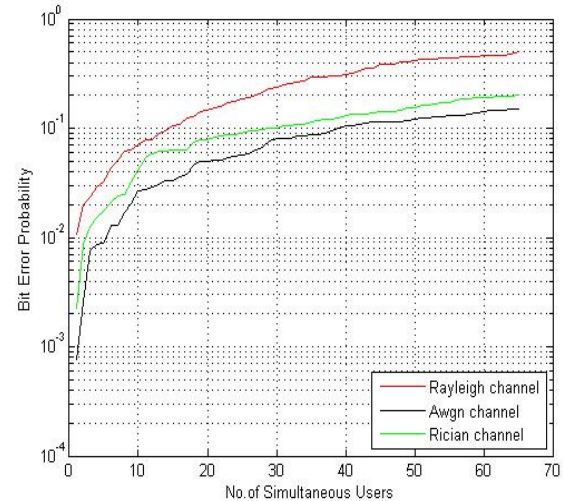


Figure 3: BEPs for AWGN, Rayleigh and Rician Fading Channels

In fig.3, the BEPs of our two-level FH-CDMA scheme under AWGN, and Rayleigh and Rician fading channels are plotted against the number of simultaneous users, where $W_m = 4$, $M_m \times L_m = 4 \times 11$, $M_h \times L_h = 11 \times 47$, $p = 13$, $p = 6$, and $\bar{E}_b/N_0 = 25 \text{ dB}$. Based on (5), (7), (9), and (10), we more accurately calculate β_0 and β_1 , which are given in fig.3. The AWGN curve always performs the best and the Rayleigh curve performs the worse, while the Rician curve is in between. Also shown in the figure is the computer-simulation result for validating our theoretical analysis.

TABLE II
SE Comparisons

Bit error probability	$P_e = 10^{-2}$	$P_e = 10^{-3}$
FH- CDMA ($k_b=2$)	$K=144$ SE=13.93%	$K=56$ SE = 5.42%
Two levwl FH- CDMA ($k_b =3$)	$K=126$ SE=18.29%	$K=53$ SE= 7.69%

To compare our partitioned two-level FH-CDMA and MFSK/FH-CDMA schemes,

$$SE = \frac{k_b K}{ML} \quad (14)$$

is another figure of merits.

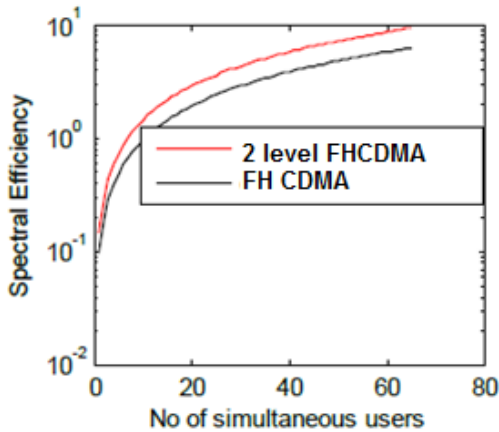


Figure 4: Spectral Efficiency Plot of two level FH CDMA & FHCDMA

SE considers the number of bits per symbol K_b , number of simultaneous users K , number of carrier frequencies M , and number of time slots L as a whole, for a given performance (i.e., BEP)

Our goal is to get the SE as large as possible for better system efficiency or utilization. Table II compares the SEs of both schemes with fixed $P_e = \{10^{-2}, 10^{-3}\}$, based on the parameters from Fig. 4. In our partitioned two-level FH-CDMA scheme, we can always increase the number of possible users by partitioning the modulation codes, thus resulting in a larger K_b than FH CDMA scheme for the same bandwidth expansion (i.e. ML). While the number of simultaneous users K of our partitioned two-level FH-CDMA scheme is only slightly less than that of FH CDMA scheme in Fig. 4, K_b results in a net gain in the SE, as shown in Table II.

Combining with higher data rate, greater SE, and Flexible selection of modulation codes and FH patterns, our two-level FH-CDMA scheme is a better choice in meeting various system operating criteria.

VI. CONCLUSIONS

In this paper, we proposed a new two-level FH-CDMA scheme. The prime/FH-CDMA and RS/FH-CDMA schemes were special cases of our scheme. The performance analyses showed that the two-level FH-

CDMA scheme provided a trade-off between performance and data rate. The partitioned two-level FH-CDMA scheme increased the number of possible users and exhibited higher data rate and greater SE than MFSK/FH-CDMA scheme. In summary, the new scheme offered more flexibility in the design of FH-CDMA systems to meet different operating requirements.

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