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A Model Based Approach of Interaction Measures for Multivariable Processes

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Abstract— **In this paper a normalized interaction measure or Gramian-Hankel interaction mea-sure for stable multivariable systems is summarized. The normalized interaction measure is a dynamic extension of Gramians and Hankel norm of the single-input-single-output (SISO) elementary subsystems built from the original multivariable system. In fact, the success of controller design depends on the input output pairings as it decides the structure and e effectiveness of the controller. Several examples, for which the proposed loop pairing criteria gives comparatively useful interaction measure over existing techniques are employed to show its effectiveness.**

Keywords— **Multivariable process, Gramian Interaction measure, Hankel interaction index array, Relative gain array, E ective relative gain array, normalized Gramian-Hankel interaction measure.**

I. Introduction

Many systems in modern engineering practices are highly complex, large in dimension and in most cases ill de ned and vague. At the problems of inter-activity and uncertainty special regards are taken in the area of multi-input-multi-output (MIMO) systems. Experienced human operators are also facing problem for smooth operation of system as complexity is growing rapidly in industrial system. Due to interactions and coupling between inputs and out-puts, it is always been difficult for the Controller design of multivariable systems. Performance, complexity, and cost of the control system design is depend on The choice of input-output pairings[7,14]. In multivariable process, the input-output pairings affects the success of controller as it decides the structure of the controller and effectiveness of the controller. Therefore a clever interaction measure would be successfully involved into solving the various problems of control system design. The relative gain array (RGA) described by [1-4] gives an interaction measure for multivariable systems. Interaction measure is most widely used in an industry for control loop configuration. Its simple calculation is the most common advantage of this technique, as it uses only steady state gain of the system. Inevitable, RGA not uses information of dynamic part of the system and hence limited to use where transient and uncertainty in the process is limited. As a result, using steady state gain alone may obtain an incorrect interaction measures and subsequently wrong loop pairing decisions. Gramian based inter-action measure approach introduced by [2] is matrices which describe certain controllability and observability properties of a given stable system. The advantage of Gramian based inter-action measure approach is applicable to both continuous time and discrete time systems. Unfortunately, Gramian controllability and the observability will depend on the chosen state space realization. An another interaction measure approach so called Hankel Interaction Index Array (HIIA) proposed by [3] is straight forward extension of gramian described by [2]. The dynamic relative gain array (DRGA) proposed by [4] is a natural extension of Bristol's RGA [1]. Although this measure may expose some hidden interaction features, but some times it can be difficult for the designer to make decisions regarding input output pairing based upon the comparative analysis of frequency responses. The research on interaction measures imply an attention of academician as well as industry era as it directly affect the productivity and product quality. Recently, an effective relative gain array (ERGA) using both steady state gain and bandwidth information of the process open loop transfer function elements are proposed by [8]. Through defining an effective gain matrix, the loop pairing procedures of popular RGA method is directly extended to method which can reflect dynamic loop interactions under finite bandwidth control. For transfer function matrices with some elements with-out phase crossover frequencies, such as first order or second order without time delay, it is difficult to obtain critical frequency array which are closely linked to system dynamic performance to calculate ERGA. To overcome one or more limitations of existing interaction measures, this paper introduced a new Gramian-Hankel Interaction Measure (GHIM) approach. To show capabilities of GHIM, we demonstrated several examples in which the normal RGA, Gramian based approach and HIIA gives comparatively an inaccurate interaction measure. Some multivariable processes are investigated using the new GHIM method. Simulation studies shows more accurate interaction assessment and the best pairings.

II. Interaction Measures of Process

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2.1 Relative Gain Array (RGA):-

The most widely used interaction measures for multivariable processes is RGA introduced by [1,4], which is a matrix of interaction measures for all possible single-input single-output (SISO) pairing. The RGA indicates the preferable variable pairings in a decentralized (multi-loop SISO) control system based on interaction considerations. The de_nition of RGA is as follows: Let K be the matrix of steady state gains of the transfer function matrix G(s) i. e.

$$\lim_{S\to 0} G(s) = K$$

Further let R be the transpose of the inverse of the matrix K, $R = [K^{-1}]^T$

RGA is obtained by carrying element-by-element multiplication of the elements of the matrix K and R. For example, RGA for 3-by-3 system is,

$$\Lambda = K \otimes R = \begin{pmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{pmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where,

$\otimes$ is element-by-element multiplication of elements of K and R. From equation (1) it is possible to describe level of interaction; large value of λ_{ij} means the there are strong interaction between corresponding input j and output i . If the value of λ_{ij} is greater than 0.5 and approaches toward unity, the interaction also leads between corresponding pairs. In general $\lambda_{ij} = 1$ is the ideal case for pairing and negative pairing should strictly be avoided. The RGA is depend on steady state gains and most suitable for nonlinear plant operating around steady state point. Observe also, that a negative diagonal element in the RGA only gives a sufficient (but not necessary) condition for instability. The next result to consider to avoid pairings that may lead to instability is Niederlinski's theorem [10]. If the compensator structure K =k/s C(s) is used and it is assumed that C(s) is diagonal and G(s) (with element g_{ij} I, j = 1, 2, ….., n) is stable, each individual control loop remains stable when any of the other loops are opened. In such case the Niederlinski Index (NI) for the control structure as above is defined as,

$$N(G)=\frac{|G(0)|}{\pi\, g_{ij}} \qquad i,j, = 1, 2, \ldots n \quad \ldots..(2)$$

where, $|G(0)|$ denotes determinant of matrix G(0) and $\pi\, g_{ij}$ denotes product of diagonal elements of G(0) for a fully decentralized control structure. It is advisory to combine the use of the RGA with a check of the Niederlinski theorem to see if the chosen pairings may be alright regarding stability.

For stability of complex nonlinear system Niederlinski index N(G) should be greater than zero. Niederlinskis index equal to one indicates that the diagonal pairs are ideal for decentralized control.

2.2 Gramian Based Measure

Gramians are matrices which describe certain controllability and observability properties of a given stable system as described by [2]. Assume stable MIMO system has state space representation given by,

$\dot{x}$ = Ax + Bu (3)

y = Cx + Du; (4)

where, $A \in R^{nxn}$, $B \in R^{nxm}$, $C \in R^{pxn}$, $D \in R^{pxm}$ are system matrix, input matrix, output matrix and direct transmission matrix respectively for m inputs and p outputs process. Controllability gramian $P \in R^{nxn}$ and observability gramian $Q \in R^{nxn}$ are symmetric non-negative definite matrices which satisfy the Lyapnunov equation,

$$AP + PA^T + BB^T = 0; \quad A^T Q + QA + C^T C = 0 \qquad \ldots.(5)$$

As shown by [2], it is possible to split the original state space model (A,B,C,D) into sub-models (A,Bj,;C,Dij),where Bj is jth column of matrix B, Ci is ith row of matrix C and Dij is ijth element of matrix D. Then for each of these sub-models, the controllability and the observability gramians are calculated. The controllability and observability Gramians for the full system will then be the sum of the Gramians for all the subsystems. Unfortunately, both the controllability and the observability gramian will depend on the chosen state space realization. However, the eigenvalues (known as Hankel Singular Values, HSV and de_ned in later subsection) of the product of P and Q will not. The state space model of subsystems (A,Bj,Ci,Dij) satisfy the eq.(5),

$$AP_j + P_j A^T + B_j B_j^T = 0;$$

$$A^T Q_i + Q_i A + C_i^T C_i = 0 \qquad \ldots\ldots\ldots\ldots\ldots (6)$$

The trace of the product $P_j\, Q_i$ is state realization independent and it is a convenient basis to measure the interaction and the ability of different controller structures to control and to observe the system state. A crucial fact is that the trace of $P_j\, Q_i$ is equal to the sum of the HSV for the elementary systems. This measure can be organized in matrix called as participation matrix defined by,

$$Q = \frac{trace(P_j Q_i)}{trace(PQ)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Note that the trace measure has been normalized by trace(PQ). This implies that the sum of all elements of Ø is equal to one. Similar to RGA, input-output pairings for decentralized control can be decided on the basis of the largest elements in the participation matrix Ø

2.3 Hankel Interaction Index Array

Hankel Interaction Index Array (HIIA) proposed by [5] is natural and straight forward extension of gramians described earlier subsection. If λ_1 , λ_2 ,……._n are eigenvalues of product of PQ, then the HSV _H i are defined as,

$$\sigma_i^H \ (G(s)) = \sqrt{\lambda i} \qquad i=1,2,\ldots n \qquad \ldots\ldots..(8)$$

The Hankel norm is thus the maximum HSV. For each elementary subsystem (A, Bj, Ci, Dij), the Hankel norm is used to quantify the ability of input u_j to control output yi. These norms were collected into a matrix $[\sum H]_{i\,j}$ where the element (i; j) of $[\sum H]_{i\,j}$ is defined as,

$$\sum[H]_{I\,j} = \ \|G\ i\ j\ (S)\|_H \qquad \ldots\ldots\ldots\ldots\ldots\ldots(9)$$

The array $\sum H]$ is very similar to the Participation Matrix Ø, defined by eq. (7), where the elements of the matrix are the trace of $P_j\ Q_i$. To get liberate of difficulties arising from scaling one can normalize the matrix in different ways. One idea to introduce a scaling is to use the same method as for the RGA, i.e. to use an index,

$$\boldsymbol{\Psi} = [\sum H] \otimes [(\sum H)^\wedge - 1]^T \qquad ..\ldots\ldots\ldots..(10)$$

This normalization results in a matrix where the sum of all elements in a row or column is equal to one, although (10) does not have the same nice physical interpretation as for the RGA. The pairing rules are same as RGA,that is, determine the input-output pairing by finding in each row ith the largest element in (i , j). Input j^{th} is then paired with output i^{th}. In HIIA, a major difficulty would be to decide whether an entry in the HIIA matrix is large enough to be relevant or not, and there are not any clear rules for this stated in the literature.

2.4 Effective Relative Gain Array (ERGA)

A dynamic loop pairing criterion for decentralized control of multivariable processes using both steady state gain and bandwidth information of the process open loop transfer function elements are proposed by [8]. Through defining an effective gain matrix, the loop pairing procedures of popular RGA method is directly extended to method which can reflect dynamic loop interactions under finite bandwidth control. The effective gain matrix is given by

$$E = G(0) \otimes \Omega = \begin{bmatrix} e11 & \cdots & e1n \\ e21 & \ddots & \vdots \\ en1 & \cdots & enn \end{bmatrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots..(11)$$

where

$$G(0) = \begin{pmatrix} g_{11}(0) & g_{12}(0) & \cdots & g_{1n}(0) \\ g_{21}(0) & e_{22}(0) & \cdots & g_{2n}(0) \\ \cdots & \cdots & \cdots & \cdots \\ g_{n1}(0) & g_{n2}(0) & \cdots & g_{nn}(0) \end{pmatrix}, and\ \Omega = \begin{pmatrix} w_{c,11} & w_{c,12} & \cdots & w_{c,1n} \\ w_{c,21} & w_{c,22} & \cdots & w_{c,2n} \\ \cdots & \cdots & \cdots & \cdots \\ w_{c,n1} & w_{c,n2} & \cdots & w_{c,nn} \end{pmatrix}$$

are the steady state gain matrix and critical frequency matrix. The ERGA matrix is given by [8],

$$\Phi = E \times E^{-T} = \begin{pmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \phi_{n1} & \phi_{n2} & \cdots & \phi_{nn} \end{pmatrix}, \qquad \ldots\ldots\ldots\ldots\ldots\ldots.(12)$$

The properties of ERGA are similar to RGA, ERGA pairing rules requires that manipulated and controlled variables in a decentralized control system be paired if ERGA element is positive and closest to 1.0. A main difficulty could arise if the process has first order or second order without time delays. In such cases, the phase cross over frequency is infinite and bandwidth matrix [11] were considered instead of critical frequency matrix.

III. A MODIFIED WAY FOR INTERACTION MEASURE

In designing centralized / decentralized PI/PID controllers it is important to have good knowledge of possible pairings. The consistent pairings impact the design of decentralized as well as the full dimensional controller. It is worth to say thousand words, if the interaction measure approach helps to design and tune the parameters of PI/PID controller. The interaction measure approach should address at least following issues to decide useful pairings:

(1) The interaction pairings should consider the dynamic as well as steady state performance of the process under consideration,

(2) Ability to quantify frequency dependent interactions,

(3) The pairing criterion should be independent on any other parameter except process model,

(4) The selected pairing should have ability to provide effective control strategies,

(5) It would be worth if the parings are supporting to design and tune the controller parameters,

(6) It should be simple and practical to use with real process.

Gramians as explained earlier are matrices of controllability and observability properties. The Gramians are applicable for stable system only. It is worth to point that the system should be stable to decide the pairings using Gramians based approach. In normalized dynamic relative gain array we have used dynamic properties of Gramians as well as properties of Hankel norm. From eq.(9)

$$[\Sigma_H]_{ij} = \begin{pmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ \beta_{n1} & \beta_{n2} & \cdots & \beta_{nn} \end{pmatrix} \quad \text{……………..(13)}$$

where, Bij is the Hankel norms collected from maximum Hankel singular values. Let the eigen values of matrix $\sum H]$ij are μ_i (i = 1,2, …n). The root of Hankel singular values can be easily calculated using,

$$\sigma_i^{\sqrt{H}} = \sqrt{\mu\, i} \qquad i= 1, 2, …..n \qquad ……(14)$$

It is straightforward to de_ne the root Hankel norm as a maximum root of Hankel singular value. Let define the root Hankel norm by

$$\sqrt[n]{H} = \|G\,i\,j\,(S)\|\,\sqrt{H} = \sqrt{\mu\,\max(\sigma_i^{\sqrt{H}})} \quad \text{……………….(15)}$$

The root Hankel norm is fundamental parameter to get the gain and dynamic complexity of the system. The root Hankel norm given by eq.(15) is used to normalize HIIA with Hankel norm matrix given by eq. (13) to yield,

$$[\sum H]_{ij} = \frac{[\sum H]_{ij}}{\sqrt[n]{H}} \quad \text{………………..(16)}$$

The properties of gramians, that is, participation matrix given by eq. (7) and eq. (16) are used to form interaction array matrix as,

$$\tau = \varphi \times [\sum H]_{ij} \quad \text{…………………….(17)}$$

It is always difficult to scaling the matrix to get normalized interaction array. The normalization of interaction array matrix eq. (17) are performed in straightforward, same as RGA, in the following ways,

$$\Gamma_{GH} = \Gamma \times \Gamma^{-T}. \quad \text{………………(18)}$$

Let the normalized version of interaction array matrix is defined as GHIM. This normalization results in a matrix where the sum of all elements in a row or column is equal to one. An expected performance for different control structures can certainly be compared by summing the elements in ΓGH. Clearly, due to the normalization, the aim is to find the simplest controller structure that gives a sum as near one as possible. When Gij(s) = 0, the gramian product PjQi will be zero and so the corresponding element in ΓGH is zero. This indicate that the structure of ΓGH is same as structure of G(s). It is important to observe that the matrix ΓGH takes the full dynamic effects of the system into account and not only the steady-state performance as the RGA or the behavior at a single frequency. The matrix ΓGH is independent on frequency responses and used for many processes having first/second/higher order dynamics with/without time delays. Since GHIM use the fundamental concepts of gramian based interaction measure and Hankel norms, the properties of same can be extended to proposed interaction measure:

(1) The sum of elements of and row or any column is 1, i. e. $\sum_{i=1}^{n} \Gamma_{GH} = \sum_{i=1}^{n} \Gamma_{GH} = 1,$

(2) An elements of ΓGH_{ij} are measure of interaction. Input uj paired with output yi if the value of ΓGH_{ij} closest one,

(3) Large value of ΓGH_{ij} should be avoided during pairing,

(4) It is better if paired values of ΓGH_{ij} lies between 0.5 to 1.5,

(5) Negative pairing should be avoided strictly.

In next section, the capacities of GHIM approach is validated for various industrial processes. The usefulness of proposed approach is compared with RGA, gramian, HIIA and ERGA.

IV. SIMULATION STUDIES

In order to evaluate the ability of the GHIM technique, it is applied to various processes and the results are compared with those of some prevalent techniques. Example 1 The Wood-Berry binary distillation column plant model introduced by [12] is a multivariable has the following transfer matrix:

$$G(s)=\begin{pmatrix}\frac{12.8}{16.7s+1}e^{-s} & \frac{-18.9}{21s+1}e^{-3s}\\ \frac{6.6}{10.9s+1}e^{-7s} & \frac{-19.4}{14.4s+1}e^{-3s}\end{pmatrix}$$

Applying four prevalent interaction measure techniques, which are given by RGA eq. (1), Gramian based eq. (7), HIIA eq. (10), and ERGA eq. (12) the interaction measure matrices are:

$$\Lambda=\begin{pmatrix}2.0094 & -1.0094\\ -1.0094 & 2.0094\end{pmatrix},$$

$$\phi=\begin{pmatrix}0.1447 & 0.3613\\ 0.0757 & 0.4182\end{pmatrix},$$

$$\Psi=\begin{pmatrix}2.6751 & -1.6751\\ -1.06751 & 2.6751\end{pmatrix},$$

$$\Phi=\begin{pmatrix}1.0909 & -0.0909\\ -0.0909 & 1.0909\end{pmatrix},$$

respectively. The interactive measure given by GHIM eq. (17) is,

$$\Gamma=\begin{pmatrix}1.3949 & -0.3949\\ -0.3949 & 1.3949\end{pmatrix}.$$

What is the conclusion for an example 1. All interaction methods are noting the pairing of y1- u1 and y2 - u2. For Wood-Berry binary distillation column plant model all interaction measure provides same information regarding pairing. This show the competency of proposed approach.

Example 2

Let us consider the process model of wet grinding circuit proposed by [13] with process transfer matrix,

$$G(s)=\begin{pmatrix}\frac{370}{500s+1} & \frac{767}{33s+1} & \frac{-50}{10s+1}\\ \frac{903}{500s+1} & \frac{-667}{166s+1}e^{-320s} & \frac{-1033}{47s+1}\\ \frac{119}{217s+1} & \frac{153}{337s+1} & \frac{-21}{10s+1}\end{pmatrix}.$$

The Bristol array i. e. RGA for the given process model is

$$\Lambda=\begin{pmatrix}-2.3616 & 2.9581 & 0.4035\\ -0.2833 & -0.0450 & 1.3283\\ 3.6449 & -1.9131 & -0.7318\end{pmatrix}.$$

From RGA, it is not clear regarding coupling and can falls into trouble to decide the pairings. Again the array shows the failure sensitivity of the process. Consider the approach of gramian as given by eq. (7)

$$\phi=\begin{pmatrix}0.0305 & 0.1309 & 0.0006\\ 0.1814 & 0.4108 & 0.2374\\ 0.0032 & 0.0052 & 0.0001\end{pmatrix}.$$

As per Gramians it is not visualize the useful pairings and difficulty arises for further analysis and design of the process. Let us consider HIIA approach for the same process model. The HIIA is,

$$\Psi=\begin{pmatrix}-1.6556 & 2.6432 & 0.0124\\ -0.2531 & 0.0662 & 1.1869\\ 2.9087 & -1.7094 & -0.1993\end{pmatrix}.$$

HIIA denotes the best pairings are y1-u2, y2-u3 and y3-u1, whereas ERGA shows the same pairings like HIIA as follows,

$$\Phi=\begin{pmatrix}-0.0750 & 0.9945 & 0.0805\\ -0.4759 & 0.0233 & 1.4526\\ 1.5509 & -0.0178 & -0.5331\end{pmatrix}$$

The interactive measure given by GHIM eq. (17) results,

$$\Gamma = \begin{pmatrix} -0.3130 & 1.3131 & -0.0002 \\ -0.0043 & 0.0008 & 1.0035 \\ 1.3173 & -0.3139 & -0.0033 \end{pmatrix}.$$

By looking all the elements of GHIM array, it simple to say that the perfect pairings are y1 - u2, y2 - u3 and y3 - u1. Look at ERGA, it shows the same pairings like GHIM, but faces difficulty, because for most of the transfer functions of process model (transfer function matrix) the crossover frequencies are infinite. The bandwidth matrix is used to obtain ERGA as suggested by [11]. However the proposed method is seems to be simple for computation and not require frequency responses.

Example 3 Consider a process described by [15] with transfer function matrix,

$$G(s) = \begin{pmatrix} \frac{-2}{10s+1}e^{-s} & \frac{1.5}{s+1}e^{-s} & \frac{1}{s+1}e^{-s} \\ \frac{1.5}{s+1}e^{-s} & \frac{1}{s+1}e^{-s} & \frac{-2}{10s+1}e^{-s} \\ \frac{1}{s+1}e^{-s} & \frac{-2}{10s+1}e^{-s} & \frac{1.5}{s+1}e^{-s} \end{pmatrix}$$

The RGA for the system is,

$$\Lambda = \begin{pmatrix} -0.9302 & 1.1860 & 0.7442 \\ 1.1860 & 0.7442 & -0.9302 \\ 0.7442 & -0.9302 & 1.1860 \end{pmatrix}$$

From RGA it is clear that the possible parings are y1 - u2, y2 - u1, y3 - u3 and y1 - u3, y2 - u2, y3 - u1 because all related RGA elements are close to

1. Therefore, RGA pairing approach fails to determine which pairing is better and which should not be consider. By applying the gramian approach,

$$\phi = \begin{pmatrix} 0.1149 & 0.1512 & 0.0672 \\ 0.1512 & 0.0672 & 0.1149 \\ 0.0672 & 0.1149 & 0.1512 \end{pmatrix}$$

It indicates the pairings as y1 - u2, y2 - u1, y3 - u3. Obviously no clear idea because y1 - u1, y2 - u3, y3 - u2 are also possible. HIIA for the process is,

$$\Psi = \begin{pmatrix} 1.0538 & 1.2268 & -1.2806 \\ 1.2268 & -1.2806 & 1.0538 \\ -1.2806 & 1.0538 & 1.2268 \end{pmatrix}$$

HIIA denotes the best pairings are y1-u1, y2-u3 and y2-u2, whereas ERGA shows the pairings are as follows,

$$\Phi = \begin{pmatrix} 0.0002 & 0.7712 & 0.2286 \\ 0.7712 & 0.2286 & 0.0002 \\ 0.2286 & 0.0002 & 0.7712 \end{pmatrix}$$

The interactive mesure given by GHIM eq. (17) results

$$\Gamma = \begin{pmatrix} 0.2514 & 0.9994 & -0.2508 \\ 0.9994 & -0.2508 & 0.2514 \\ -0.2508 & 0.2514 & 0.9994 \end{pmatrix}$$

shows the perfect pairings as y1 - u2, y2 - u1 and y3 - u3. This was also guaranteed by generalized dynamic relative gain (GDRG) [15] and ERGA using bandwidth by [11]. However the proposed method indicate the only possible pairings are y1- u2, y2-u1 and y3-u3. This is perfectness and most important ability of proposed approach because all pairing elements of GHIM array matrix are close to 1 (0.9994).

Example 4 Consider a 4-by-4 process model described by [16] with transfer function matrix,

$$G(s) = \begin{pmatrix} \frac{2.22}{(36s+1)(25s+1)}e^{-2.5s} & \frac{-2.94(7.9s+1)}{(23.7s+1)^2}e^{-0.05s} & \frac{0.017}{(31.6s+1)(7s+1)}e^{-0.2s} & \frac{-0.64}{(29s+1)^2}e^{-20s} \\ \frac{-2.33}{(35s+1)^2}e^{-5s} & \frac{3.46}{32s+1}e^{-1.01s} & \frac{-0.51}{(32s+1)^2}e^{-7.5s} & \frac{1.68}{(28s+1)^2}e^{-2s} \\ \frac{-1.06}{(17s+1)^2}e^{-22s} & \frac{3.511}{(12s+1)^2}e^{-13s} & \frac{4.41}{16.2s+1}e^{-1.01s} & \frac{-5.38}{17s+1}e^{-0.5s} \\ \frac{-5.73}{(8s+1)(50s+1)}e^{-2.5s} & \frac{4.32(25s+1)}{(50s+1)(5s+1)}e^{-0.01s} & \frac{-1.25}{(43.6s+1)(9s+1)}e^{-2.8s} & \frac{4.78}{(48s+1)(5s+1)}e^{-1.15s} \end{pmatrix}$$

The RGA for the this process is,

$$\Lambda = \begin{pmatrix} 3.5280 & -1.0766 & 0.0502 & -1.5016 \\ -3.3628 & 2.7685 & -0.7028 & 2.2971 \\ -0.1203 & 0.1305 & 2.2635 & -1.2736 \\ 0.9552 & -0.8224 & -0.6109 & 1.4781 \end{pmatrix}$$

From RGA it is difficult to decide the possible parings because all related GA elements are not close to 1. Therefore, RGA pairing approach fails to indicate which pairing is better. The gramian measure is,

$$\phi = \begin{pmatrix} 0.0345 & 0.0688 & 0.0000 & 0.0040 \\ 0.0398 & 0.0568 & 0.0020 & 0.0198 \\ 0.0140 & 0.1420 & 0.0976 & 0.1369 \\ 0.1942 & 0.0559 & 0.0097 & 0.1239 \end{pmatrix}$$

Thus gramians does not gives any measure information. HIIA for the process

$$\Psi = \begin{pmatrix} 3.1549 & -0.5297 & 0.0403 & -1.6654 \\ -2.5107 & 2.3104 & -1.1640 & 2.3643 \\ 0.1705 & -0.1812 & 2.4180 & -1.4074 \\ 0.1852 & -0.5995 & -0.2943 & 1.7085 \end{pmatrix}$$

HIIA denotes the best pairings are y1-u1, y2-u2, y3-u3 and y4- u4. ERGA interaction matrix is,

$$\Phi = \begin{pmatrix} 0.9927 & 0.0073 & 0.0000 & -0.0000 \\ 0.0120 & -0.0541 & -0.0000 & 1.0421 \\ 0.0000 & -0.0000 & 1.0000 & -0.0000 \\ -0.0046 & 1.0467 & -0.0000 & -0.0421 \end{pmatrix}$$

ERGA shows the possible pairings are y1 - u1, y2 - u4, y3 - u3 and y4 - u2. But by RGA recommends that the negative pairing should be avoided strictly (The fourth pairing element of RGA array, that is y4 - u2 is negative). The interactive measure given by GHIM is,

$$\Gamma = \begin{pmatrix} 1.6123 & -0.4894 & 0.0000 & -0.1228 \\ -1.4974 & 1.5843 & -0.0683 & 0.9814 \\ 0.0003 & -0.0060 & 1.0654 & -0.0597 \\ 0.8848 & -0.0888 & 0.0028 & 0.2012 \end{pmatrix}$$

GHIM shows the pairings as y1 - u1, y2 - u2, y3 - u3 and y4 - u4. Selection of pairings play a crucial role while designing an effective controller. Design of decentralized PI control systems based on Nyquist stability analysis using the diagonal pairing are confirmed by [17].

V. Conclusions

In this paper, a full dimensional Gramian-Hankel interaction measure approach is discussed for multivariable interactive processes. It uses dynamic interactions, controllability and observability matrices of subsystems. Properties of GHIM shows it can be applicable for any process and has no constrains of number of inputs and outputs. The method is very simple, straightforward and easy to be understood and embedded into the computer control systems. The algorithm has straightforward extension for higher order processes having complex dynamics and considerably time delays. As the approach uses the ability of gramians and Hankel norms, thus it makes the simplicity to observe and control the systems state, and dynamic interaction. The proposed measure has related rules for pairings and designer can take benefit to design all types of controller structures. The approach has been compared with the traditional RGA, HIIA, Gramian based measure and ERGA. The new interaction measure approach exhibits a superior performance when the multivariable interaction has a non-monotonic behavior in frequency. It is also very simpler which measure dynamic interaction, since it does not require complex analysis of a number of frequency responses.

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