ON SOFT $g^{\#}\alpha$-SEPARATION AXIOMS

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***Abstract*— In this paper, we introduce soft $g^{\#}\alpha$-closed sets in soft topological spaces and discuss some of its properties. We also introduce four new type of spaces namely soft T^a, soft T^b, soft T^c, soft T^d spaces and soft $g^{\#}\alpha$-continuous functions and soft $g^{\#}\alpha$-irresolute functions and study some of their properties.**

***Keywords*—soft $g^{\#}\alpha$-closed sets , soft T^a, soft T^b, soft T^c, soft T^d spaces , soft $g^{\#}\alpha$-continuous functions and soft $g^{\#}\alpha$-irresolute functions.**

I. INTRODUCTION

Molodtsov (1999) introduced the concept of soft sets, as a general mathematical tool for dealing with uncertain objects. A soft set is a collection of approximate description of an object in universe set U and which contains parameters like words, real numbers, and functions and so on.

Soft set theory has a rich potential for applications in several directions which include the smoothness of function, game theory, operations research, Riemann integration, Perron integration, probability theory and measurement theory etc.

In 2003 Maji et al., presented soft set theory with some implementation in their work. Roy & Maji (2007) presented a novel method of object recognition from an imprecise multi observer data in decision making problem.

Aktas and Cagman (2007) introduced the basic properties of soft sets related to the concept of fuzzy sets and rough sets and then they gave the notion of soft group and derived some basic properties of it.

Kannan.K [10] introduced and studied the properties of soft generalized-closed sets and the concepts of soft $T_{1/2}$-spaces. Arockiarani.I and Arokialancy.A [4] introduced soft β-separation axioms. Arockiarani.I and Arokialancy.A [3] introduced soft gβ-closed sets and soft gsβ-closed sets in soft topological spaces and they obtained some properties for these sets. Nono.K, Devi.R, Devipriya.M. Muthukumaraswamy.K and Maki.H [16] introduced the concepts of $g^{\#}\alpha$-closed sets in topological spaces—.

In this paper, we introduce soft $g^{\#}\alpha$-closed sets and study some of its properties in soft topological spaces. Moreover we introduce soft $g^{\#}\alpha$-separation axioms soft T^a, soft T^b, soft T^c, soft T^d spaces and soft $g^{\#}\alpha$-continuous functions and soft $g^{\#}\alpha$-irresolute functions.

Throughout this paper, the spaces X and Y stand for soft topological spaces with ((X,τ,E) and (Y,γ,K)) assumed unless otherwise stated. Moreover, throughout this paper, a soft mapping f : (X,τ,E) → (Y,γ,K) stands for a mapping, where f : (X,τ,E) → (Y,γ,K), u : X → Y and p : E → K are assumed mappings unless otherwise stated. Soft operations denoted by usual set theoretical operations with '~' symbol over.

The family of all soft $g^{\#}\alpha$-closed sets (resp. soft α-closed sets, soft αg-closed sets, soft g-closed sets, soft g^*-closed sets) is denoted by $\tilde{s}G^{\#}\alpha CS(X,\tau,E)$, (resp. $\tilde{s}\alpha CS(X,\tau,E)$, $\tilde{s}\alpha GCS(X,\tau,E)$, $\tilde{s}GCS(X,\tau,E)$, $\tilde{s}G^*CS(X,\tau,E)$) and the family of all soft $g^{\#}\alpha$-open sets (resp. soft α-open sets, soft αg-open sets, soft g-open sets, soft g^*-open sets) is denoted by $\tilde{s}G^{\#}\alpha OS(X,\tau,E)$, (resp.$\tilde{s}\alpha OS(X,\tau,E)$,$\tilde{s}\alpha GOS(X,\tau,E)$,$\tilde{s}GOS(X,\tau,E)$,$\tilde{s}G^*OS(X,\tau,E)$).

II. PRELIMINARIES

Let X be an initial universe and let E be the set of parameters. Let P(X) denote the power set of X and let A be a non-empty subset of E.

DEFINITION: 1 [10]

A pair (F,A) is called a soft set over X, where F is a mapping given by F: A → P(X). In other words, a soft set over X is a parameterized family of subsets of the universe X.

DEFINITION: 2 [17]

Let (X,τ, E) be a soft topological space over X, (G,A) be a soft set over X and x ∈ X. Then x is said to be a soft interior point of (G, A) and (G,A) is said to be a soft neighborhood of x, if there exists a soft open set (F,A) such that x ∈ (F,A) $\tilde{\subseteq}$ (G,A).

DEFINITION: 3 [14]

A soft set (F,A) of a soft topological space (X,τ,E) is called soft α-open set, if (F,A) $\tilde{\subseteq}$ $\tilde{s}$int($\tilde{s}$cl($\tilde{s}$int(F,A))). The complement of soft α-open set is called soft α-closed set (briefly $\tilde{s}$α-closed).

DEFINITION: 4 [5]

A soft set (F,A) in a soft topological space (X,τ,E) will be termed soft semi-open if and only if there exists a soft open set (O,A) such that (O,A) $\tilde{\subseteq}$ (F,A) $\tilde{\subseteq}$ $\tilde{s}$cl(O,A).

DEFINITION: 5 [5]

A soft set (F,A) in a soft topological space (X,τ,E) will be termed soft semi-closed if and only if there exists a soft closed set (O,A) such that $\tilde{s}$int(O,A) $\tilde{\subseteq}$ (F,A) $\tilde{\subseteq}$ (O,A).

DEFINITION: 6 [10]

A subset (A,E) of a soft topological space (X,τ,E) is called soft generalized closed set (briefly soft g-closed), if $\tilde{s}$cl(A,E) $\tilde{\subseteq}$ (U,E), whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft open in X.

DEFINITION: 7 [10]

A subset (A,E) of a soft topological space (X,τ,E) is called soft semi-generalized closed set (briefly soft sg-closed), if $\tilde{s}$scl(A,E) $\tilde{\subseteq}$ (U,E), whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft semi-open in X.

DEFINITION: 8 [3]

A subset (A,E) of a soft topological space (X,τ,E) is called soft α-generalized closed set (briefly soft αg-closed), if $\tilde{s}$αcl(A,E) $\tilde{\subseteq}$ (U,E), whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft open in X.

DEFINITION: 9 [3]

A subset (A,E) of a soft topological space (X,τ,E) is called soft generalized semi-closed set (briefly soft gs-closed), if $\tilde{s}$scl(A,E) $\tilde{\subseteq}$ (U,E), whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft open in X.

DEFINITION: 10 [3]

A subset (A,E) of a soft topological space (X,τ,E) is called soft g^*closed set (briefly soft g^*-closed), if $\tilde{s}$cl(A,E) $\tilde{\subseteq}$ (U,E), whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft g-open in X.

DEFINITION: 11 [3]

A subset (A,E) of a soft topological space (X,τ,E) is said to be clopen, if (A,E) is closed and open in X.

DEFINITION: 12 [10]

A soft topological space (X,τ, E) is a soft $T_{1/2}$ -space if every soft g-closed set is soft closed in X.

DEFINITION: 13 [4]

Let (X,τ,E) be a soft topological space over X and x,y ∈ X such that x ≠ y. If there exist soft open sets (F,A) and (G,A) such that x $\tilde{\in}$ (F,A) and y $\tilde{\notin}$ (F,A) or y $\tilde{\in}$ (G,A) and x $\tilde{\notin}$ (G,A), then (X,τ,E) is called a soft T_0 space.

DEFINITION: 14 [4]

Let (X,τ,E) be a soft topological space over X and x,y ∈ X such that x ≠ y. If there exist soft open sets (F,A) and (G,A) such that x $\tilde{\in}$ (F,A) and y $\tilde{\notin}$ (F,A) and y $\tilde{\in}$ (G,A) and x $\tilde{\notin}$ (G,A), then (X,τ,E) is called a soft T_1 space.

DEFINITION: 15 [4]

Let (X,τ,E) be a soft topological space over X and x,y ∈ X such that x ≠ y. If there exist soft open sets (F,A) and (G,A) such that x $\tilde{\in}$ (F,A) and y $\tilde{\in}$ (G,A) and (F,A) $\tilde{\cap}$ (G,A) = Φ. Then (X,τ,E) is called a soft T_2 space.

DEFINITION: 16 [13]

A soft mapping f : X → Y is called soft α-continuous (resp.soft β-continuous, soft pre-continuous, soft semi-continuous) if the inverse image of each soft open set in Y is $\tilde{s}$α-open (resp. $\tilde{s}$β-open, $\tilde{s}$p-open, $\tilde{s}$s-open) set in X.

DEFINITION: 17 [14]

A function f: (X,τ,E) → (Y,γ,K) is called soft α-continuous if the inverse image is a soft α-open set of (X,τ,E) for every soft open set of (Y,γ,K).

III.SOFT $g^{\#}$α-CLOSED SETS

In this section, we introduce the concept of soft $g^{\#}$α-closed sets in a soft topological space and examine some of their properties.

DEFINITION: 18

A soft set (A,E) is called soft $g^{\#}$α-closed in a soft topological space (X,τ,E), if $\tilde{s}$αcl(A,E) $\tilde{\subseteq}$ (U,E), whenever (A,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft g-open in X.

THEOREM: 19

Every soft closed set is soft $g^{\#}$α-closed.

PROOF:

Suppose (A,E) is a soft closed set, (U,E) be soft g-open in X such that (A,E) $\tilde{\subseteq}$ (U,E), $\tilde{s}$cl(A,E) = (A,E) $\tilde{\subseteq}$ (U,E). Thus $\tilde{s}\alpha$cl(A,E) $\tilde{\subseteq}$ $\tilde{s}$cl(A,E) $\tilde{\subseteq}$ (U,E). Hence (A,E) is soft $g^{\#}\alpha$-closed.

The converse need not be true as shown by the following example

EXAMPLE: 20

Let X={a,b}, A = {e_1,e_2} and Let τ = {$(F,A)_1$, $(F,A)_6$, $(F,A)_{16}$}. The soft sets over X are {$(F,A)_1$, $(F,A)_2$, $(F,A)_3$, $(F,A)_4$, $(F,A)_5$, $(F,A)_6$, $(F,A)_7$, $(F,A)_8$, $(F,A)_9$, $(F,A)_{10}$, $(F,A)_{11}$, $(F,A)_{12}$, $(F,A)_{13}$, $(F,A)_{14}$, $(F,A)_{15}$, $(F,A)_{16}$} which are defined as

$(F,A)_1$ = {(e_1,∅),(e_2,∅)}
$(F,A)_2$ = {(e_1,∅),(e_2,{a})
$(F,A)_3$ = {(e_1,∅),(e_2,{b})}
$(F,A)_4$ = {(e_1, ∅),(e_2,{a,b})}
$(F,A)_5$ = {(e_1,{a}),(e_2, ∅)}
$(F,A)_6$ = {(e_1,{a}),(e_2,{a})}
$(F,A)_7$ = {(e_1,{a}),(e_2,{b})}
$(F,A)_8$ = {(e_1,{a}),(e_2,{a,b})}
$(F,A)_9$ = {(e_1,{b}),(e_2,∅)}
$(F,A)_{10}$ = {(e_1,{b}),(e_2,{a})}
$(F,A)_{11}$ = {(e_1,{b}),(e_2,{b})}
$(F,A)_{12}$ = {(e_1,{b}),(e_2,{a,b})}
$(F,A)_{13}$ = {(e_1,{a,b}),(e_2, ∅)}
$(F,A)_{14}$ = {(e_1,{a,b}),(e_2,{a})}
$(F,A)_{15}$ = {(e_1,{a,b}),(e_2,{b})}
$(F,A)_{16}$ ={(e_1,{a,b}),(e_2,{a,b})}

The soft set $(F,A)_{15}$ is soft $g^{\#}\alpha$-closed in (X,τ,E) but it is not soft closed.

THEOREM: 21

Every soft α-closed set is soft $g^{\#}\alpha$-closed.

PROOF:

Suppose (A,E) is soft α-closed set, (U,E) be soft g-open in X, such that (A,E) $\tilde{\subseteq}$ (U,E). For every soft α-closed set, $\tilde{s}\alpha$cl(A,E) = (A,E). Since (A,E) $\tilde{\subseteq}$ (U,E), we have $\tilde{s}\alpha$cl(A,E) $\tilde{\subseteq}$ (U,E). Hence (A,E) is soft $g^{\#}\alpha$-closed.

The converse need not be true as shown by the following example

EXAMPLE: 22

Let X = {a,b}, A={e_1,e_2} and Let τ = {$(F,A)_1$,$(F,A)_6$, $(F,A)_{16}$}. The soft sets over X are defined as in example 20. The soft set $(F,A)_{15}$ is $\tilde{s}g^{\#}\alpha$-closed but it is not $\tilde{s}\alpha$-closed.

THEOREM: 23

Every soft g^{*}-closed set is soft $g^{\#}\alpha$-closed.

PROOF:

Suppose (A,E) is a soft g^{*}-closed set, (U,E) be soft g-open in X, such that (A,E) $\tilde{\subseteq}$ (U,E). Since (A,E) is a soft g^{*}-closed set, $\tilde{s}$cl(A,E) $\tilde{\subseteq}$ (U,E), we have $\tilde{s}\alpha$cl(A,E) $\tilde{\subseteq}$ $\tilde{s}$cl(A,E) $\tilde{\subseteq}$ (U,E). Thus $\tilde{s}\alpha$cl(A,E) $\tilde{\subseteq}$ (U,E). Hence (A,E) is soft $g^{\#}\alpha$-closed.

The converse need not be true as shown by the following example

EXAMPLE: 24

Let X={a,b}, A = {e_1,e_2} and Let τ = {$(F,A)_1$, $(F,A)_6$, $(F,A)_{16}$} . The soft sets over X are defined as in example 20. The soft set $(F,A)_3$ is soft $g^{\#}\alpha$-closed in (X,τ,E) but it is not soft g^{*}-closed.

THEOREM: 25

Every soft $g^{\#}\alpha$-closed set is soft αg-closed.

PROOF:

Suppose (A,E) is a soft $g^{\#}\alpha$-closed set, (U,E) be soft open in X, such that (A,E) $\tilde{\subseteq}$ (U,E). Since (A,E) is a soft $g^{\#}\alpha$-closed set, $\tilde{s}\alpha$cl(A,E) $\tilde{\subseteq}$ (U,E). Hence (A,E) is soft αg-closed.

The converse need not be true as shown by the following example

EXAMPLE: 26

Let X={a,b}, A = {e_1,e_2} and Let τ = {$(F,A)_1$, $(F,A)_6$, $(F,A)_{16}$} . The soft sets over X are defined as in example 20. The soft set $(F,A)_4$ is soft αg-closed in (X,τ,E), but it is not soft $g^{\#}\alpha$-closed.

THEROM: 27

Every soft $g^{\#}\alpha$-closed set is soft gs-closed.

PROOF:

Suppose (A,E) is a soft $g^{\#}\alpha$-closed set, (U,E) be soft open in X, such that (A,E) $\tilde{\subseteq}$ (U,E). Since (A,E) is a soft $g^{\#}\alpha$-closed set, $\tilde{s}\alpha$cl(A,E) $\tilde{\subseteq}$ (U,E), we have $\tilde{s}$scl(A,E) $\tilde{\subseteq}$ $\tilde{s}\alpha$cl(A,E) $\tilde{\subseteq}$ (U,E). Thus $\tilde{s}$scl(A,E) $\tilde{\subseteq}$ (U,E). Hence (A,E) is soft gs-closed.

The converse need not be true as shown by the following example

EXAMPLE: 28

Let $X = \{a,b\}$, $A = \{e_1,e_2\}$ and Let $\tau = \{(F,A)_1, (F,A)_6, (F,A)_{16}\}$. The soft sets over X are defined as in example 20. The soft sets $(F,A)_4$ is soft gs-closed in (X,τ,E) but not soft $g^{\#}\alpha$-closed.

REMARK: 29

The concepts of the soft g-closed sets and soft $g^{\#}\alpha$-closed sets are independent to each other which can be seen from the following examples.

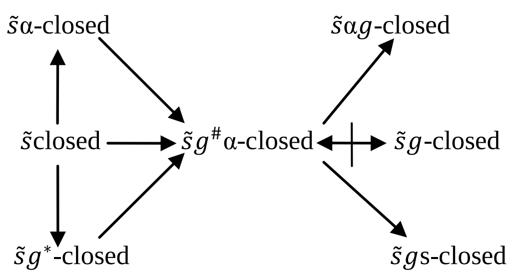
EXAMPLE: 30

Let $X = \{a,b\}$, $A = \{e_1,e_2\}$ and Let $\tau = \{(F,A)_1, (F,A)_6, (F,A)_{16}\}$. The soft sets over X are defined as in example 20.
Soft g-closed sets are $(F,A)_1$, $(F,A)_3$, $(F,A)_4$, $(F,A)_7$, $(F,A)_8$, $(F,A)_9$, $(F,A)_{10}$, $(F,A)_{11}$, $(F,A)_{12}$, $(F,A)_{13}$, $(F,A)_{14}$, $(F,A)_{15}$, $(F,A)_{16}$.
Soft $g^{\#}\alpha$-closed sets are $(F,A)_1$,$(F,A)_3$, $(F,A)_9$, $(F,A)_{11}$, $(F,A)_{12}$, $(F,A)_{15}$,$(F,A)_{16}$.
Here $(F,A)_4$ is soft g-closed but not soft $g^{\#}\alpha$-closed.

EXAMPLE: 31

Let $X = \{a,b\}$, $A = \{e_1,e_2\}$ and Let $\tau =\{(F,A)_1, (F,A)_5, (F,A)_7, (F,A)_8, (F,A)_{16}$. The soft sets over X are defined as in example 20.
Soft g-closed sets = $\{(F,A)_1, (F,A)_9, (F,A)_{10}, (F,A)_{11}, (F,A)_{12}, (F,A)_{13}, (F,A)_{14}, (F,A)_{15}, (F,A)_{16}\}$
Soft $g^{\#}\alpha$-closed sets = $\{(F,A)_1, (F,A)_2, (F,A)_3, (F,A)_4, (F,A)_9, (F,A)_{10}, (F,A)_{11}, (F,A)_{12}, (F,A)_{13}, (F,A)_{14}, (F,A)_{15}, (F,A)_{16}\}$
Here $(F,A)_5$ is soft $g^{\#}\alpha$-closed but not soft g-closed.

From the above results the following implication is made:



THEOREM: 32

If (A,E) and (B,E) are soft $g^{\#}\alpha$-closed sets then so is $(A,E) \cup (B,E)$.

PROOF:

Suppose that (A,E) and (B,E) are soft $g^{\#}\alpha$-closed sets. Let $(A,E) \cup (B,E) \tilde{\subseteq} (U,E)$ and (U,E) is soft g-open in X. Since $(A,E) \cup (B,E) \tilde{\subseteq} (U,E)$, we have $(A,E) \tilde{\subseteq} (U,E)$ and $(B,E) \tilde{\subseteq} (U,E)$. Since (U,E) is soft g-open in X and (A,E) and (B,E) are soft $g^{\#}\alpha$-closed sets, we have $\tilde{s}\alpha cl(A,E) \tilde{\subseteq} (U,E)$ and $\tilde{s}\alpha cl(B,E) \tilde{\subseteq} (U,E)$.Therefore $\tilde{s}\alpha cl((A,E) \cup (B,E)) = \tilde{s}\alpha cl(A,E) \cup \tilde{s}\alpha cl(B,E) \tilde{\subseteq} (U,E)$.

REMARK: 33

The intersection of two soft $g^{\#}\alpha$-closed sets is need not be soft $g^{\#}\alpha$-closed.

THEOREM: 34

If (A,E) is soft $g^{\#}\alpha$-closed in X and $(A,E) \tilde{\subseteq} (B,E) \tilde{\subseteq} \tilde{s}\alpha cl(A,E)$, then (B,E) is soft $g^{\#}\alpha$-closed.

PROOF:

Suppose that (A,E) is soft $g^{\#}\alpha$-closed in X and $(A,E) \tilde{\subseteq} (B,E) \tilde{\subseteq} \tilde{s}\alpha cl(A,E)$. Let $(B,E) \tilde{\subseteq} (U,E)$ and (U,E) is soft g-open in X. Since $(A,E) \tilde{\subseteq} (B,E)$ and $(B,E) \tilde{\subseteq} (U,E)$, we have $(A,E) \tilde{\subseteq} (U,E)$. Moreover (A,E) is $\tilde{s}g^{\#}\alpha$-closed, $\tilde{s}\alpha cl(A,E) \tilde{\subseteq} (U,E)$. Since $(B,E) \tilde{\subseteq} \tilde{s}\alpha cl(A,E)$, we have $\tilde{s}\alpha cl(B,E) \tilde{\subseteq} \tilde{s}\alpha cl(A,E) \tilde{\subseteq} (U,E)$. Therefore (B,E) is soft $g^{\#}\alpha$-closed.

THEOREM: 35

If a set (A,E) is soft $g^{\#}\alpha$-closed in X, then $\tilde{s}\alpha cl(A,E) \setminus (A,E)$ contains only null soft g-closed set.

PROOF:

Suppose that (A,E) is soft $g^{\#}\alpha$-closed in X. Let (F,E) be soft g-closed and $(F,E) \tilde{\subseteq} \tilde{s}\alpha cl(A,E) \setminus (A,E)$. Since F is soft g-closed. We have its relative complement F^c is soft g-open. Since $(F,E) \tilde{\subseteq} \tilde{s}\alpha cl(A,E) \setminus (A,E)$. We have $(F,E) \tilde{\subseteq} \tilde{s}\alpha cl(A,E)$ and $(F,E) \tilde{\subseteq} (A,E)^c$. Hence $(A,E) \tilde{\subseteq} (F,E)^c$. Consequently $\tilde{s}\alpha cl(A,E) \tilde{\subseteq} (F,E)^c$ as (A,E) is soft $g^{\#}\alpha$-closed in X. Therefore $(F,E) \tilde{\subseteq} \tilde{s}\alpha cl(A,E)^c$. Hence $(F,E) = \Phi$. Hence $\tilde{s}\alpha cl(A,E) \setminus (A,E)$ contains only null soft g-closed.

THEOREM: 36

If (A,E) is soft g-open and soft $g^{\#}\alpha$-closed, then (A,E) is soft α-closed.

PROOF:

Suppose (A,E) is soft g-open and soft $g^{\#}\alpha$-closed, then by definition $\tilde{s}\alpha cl(A,E) \tilde{\subseteq} (A,E)$ but we know $(A,E) \tilde{\subseteq} \tilde{s}\alpha cl(A,E)$. Thus $(A,E) = \tilde{s}\alpha cl(A,E)$. Moreover every soft α-closed set is soft $g^{\#}\alpha$-closed. Hence (A,E) is soft α-closed.

IV.SOFT $g^{\#}\alpha$- SEPARATION AXIOMS

In this section, we introduce new type of spaces namely soft T^a, soft T^b, soft T^c, soft T^d spaces.

DEFINITION: 37

A soft topological space (X,τ,E) is soft T^a space if every soft $g^{\#}\alpha$-closed set is soft α-closed.

DEFINITION: 38

A soft topological space (X,τ,E) is soft T^b space if every soft αg-closed set is soft $g^{\#}\alpha$-closed.

DEFINITION: 39

A soft topological space (X,τ,E) is soft T^c space if every soft gs-closed set is soft $g^{\#}\alpha$-closed.

DEFINITION: 40

A soft topological space (X,τ,E) is soft T^d space if every soft $g^{\#}\alpha$-closed set is soft g^*-closed.

REMARK: 41

The concepts of the soft T^a space and soft T^d space are independent to each other which can be seen from the following example.

EXAMPLE: 42

Let X = {a,b}, A = $\{e_1,e_2\}$ and Let τ= $\{(F,A)_1, (F,A)_3, (F,A)_8, (F,A)_{11}, (F,A)_{16}\}$.
The soft sets over X are defined as in example 20.
Now $\tilde{s}G^{\#}\alpha CS(X,\tau,E) = \tilde{s}\alpha CS(X,\tau,E) = \{(F,A)_1, (F,A)_2, (F,A)_6, (F,A)_9, (F,A)_{10}, (F,A)_{13}, (F,A)_{16}\}$. But $\tilde{s}G^*CS(X,\tau,E) = \{(F,A)_1, (F,A)_6, (F,A)_9, (F,A)_{13}, (F,A)_{14}, (F,A)_{16}\}$. Hence X is a soft T^a space but not soft T^d space.

EXAMPLE: 43

Let X = {a,b}, A = $\{e_1,e_2\}$ and Let τ = $(F,A)_1,(F,A)_{12}, (F,A)_{16}\}$.
The soft sets over X are defined as in example 20.
Now $\tilde{s}G^{\#}\alpha CS(X,\tau,E) = \tilde{s}G^*CS(X,\tau,E) = \{(F,A)_1,(F,A)_5, (F,A)_6, (F,A)_7, (F,A)_8, (F,A)_{13}, (F,A)_{14}, (F,A)_{15}, (F,A)_{16}\}$.
But $\tilde{s}\alpha CS(X,\tau,E) = \{(F,A)_1, (F,A)_5, (F,A)_{16}\}$.
Hence X is a soft T^d space but not soft T^a space.

REMARK: 44

The concepts of the soft T^c space and soft T^d space are independent to each other which can be seen from the following example.

EXAMPLE: 45

Let X = {a,b}, A = $\{e_1,e_2\}$ and Let τ= $\{(F,A)_1, (F,A)_5, (F,A)_7, (F,A)_8, (F,A)_{16}$.
The soft sets over X are defined as in example 20.
Now $\tilde{s}GSCS(X,\tau,E) = \tilde{s}G^{\#}\alpha CS(X,\tau,E) = \{(F,A)_1,(F,A)_2, (F,A)_3, (F,A)_4, (F,A)_9, (F,A)_{10}, (F,A)_{11}, (F,A)_{12}, (F,A)_{13}, (F,A)_{14}, (F,A)_{15}, (F,A)_{16}\}$. But $\tilde{s}G^*CS(X,\tau,E) = \{(F,A)_1, (F,A)_9, (F,A)_{10}, (F,A)_{11}, (F,A)_{12}, (F,A)_{13}, (F,A)_{14}, (F,A)_{15}, (F,A)_{16}\}$.
Hence X is a soft T^c space but not T^d space.

EXAMPLE: 46

Let X = {a,b}, A = $\{e_1,e_2\}$ and Let τ= $\{(F,A)_1, (F,A)_2, (F,A)_7, (F,A)_8, (F,A)_{16}\}$.
The soft sets over X are defined as in example 20.
Now $\tilde{s}G^{\#}\alpha CS(X,\tau,E) = \tilde{s}G^*CS(X,\tau,E) = \{(F,A)_1,(F,A)_9, (F,A)_{10}, (F,A)_{11}, (F,A)_{12}, (F,A)_{16}\}$. But $\tilde{s}GSCS(X,\tau,E) = \{(F,A)_1,(F,A)_2, (F,A)_3, (F,A)_5, (F,A)_7, (F,A)_9, (F,A)_{10}, (F,A)_{11}, (F,A)_{12}, (F,A)_{13}, (F,A)_{14}, (F,A)_{15}, (F,A)_{16}\}$.
Hence X is a soft T^d space but not soft T^c space.

DEFINITION: 47

Let (X,τ,E) be a soft topological space over X and x,y $\tilde{\in}$ X such that x ≠ y. If there exist soft $g^{\#}\alpha$-open sets (F,A) and (G,A) such that x $\tilde{\in}$ (F,A) and y $\tilde{\notin}$ (F,A) or y $\tilde{\in}$ (G,A) and x $\tilde{\notin}$ (G,A), then (X,τ,E) is called a soft $g^{\#}\alpha_0$-space.

DEFINITION: 48

Let (X,τ,E) be a soft topological space over X and x,y $\tilde{\in}$ X such that x ≠ y. If there exist soft $g^{\#}\alpha$-open sets (F,A) and (G,A) such that x $\tilde{\in}$ (F,A) and y $\tilde{\notin}$ (F,A) and y $\tilde{\in}$ (G,A) and x $\tilde{\notin}$ (G,A), then (X,τ,E) is called a soft $g^{\#}\alpha_1$-space.

DEFINITION: 49

Let (X,τ,E) be a soft topological space over X and x,y $\tilde{\in}$ X such that x ≠ y. If there exist soft $g^{\#}\alpha$-open sets (F,A) and (G,A) such that x $\tilde{\in}$ (F,A) and y $\tilde{\in}$ (G,A) and (F,A) $\tilde{\cap}$ (G,A) = Φ, then (X,τ,E) is called a soft $g^{\#}\alpha_2$-space.

THEOREM: 50

Every soft $g^{\#}\alpha_1$-space is soft $g^{\#}\alpha_0$-space.

PROOF:

Let (X,τ,E) be a soft topological space over X and x,y $\tilde{\in}$ X such that x $\neq$ y. If there exist soft g-open sets (F,A) and (G,A) such that x $\tilde{\in}$ (F,A) and y $\tilde{\notin}$ (F,A) and y $\tilde{\in}$ (G,A) and x $\tilde{\notin}$ (G,A), which implies soft $g^{\#}\alpha_0$-space.

The converse need not be true as shown by the following example.

EXAMPLE: 51

Let X = {a,b} and x = {a}, y = {b}, A = {e_1,e_2} and τ = {$(F,A)_1$, $(F,A)_5$, $(F,A)_7$, $(F,A)_8$, $(F,A)_{16}$}. The soft sets over X are defined as in example 20. Now let us take soft $g^{\#}\alpha$-open sets $(F,A)_5$ and $(F,A)_7$. By the definition, x $\tilde{\in}$ $(F,A)_5$ and y $\tilde{\notin}$ $(F,A)_5$. Then (X,τ,E) is called soft $g^{\#}\alpha_0$-space but not soft $g^{\#}\alpha_1$-space.

THEOREM: 52

Every soft T_0-space is soft $g^{\#}\alpha_0$-space.

PROOF:

For a soft T_0-space, there exist two distinct points x,y such that x $\tilde{\in}$ (F,A) and y $\tilde{\notin}$ (F,A), where (F,A) is soft open. Every soft open is soft $g^{\#}\alpha$-open. Hence which is soft $g^{\#}\alpha_0$-space.

The converse need not be true as shown by the following example.

EXAMPLE: 53

Let X = {a,b} and x = {a}, y = {b}, A = {e_1,e_2} and τ = {$(F,A)_1$, $(F,A)_4$, $(F,A)_8$, $(F,A)_{16}$}. The soft sets over X are defined as in example 20. Now let us take soft $g^{\#}\alpha$-open sets $(F,A)_2$ and $(F,A)_9$. By the definition x $\tilde{\in}$ $(F,A)_2$ and y $\tilde{\notin}$ $(F,A)_2$. Then (X,τ,E) is called soft $g^{\#}\alpha_0$-space but not soft T_0-space.

THEOREM: 54

Every soft T_1-space is soft $g^{\#}\alpha_0$-space.

PROOF:

For a soft T_1-space, there exist two distinct points x,y such that x $\tilde{\in}$ (F,A) and y $\tilde{\notin}$ (F,A) and y $\tilde{\in}$ (G,A) and x $\tilde{\notin}$ (G,A), where (F,A) is soft open. Every soft open is soft $g^{\#}\alpha$-open. Hence it is soft $g^{\#}\alpha_1$-space.

The converse need not be true as shown by the following example

EXAMPLE: 55

Let X = {a,b} and x = {a}, y = {b}, A = {e_1,e_2} and τ = {$(F,A)_1$, $(F,A)_4$, $(F,A)_8$, $(F,A)_{16}$}. The soft sets over X are defined as in example 20. Now let us take soft $g^{\#}\alpha$-open sets $(F,A)_5$ and $(F,A)_3$. By the definition x $\tilde{\in}$ $(F,A)_5$ and y $\tilde{\notin}$ $(F,A)_5$. Then (X,τ,E) is called soft $g^{\#}\alpha_0$-space but not soft T_1-space.

THEOREM: 56

Let (X,τ,E) be a soft topological space over X . If (x,E) is a soft $g^{\#}\alpha$-closed set in (X,τ,E), for x $\tilde{\in}$ X, then (X,τ,E) is a soft $g^{\#}\alpha_1$-space.

PROOF:

Since (x,E) is a soft $g^{\#}\alpha$-closed set in (X,τ,E), then $(x,E)^c$ is a soft $g^{\#}\alpha$-open set. Let x,y $\tilde{\in}$ X such that x $\neq$ y. For x $\tilde{\in}$ X, $(x,E)^c$ is a soft $g^{\#}\alpha$-open set such that y $\tilde{\in}$ $(x,E)^c$ and x $\tilde{\notin}$ $(x,E)^c$. Similarly $(y,E)^c$ is a soft $g^{\#}\alpha$-open set such that x $\tilde{\in}$ $(y,E)^c$ and y $\tilde{\notin}$ $(y,E)^c$. Thus (X,τ,E) is a soft $g^{\#}\alpha_1$-space over X.

THEOREM: 57

A soft topological space X is a soft $g^{\#}\alpha_1$-space if and only if each singleton set is soft $g^{\#}\alpha$- closed set in X.

PROOF:

Let x $\tilde{\in}$ X, since X is a soft $g^{\#}\alpha_1$-space, for each x $\neq$ y in X. We can choose a soft $g^{\#}\alpha$-open nbd (X,E) of y such that x $\tilde{\notin}$ (X,E). The union of all these $g^{\#}\alpha$-open sets is $g^{\#}\alpha$-open and is the complement of {x} in X. Hence {x} is soft $g^{\#}\alpha$-open.

Conversely, suppose that each singleton set is soft $g^{\#}\alpha$-open in X. Let x,y $\tilde{\in}$ X with x $\neq$ y. Then X-{x} is a soft $g^{\#}\alpha$-open set containing y but not x. Similarly X-{y} is a soft $g^{\#}\alpha$-open set containing x but not y. Thus X is soft $g^{\#}\alpha_1$-space.

V.SOFT $g^{\#}\alpha$-CONTINUOUS FUNCTIONS

In this section, we introduce the concept of soft $g^{\#}\alpha$-continuous functions and soft $g^{\#}\alpha$-irresolute functions and discuss some of their properties.

DEFINITION: 58

A function f: (X,τ,E) $\rightarrow$ (Y,γ,K) is called soft $g^{\#}\alpha$-continuous if the inverse image of each soft open set in Y is a soft $g^{\#}\alpha$-open set in X.

THEOREM: 59

Every soft α-continuous function is soft $g^{\#}\alpha$-continuous.

PROOF:

Let f: (X,τ,E) $\rightarrow$ (Y,γ,K) be a soft α-continuous function. Let (G,B) be a soft open in (Y,γ,K). Since f is soft α-continuous, f^{-1}(G,B) is soft α-open in (X,τ,E). Since every soft α-open is soft $g^{\#}\alpha$-open. Thus f^{-1}(G,B) is soft $g^{\#}\alpha$-open in (X,τ,E). Hence f is soft $g^{\#}\alpha$-continuous.

The converse need not be true as shown by the following example

EXAMPLE : 60

Let X = {a,b}, Y = {r,s}, E = {e_1,e_2,e_3} and K = {k_1,k_2,k_3}, A = {e_1,e_2}, B = {k_1,k_2}. Let (X,τ,E) and (Y,γ,K) be soft topological spaces. Define u : X → Y and p : E → K as u(a) = r , u(b) = s and p(e_1) = k_1, p(e_2) = k_2, p(e_3) = k_3. Let us consider τ = {$(F,A)_1$, $(F,A)_5$, $(F,A)_7$, $(F,A)_8$, $(F,A)_{16}$}. Also consider γ = {$(G,B)_1$, $(G,B)_4$, $(G,B)_{16}$}.
Let f: (X,τ,E) → (Y,γ,K) be a soft mapping. Then inverse images of soft open sets in Y are soft $g^{\#}\alpha$-open in X, but not soft α-open in X. Hence f is soft $g^{\#}\alpha$-continuous but not soft α-continuous.

THEOREM : 61

Every soft continuous function is soft $g^{\#}\alpha$-continuous function.

PROOF :
Let f: (X,τ,E) → (Y,γ,K) be a soft continuous function. Let (G,B) be a soft open set in (Y,γ,K). Since f is soft continuous, f^{-1}((G,B)) is soft open in (X,τ,E). And so f^{-1}((G,B)) is soft $g^{\#}\alpha$-open set in (X,τ,E). Therefore, f is soft $g^{\#}\alpha$-continuous function.

The converse need not be true which is shown below

EXAMPLE : 62

Let X = {a,b}, Y = {r,s}, E = {e_1,e_2, e_3}, and K = {k_1,k_2, k_3}, A = {e_1,e_2}, B = {k_1,k_2}. Let (X,τ,E) and (Y,γ,K) be soft topological spaces. Define u : X → Y and p : E → K as u(a) = r , u(b) = s and p(e_1) = k_1, p(e_2) = k_2, p(e_3) = k_3. Let us consider τ = {$(F,A)_1$, $(F,A)_5$, $(F,A)_7$, $(F,A)_8$, $(F,A)_{16}$}.Also consider γ = {$(G,B)_1$, $(G,B)_6$, $(G,B)_{16}$}.
Let f: (X,τ,E) → (Y,γ,K) be a soft mapping.Then inverse images of soft open sets in Yare soft $g^{\#}\alpha$-open in X, but not soft open in X. Hence f is a soft $g^{\#}\alpha$-continuous function but not soft continuous function.

DEFINITION : 63

A mapping f : (X,τ,E) → (Y,γ,K) is soft $g^{\#}\alpha$-irresolute mapping if the inverse image of every soft $g^{\#}\alpha$-open set in (Y,γ,K) is soft $g^{\#}\alpha$-open set in (X,τ,E).

THEOREM : 64

Every soft $g^{\#}\alpha$-irresolute mapping is soft $g^{\#}\alpha$-continuous mapping.

PROOF :
Let f: (X,τ,E) → (Y,γ,K) is soft $g^{\#}\alpha$-irresolute mapping. Let (G,B) be a soft closed set in (Y,γ,K), then (G,B) is soft $g^{\#}\alpha$-closed set in (Y,γ,K). Since f is soft $g^{\#}\alpha$-irresolute mapping, f^{-1}((G,B)) is a soft $g^{\#}\alpha$-closed set in (X,τ,E). Hence f is soft $g^{\#}\alpha$-continuous mapping.

EXAMPLE : 65

Let X = {a,b}, Y = {r,s}, E = {e_1,e_2, e_3}, and K = {k_1,k_2, k_3}, A = {e_1,e_2}, B = {k_1,k_2}. Let (X,τ,E) and (Y,γ,K) be soft topological spaces. Define u : X → Y and p : E → K as u(a) = r , u(b) = s and p(e_1) = k_1, p(e_2) = k_2, p(e_3) = k_3. Let us consider τ = {$(F,A)_1$, $(F,A)_3$, $(F,A)_5$, $(F,A)_7$, $(F,A)_{11}$, $(F,A)_{12}$, $(F,A)_{15}$, $(F,A)_{16}$}. Also consider γ = {$(G,B)_1$, $(G,B)_5$, $(G,B)_7$, $(G,B)_8$, $(G,B)_{16}$}
Let f: (X,τ,E) → (Y,γ,K) be a soft mapping.
Then the inverse images of soft open sets of Y are soft $g^{\#}\alpha$-open in X and the inverse images of soft $g^{\#}\alpha$-open sets of Y are not soft $g^{\#}\alpha$-open in X. Hence f is a soft $g^{\#}\alpha$-continuous, but not soft $g^{\#}\alpha$-irresolute function.

THEOREM : 66

Let f: (X,τ,E) → (Y,γ,K), g: (Y,γ,K) → (Z,σ,T) be two functions. Then
(i) $g \circ f$: (X,τ,E) → (Z,σ,T) is soft $g^{\#}\alpha$-continuous, if f is soft $g^{\#}\alpha$-continuous and g is soft continuous.
(ii) $g \circ f$: (X,τ,E) → (Z,σ,T) is soft $g^{\#}\alpha$-irresolute, if f and g is soft $g^{\#}\alpha$-irresolute functions.
(iii) $g \circ f$: (X,τ,E) → (Z,σ,T) is soft $g^{\#}\alpha$-continuous if f is soft $g^{\#}\alpha$-irresolute and g is soft $g^{\#}\alpha$-continuous.

PROOF :

i. Let (F,A) be soft closed set of (Z,σ,T). Since g: (Y,γ,K) → (Z,σ,T) is soft continuous, by definition is g^{-1}((H,T)) soft closed set of (Z,σ,T). Now f: (X,τ,E) → (Y,γ,K) is soft $g^{\#}\alpha$-continuous and g^{-1}((H,T)) is soft closed set of (Y,γ,K) , so by the definition, $f^{-1}(g^{-1}$((H,T))) = $(g \circ f)^{-1}$((H,T)) is soft $g^{\#}\alpha$-closed in (X,τ,E).Hence $g \circ f$: (X,τ,E) → (Z,σ,T) is soft $g^{\#}\alpha$-continuous.
ii. Let : (Y,γ,K) → (Z,σ,T) is soft $g^{\#}\alpha$-irresolute and let (H,T) be soft $g^{\#}\alpha$-closed set of (Z,σ,T). Since g is soft $g^{\#}\alpha$-irresolute by the definition, g^{-1}((H,T)) is $\tilde{s}$α-closed set of (Y,γ,K). Also f: (X,τ,E) → (Y,γ,K) is soft $g^{\#}\alpha$-irresolute, so $f^{-1}(g^{-1}$((H,T))) = $(g \circ f)^{-1}$ ((H,T)) is soft $g^{\#}\alpha$-closed.
Thus, $g \circ f$: (X,τ,E) → (Z,σ,T) is soft $g^{\#}\alpha$-irresolute.
iii. Let (H,T) be soft $g^{\#}\alpha$-closed set of (Z,σ,T). Since g : (Y,γ,K) → (Z,σ,T) is $\tilde{s}$α-continuous, g^{-1}((H,T)) is $\tilde{s}$α-closed set of (Y,γ,K). Also f: (X,τ,E) → (Y,γ,K) is

soft $g^{\#}\alpha$-irresolute, so every soft $g^{\#}\alpha$-closed set of (Y,γ,K) is soft$g^{\#}\alpha$-closed in (X,τ,E). Therefore, $f^{-1}(g^{-1}$((H,T))) = $(g \circ f)^{-1}$((H,T)) is soft $g^{\#}\alpha$-closed set of (X,τ,E). Thus, is soft $g^{\#}\alpha$-continuous.

VI. CONCLUSION

In this paper, we have introduced soft $g^{\#}$α-closed sets and some of their properties in soft topological spaces. Moreover, we introduced soft $g^{\#}$α-separation axioms soft T^a, soft T^b, soft T^c, soft T^d spaces and soft $g^{\#}$α-continuous functions and soft $g^{\#}$α-irresolute functions were defined.

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