SOFT #gα-CLOSED SETS AND SOFT #gα-CONTINUOUS FUNCTIONS

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Abstract **− We introduce a new class of soft sets namely soft #gα-closed sets and study some of their properties in soft topological spaces. We introduce a new space namely soft $T^{\#a}$ space using soft #gα-closed sets. We also investigate the concept of soft #gα-continuous functions and soft #gα-irresolute functions.**

Keywords **−soft #gα-closed set, soft #gα-continuous function, soft #gα-irresolute function, soft $T^{\#a}$ space.**

I. INTRODUCTION

Russian researcher Molodtsov [16] (1999) introduced the concept of soft set theory, as a general mathematical tool for dealing with uncertain objects. A soft set gives an approximate description of an object in universe set X and we can use any parameters including words, real number and so on which we prefer. He presented the fundamental results of soft set theory and applied it in various directions such as game theory, operations research, theory of probability etc.

Further, Soft set theory was investigated from different aspects. These studies were started by Maji et al. in 2000.They discussed theoretical aspects and practical applications of soft sets in decision making problems.

In 2001, Maji et al. [11] introduced the concept of fuzzy soft set and the basic properties of fuzzy soft sets. In 2002, Maji et al. [12] applied soft sets to solve the problems in decision making. Then they introduced some new operations in soft set theory in 2003 [13]. In 2007, Aktas and Cagman [3] introduced the basic properties of soft sets and also they gave the notion of soft group and derived some basic properties of those sets.

Similarity measure in soft set theory was studied by Majumdar and Samanta [10] in 2008, Ali et al. in 2009. In 2010, Shabir and Naz [17] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. They also introduced and investigated the notions of soft open sets, soft closed sets, soft closure, soft interior, soft neighbourhood of a point and soft separation axioms.

Athar Kharal and Ahmad [2] defined the notion of a mapping on soft classes and studied several properties of images and inverse images of soft sets.

Kannan [7] introduced and studied the properties of soft g-closed sets and the concepts of soft $T^{1/2}$ spaces. Akdag and Ozkan [18] defined soft α-open (resp. soft α-closed) sets. Mahanta and Das [9] introduced and characterized various forms of soft functions. Arockiarani and Lancy [1] introduced soft gβ-closed sets and soft gsβ-closed sets in soft topological spaces. Devi.R, Maki.H and Kokilavani.V [5] introduced the concept of #gα-closed sets in Topological spaces.

In this paper, we introduce a new class of soft sets namely soft #gα-closed sets in Soft topological spaces using the concept of #gα-closed sets in Topological spaces [6] and we study some basic properties of soft #gα-closed sets. Also we introduce a new space namely soft $T^{\#a}$ space. We also introduce the concept of soft #gα-continuous functions and soft #gα-irresolute functions and examine their characteristics.

Throughout this paper, the spaces X and Y stand for soft topological spaces with ((X, τ, E) and (Y, υ, K)) assumed unless otherwise stated. Moreover, the soft mapping $f : X \longrightarrow Y$ stands for a mapping where $f : (X, \tau, E) \longrightarrow (Y, \upsilon, K)$, $u : X \longrightarrow Y$, $p : E \longrightarrow K$ are assumed mappings unless otherwise stated. Soft operations are denoted by usual set theoretical operations with '~' symbol above.

The family of all soft #gα-closed sets (resp. soft α-closed sets, soft αg-closed sets and soft gs-closed sets) is denoted by $\tilde{s}^{\#}$GαCS(X,τ,E), (resp. $\tilde{s}$αCS(X,τ,E), $\tilde{s}$αGCS(X,τ,E) and $\tilde{s}$GSCS(X,τ,E)) and the family of all soft #gα-open sets (resp. soft α-open sets, soft αg-open sets and soft gs-open sets) is denoted by $\tilde{s}^{\#}$GαOS(X,τ,E),(resp.$\tilde{s}$αOS(X,τ,E), $\tilde{s}$αGOS(X,τ,E) and $\tilde{s}$GSOS(X,τ,E).

II. PRELIMINARIES

Definition 1 [4]:

Let X be an initial universe set and E be a collection of all possible parameters with respect to X, where parameters are the characteristics or properties of objects in X. We denote the universe set of parameters with respect to X by E. A pair (F,A) is called a soft set over X if A is a non-empty subset of E and F : A → P(X), where P(X) is the set of all subsets of X.

Definition 2 [14]:

A soft set (F,A) of a soft topological space (X,τ,E) is said to be

(1) soft open if its complement is soft closed.

(2) soft α-open if (F,A) ⊆̃ $\tilde{s}$int($\tilde{s}$cl($\tilde{s}$int((F,A)))), soft α-closed if $\tilde{s}$cl($\tilde{s}$int($\tilde{s}$cl((F,A)))) ⊆̃ (F,A).

(3) soft pre-open if (F,A) ⊆̃ $\tilde{s}$int($\tilde{s}$cl((F,A))), soft pre-closed if $\tilde{s}$cl($\tilde{s}$int((F,A))) ⊆̃ (F,A).

(4) soft semi-open if (F,A) ⊆̃ $\tilde{s}$cl($\tilde{s}$int((F,A))), soft semi-closed if $\tilde{s}$int($\tilde{s}$cl((F,A))) ⊆̃ (F,A).

Definition 3:

A soft set (A,E) of a soft topological space (X,τ,E) is called

(1) a soft g-closed set [7] if $\tilde{s}$cl(A,E) ⊆̃ (U,E), whenever (A,E) ⊆̃ (U,E) and (U,E) is soft open in X.

(2) a soft αg-closed set [1] if $\tilde{s}$αcl(A,E) ⊆̃ (U,E), whenever (A,E) ⊆̃ (U,E) and (U,E) is soft open in X.

(3) a soft gs-closed set [1] if $\tilde{s}$scl(A,E) ⊆̃ (U,E), whenever (A,E) ⊆̃ (U,E) and (U,E) is soft open in X.

(4) a soft $g^{\#}$α-closed set [8] if $\tilde{s}$αcl(A,E) ⊆̃ (U,E), whenever (A,E) ⊆̃ (U,E) and (U,E) is soft g-open in X.

Definition 4 [6]:

Let (X,τ,E) be a soft topological space over X and x,y ∈̃ X such that x and y are soft distinct points. Then (X,τ,E) is called

(1) soft T_0 – space if there exist soft open sets (F,A) and (G,A) such that x ∈̃ (F,A) and y ∉̃ (F,A) or y ∈̃ (G,A) and x ∉̃ (G,A).

(2) soft T_1 – space if there exist soft open sets (F,A) and (G,A) such that x ∈̃ (F,A) and y ∉̃ (F,A) and y ∈̃ (G,A) and x ∉̃ (G,A).

(3) soft T_2 – space if there exist soft open sets (F,A) and (G,A) such that x ∈̃ (F,A) and y ∈̃ (G,A) and (F,A) ∩̃ (G,A) = Φ.

Definition 5 [14]:

A soft mapping f : X → Y is said to be

(1) soft continuous if the inverse image of each soft open set in Y is a soft open set in X.

(2) soft α-continuous if the inverse image of each soft open set in Y is a soft α-open set in X.

(3) soft $g^{\#}$α-continuous if the inverse image of each soft open set in Y is a soft $g^{\#}$α-open set in X.

III. SOFT $^{\#}$gα-CLOSED SETS

Definition 6:

A soft set (A,E) is called soft $^{\#}$gα-closed in a soft topological space (X,τ,E), if $\tilde{s}$αcl(A,E) ⊆̃ (U,E), whenever (A,E) ⊆̃ (U,E) and (U,E) is soft $g^{\#}$α-open in X.

Theorem 7: Every soft closed set is soft $^{\#}$gα-closed.

Proof:

Let (U,E) be soft $g^{\#}$α-open in X and let (A,E) be a soft closed set such that (A,E) ⊆̃ (U,E). For every soft closed set, $\tilde{s}$cl(A,E) = (A,E). Since $\tilde{s}$αcl(A,E) ⊆̃ $\tilde{s}$cl(A,E) and $\tilde{s}$cl(A,E) = (A,E) ⊆̃ (U,E). We have $\tilde{s}$αcl(A,E) ⊆̃ (U,E). Hence (A,E) is soft $^{\#}$gα-closed.

Converse of the above theorem need not be true as shown in the following example.

Example 8: Let X = {a,b}, A = {e_1,e_2}.Then the soft sets over X are defined as follows

$(F,A)_1$ = {(e_1,∅),(e_2,∅)}
$(F,A)_2$ = {(e_1,∅),(e_2,{a})}
$(F,A)_3$ = {(e_1,∅),(e_2,{b})}
$(F,A)_4$ = {(e_1,∅),(e_2,{a,b})}
$(F,A)_5$ = {(e_1,{a}),(e_2,∅)}
$(F,A)_6$ = {(e_1,{a}),(e_2,{a})}
$(F,A)_7$ = {(e_1,{a}),(e_2,{b})}
$(F,A)_8$ = {(e_1,{a}),(e_2,{a,b})}
$(F,A)_9$ = {(e_1,{b}),(e_2,∅)}
$(F,A)_{10}$ = {(e_1,{b}),(e_2,{a})}
$(F,A)_{11}$ = {(e_1,{b}),(e_2,{b})}
$(F,A)_{12}$ = {(e_1,{b}),(e_2,{a,b})}
$(F,A)_{13}$ = {(e_1,{a,b}),(e_2,∅)}
$(F,A)_{14}$ = {(e_1,{a,b}),(e_2,{a})}
$(F,A)_{15}$ = {(e_1,{a,b}),(e_2,{b})}
$(F, A)_{16}$ = {(e_1,{a,b}),(e_2,{a,b})}

Let τ = {$(F,A)_1$, $(F,A)_6$, $(F,A)_{16}$}.A soft subset $(F,A)_{15}$ is soft $^{\#}$gα-closed in (X, τ ,E) but not soft closed.

Theorem 9: Every soft α-closed set is soft $^{\#}$gα-closed.

Proof:

Let (U,E) be soft $g^{\#}$α-open in X and let (A,E) be a soft α-closed set, such that (A,E) ⊆̃ (U,E). For every soft α-closed set, $\tilde{s}$αcl(A,E) = (A,E).Thus we have $\tilde{s}$αcl(A,E) = (A,E) ⊆̃ (U,E). Hence $\tilde{s}$αcl(A,E) ⊆̃ (U,E). Thus, (A,E) is soft $^{\#}$gα-closed.

Converse of the above theorem need not be true by the following example.

Example 10: Let X = {a,b} , A = {e_1,e_2}.Then the soft sets over X are defined as in example 8. Let τ = {$(F,A)_1$, $(F,A)_6$, $(F,A)_{16}$}.A soft subset $(F,A)_{15}$ is soft $^{\#}$gα-closed in (X, τ ,E) but not soft α-closed.

Theorem 11: Every soft $^{\#}$gα-closed set is soft αg-closed.

Proof:

Let (U,E) be soft open in X and let (A,E) be a soft $^{\#}$gα-closed set, such that (A,E) ⊆̃ (U,E). Since (A,E) is soft $^{\#}$gα-closed, $\tilde{s}$αcl (A,E) ⊆̃ (U,E). Hence (A,E) is soft αg-closed.

Converse of the above theorem need not be true by the following example.

Example 12: Let X = {a,b} , A = $\{e_1,e_2\}$. Then the soft sets over X are defined as in example 8. Let τ = $\{(F,A)_1, (F,A)_6, (F,A)_{16}\}$. A soft subset $(F,A)_4$ is soft αg -closed in (X, τ ,E) but not soft $^{\#}$gα-closed.

Theorem 13: Every soft $^{\#}$gα-closed set is soft gs-closed.

Proof:

Let (U,E) be soft open in X and let (A,E) be a soft $^{\#}$gα-closed set, such that (A,E) $\tilde{\subseteq}$ (U,E). Since (A,E) is soft $^{\#}$gα-closed, $\tilde{s}\alpha cl$(A,E) $\tilde{\subseteq}$ (U,E). We have, $\tilde{s}scl$(A,E) $\tilde{\subseteq}$ $\tilde{s}\alpha cl$(A,E). Thus, $\tilde{s}scl$(A,E) $\tilde{\subseteq}$ (U,E). Hence (A,E) is soft gs-closed.

The following example shows that converse of the above theorem need not be true.

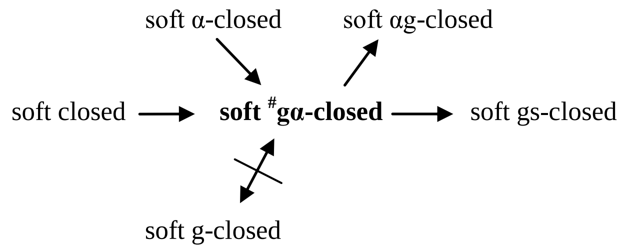
Example 14: Let X = {a,b} , A = $\{e_1,e_2\}$. Then the soft sets over X are defined as in example 8. Let τ = $\{(F,A)_1, (F,A)_6, (F,A)_{16}\}$. A soft subset $(F,A)_4$ is soft gs-closed in (X, τ ,E) but not soft $^{\#}$gα-closed.

Remark 15: The concepts of soft g-closed sets and soft $^{\#}$gα-closed sets are independent to each other.

Let τ = $\{(F,A)_1, (F,A)_5, (F,A)_7, (F,A)_8, (F,A)_{16}\}$.Then the soft g-closed sets of X are $(F,A)_1$, $(F,A)_9$, $(F,A)_{10}$, $(F,A)_{11}$, $(F,A)_{12}$, $(F,A)_{13}$, $(F,A)_{14}$, $(F,A)_{15}$, $(F,A)_{16}$.Soft $^{\#}$gα-closed sets of X are $(F,A)_1$, $(F,A)_2$, $(F,A)_3$, $(F,A)_4$, $(F,A)_9$, $(F,A)_{10}$, $(F,A)_{11}$, $(F,A)_{12}$, $(F,A)_{16}$.

Here the soft set $(F,A)_{13}$ is soft g-closed but not soft $^{\#}$gα-closed and the soft set $(F,A)_2$ is soft $^{\#}$gα-closed but not soft g-closed.

The following implication is made from the above results:



Theorem 16: If (A,E) and (B,E) are soft $^{\#}$gα-closed sets then (A,E) $\tilde{\cup}$ (B,E) is soft $^{\#}$gα-closed.

Proof:

Suppose that (A,E) and (B,E) are soft $^{\#}$gα-closed sets and (U,E) be soft g$^{\#}$α-open such that (A,E) $\tilde{\cup}$ (B,E) $\tilde{\subseteq}$ (U,E). Since (A,E) $\tilde{\cup}$ (B,E) $\tilde{\subseteq}$ (U,E),we have (A,E) $\tilde{\subseteq}$ (U,E) and (B,E) $\tilde{\subseteq}$ (U,E). Since (U,E) is soft g$^{\#}$α-open and (A,E) and (B,E) are soft $^{\#}$gα-closed sets, we have $\tilde{s}\alpha cl$(A,E) $\tilde{\subseteq}$ (U,E) and $\tilde{s}\alpha cl$(B,E) $\tilde{\subseteq}$ (U,E). Therefore,
$\tilde{s}\alpha cl$((A,E) $\tilde{\cup}$ (B,E)) = $\tilde{s}\alpha cl$(A,E) $\tilde{\cup}$ $\tilde{s}\alpha cl$(B,E) $\tilde{\subseteq}$ (U,E).

Remark 17: The intersection of two soft $^{\#}$gα-closed sets need not be soft $^{\#}$gα-closed.

Theorem 18: If (A,E) is soft $^{\#}$gα-closed in X and (A,E) $\tilde{\subseteq}$ (B,E) $\tilde{\subseteq}$ $\tilde{s}\alpha cl$(A,E), then (B,E) is soft $^{\#}$gα closed.

Proof:

Suppose that (A,E) is soft $^{\#}$gα-closed in X and (A,E) $\tilde{\subseteq}$ (B,E) $\tilde{\subseteq}$ $\tilde{s}\alpha cl$(A,E) where (B,E) is soft $^{\#}$gα-closed. Let (B,E) $\tilde{\subseteq}$ (U,E) and (U,E) is soft g$^{\#}$α-open in X. Since (A,E) $\tilde{\subseteq}$ (B,E) and (B,E) $\tilde{\subseteq}$ (U,E), we have (A,E) $\tilde{\subseteq}$ (U,E). Hence $\tilde{s}\alpha cl$(A,E) $\tilde{\subseteq}$ (U,E) as (A,E) is soft $^{\#}$gα-closed. Thus, we have $\tilde{s}\alpha cl$(B,E) $\tilde{\subseteq}$ $\tilde{s}\alpha cl$(A,E) $\tilde{\subseteq}$ (U,E). Therefore, (B,E) is soft $^{\#}$gα-closed.

Theorem 19: If a set (A,E) is soft $^{\#}$gα-closed in X then $\tilde{s}\alpha cl$(A,E) \ (A,E) contains only null soft $^{\#}$gα-closed set.

Proof:

Suppose that (A,E) is soft $^{\#}$gα-closed in X. Let (F,E) be soft g$^{\#}$α-closed such that (F,E) $\tilde{\subseteq}$ $\tilde{s}\alpha cl$(A,E) \ (A,E).

Since (F,E) $\tilde{\subseteq}$ $\tilde{s}\alpha cl$(A,E) \ (A,E), we have (F,E) $\tilde{\subseteq}$ $\tilde{s}\alpha cl$(A,E) and (F,E) $\tilde{\subseteq}$ $(A,E)^c$. Hence (A,E) $\tilde{\subseteq}$ $(F,E)^c$, also $\tilde{s}\alpha cl$(A,E) $\tilde{\subseteq}$ $(F,E)^c$ as (A,E) is soft $^{\#}$gα-closed. Therefore, (F,E) $\tilde{\subseteq}$ $\tilde{s}\alpha cl(A,E)^c$ and it is clear that (F,E) = Φ. Hence, $\tilde{s}\alpha cl$(A,E) \ (A,E) contains only null soft $^{\#}$gα-closed set.

Theorem 20: If (A,E) is soft g$^{\#}$α-open and soft $^{\#}$gα-closed then (A,E) is soft α-closed.

Proof:

Suppose (A,E) is soft g$^{\#}$α-open and soft $^{\#}$gα-closed then by the definition, $\tilde{s}\alpha cl$(A,E) $\tilde{\subseteq}$ (A,E).But (A,E) $\tilde{\subseteq}$ $\tilde{s}\alpha cl$(A,E) as every soft α-closed set is soft $^{\#}$gα-closed. Thus,(A,E) = $\tilde{s}\alpha cl$(A,E).Therefore, (A,E) is soft α-closed.

IV. NEW SOFT SEPARATION AXIOMS

Definition 21:

A soft topological space (X, τ, E) is said to be soft $T^{\#a}$ space if every soft $^{\#}$gα-closed set is soft α-closed.

***Theorem 22*:** For a soft topological space (X, τ, E), the following conditions are equivalent.

1. X is a soft $T^{\#a}$ space.
2. Every singleton of X is soft g$^{\#}$α-closed or soft α-open.
3. Every singleton of X is soft g$^{\#}$α-closed or soft open.

Proof:

(1) ⟹ (2): Let x $\tilde{\in}$ X. Suppose that {x} is not soft g$^{\#}$α-closed set of (X, τ, E).Then X \ {x} is not soft g$^{\#}$α-open.

Thus X is the only soft $g^{\#}\alpha$-open set containing X \ {x}. Hence X \ {x} is soft $^{\#}g\alpha$-closed set of (X, τ, E). By (1), X \ {x} is soft α-closed set of (X,τ,E) or equivalently {x} is soft α-open.

Therefore {x} is soft $g^{\#}\alpha$-closed or soft α-open.

(2) ⟹ (3): Since we know that {x} is soft α-open if and only if {x} is soft open and hence the proof follows.

(3) ⟹ (2): Since {x} is soft open ⟹{x} is soft α-open, proof is obvious.

(3) ⟹ (1):Let (A,E) be a soft $^{\#}g\alpha$-closed set. Trivially (A,E) $\tilde{\subseteq}$ $\tilde{s}\alpha cl$(A,E).

Let x $\tilde{\in}$ $\tilde{s}\alpha cl$(A,E). By (2) {x} is either $g^{\#}\alpha$-closed or soft α-open.

We consider the following two cases:

Case (1): Let {x} be soft $g^{\#}\alpha$-closed. If a set (A,E) is soft $^{\#}g\alpha$-closed in X then $\tilde{s}\alpha cl$(A,E) \ (A,E) contains only null soft $^{\#}g\alpha$-closed set. If x $\tilde{\notin}$ (A,E), then $\tilde{s}\alpha cl$(A,E) \ (A,E) contains a nonempty soft $g^{\#}\alpha$-closed set of {x} which is a contradiction. Hence (A,E) is soft $^{\#}g\alpha$-closed set and x $\tilde{\in}$ (A,E).

Case (2): Let {x} be soft α-open. Since x $\tilde{\in}$ $\tilde{s}\alpha cl$(A,E), {x} $\tilde{\cap}$ (A,E) ≠ Φ. Hence x $\tilde{\in}$(A,E) which implies $\tilde{s}\alpha cl$(A,E) $\tilde{\subseteq}$ (A,E).Thus $\tilde{s}\alpha cl$(A,E) = (A,E).Therefore (A,E) is soft α-closed. Hence X is a soft $T^{\#a}$ space.

Definition 23:
Let (X,τ,E) be a soft topological space over X and x,y $\tilde{\in}$ X such that x and y are soft distinct points. If there exist soft $^{\#}g\alpha$-open sets (F,A) and (G,A) such that x $\tilde{\in}$(F,A) and y $\tilde{\notin}$ (F,A) or y $\tilde{\in}$ (G,A) and x $\tilde{\notin}$ (G,A) then (X,τ,E) is called soft $^{\#}g\alpha_0$ -space.

Definition 24:
Let (X,τ,E) be a soft topological space over X and x,y $\tilde{\in}$ X such that x and y are soft distinct points. If there exist soft $^{\#}g\alpha$-open sets (F,A) and (G,A) such that x $\tilde{\in}$(F,A) and y $\tilde{\notin}$ (F,A) and y $\tilde{\in}$ (G,A) and x $\tilde{\notin}$ (G,A) then (X,τ,E) is called soft $^{\#}g\alpha_1$ -space.

Definition 25:
Let (X,τ,E) be a soft topological space over X and x,y $\tilde{\in}$ X such that x and y are soft distinct points. If there exist soft $^{\#}g\alpha$-open sets (F,A) and (G,A) such that x $\tilde{\in}$(F,A) and y $\tilde{\in}$ (G,A) and (F,A) $\tilde{\cap}$ (G,A) = Φ, then (X,τ,E) is called soft $^{\#}g\alpha_2$ -space.

Theorem 26: Every soft $^{\#}g\alpha_1$ -space is soft $^{\#}g\alpha_0$ -space.

Proof:
Let (X, τ, E) be a soft topological space over X and a,b $\tilde{\in}$ X such that a and b are soft distinct points. If there exist soft $^{\#}g\alpha$-open sets (F,A) and (G,A) such that a $\tilde{\in}$ (F,A) and b $\tilde{\notin}$ (F,A) or b $\tilde{\in}$ (G,A) and a $\tilde{\notin}$ (G,A) which implies soft $^{\#}g\alpha_0$ -space.

The following example shows that converse of the above theorem need not be true

Example 27: Let X = {a,b} , A = $\{e_1,e_2\}$ and the soft sets of X are defined as in example 8.Let τ = $\{(F,A)_1, (F,A)_5, (F,A)_7, (F,A)_8, (F,A)_{16}\}$. Then the soft $^{\#}g\alpha$-open sets over (X, τ, E) are $(F,A)_1$, $(F,A)_5$, $(F,A)_6$, $(F,A)_7$, $(F,A)_8$, $(F,A)_{13}$, $(F,A)_{14}$, $(F,A)_{15}$, $(F,A)_{16}$. Now consider the soft $^{\#}g\alpha$-open sets $(F,A)_5$ and $(F,A)_7$. Then by the definition, a $\tilde{\in}$ $(F,A)_5$ and b $\tilde{\notin}$ $(F,A)_5$ and a,b $\tilde{\in}$ $(F,A)_7$. Hence X is soft $^{\#}g\alpha_0$ –space but not soft $^{\#}g\alpha_1$ –space

Theorem 28: Every soft T_0 -space is soft $^{\#}g\alpha_0$ -space.

Proof:
Let the soft topological space (X, τ, E) be a soft T_0 -space. By the definition, for a soft T_0 -space there exists two soft distinct points such that a $\tilde{\in}$ (F,A) and b $\tilde{\notin}$ (F,A) where (F,A) is soft open, we know that every soft open set is soft $^{\#}g\alpha$-open. Therefore, (X, τ, E) is a soft $^{\#}g\alpha_0$ -space.

Converse of the above theorem need not be true from the following example.

Example 29: Let X = {a,b} , A = $\{e_1,e_2\}$ and the soft sets of X are defined as in example 8. Let τ = $\{(F,A)_1, (F,A)_7, (F,A)_{10}, (F,A)_{16}\}$. Now, consider the soft $^{\#}g\alpha$-open sets $(F,A)_5$ and $(F,A)_7$.Then by the definition, a $\tilde{\in}$ $(F,A)_5$ and b $\tilde{\notin}$ $(F,A)_5$. Hence X is soft $^{\#}g\alpha_0$ –space. Now, consider the soft open sets $(F,A)_7$ and $(F,A)_{10}$ .Here, a,b $\tilde{\in}$ $(F,A)_7$ and a,b $\tilde{\in}$ $(F,A)_{10}$. Hence X is not soft T_0 –space.

Theorem 30: Every soft T_1 -space is soft $^{\#}g\alpha_1$ -space.

Proof:
Let the soft topological space (X, τ, E) be a soft T_1 -space. By the definition, for a soft T_1 -space there exists two soft distinct points such that a $\tilde{\in}$ (F,A) and b $\tilde{\notin}$ (F,A) and b $\tilde{\in}$ (G,A) and a $\tilde{\notin}$ (G,A) where (F,A) and (G,A) are soft open sets, we know that every soft open set is soft $^{\#}g\alpha$-open. Therefore, (X, τ, E) is a soft $^{\#}g\alpha_1$ -space.

Converse of the above theorem need not be true which is shown in the following example.

Example 31: Let X = {a,b} , A = $\{e_1,e_2\}$ and the soft sets of X are defined as in example 8.Let τ = $\{(F,A)_1, (F,A)_7, (F,A)_{10}, (F,A)_{16}\}$.Now, consider the soft $^{\#}g\alpha$-open sets $(F,A)_5$ and $(F,A)_9$. By the definition, a $\tilde{\in}$ $(F,A)_5$ and b $\tilde{\notin}$ $(F,A)_5$ and b $\tilde{\in}$ $(F,A)_9$ and a $\tilde{\notin}$ $(F,A)_9$.Hence X is soft $^{\#}g\alpha_1$ –space. Now, consider the soft open sets $(F,A)_7$ and $(F,A)_{10}$ .Here, a,b $\tilde{\in}$ $(F,A)_7$ and a,b $\tilde{\in}$ $(F,A)_{10}$. Hence X is not soft T_1 –space.

Theorem 32: Let (X, τ, E) be a soft topological space over X. If (x,E) is a soft #gα-closed set in (X, τ, E) for x $\tilde{\in}$ X then (X, τ, E) is a soft #$g\alpha_1$–space.

Proof:

Since (x,E) is a soft #gα-closed in (X, τ, E) then $(x,E)^c$ is a soft #gα-open set. Let x,y $\tilde{\in}$ X such that such that x and y are soft distinct points. For x $\tilde{\in}$ X, $(x,E)^c$ is a soft #gα-open set such that y $\tilde{\in}$ $(x,E)^c$ and x $\tilde{\notin}$ $(x,E)^c$. Similarly $(y,E)^c$ is a soft #gα-open set such that x $\tilde{\in}$ $(y,E)^c$ and y $\tilde{\notin}$ $(y,E)^c$.

Thus (X, τ, E) is a soft #$g\alpha_1$–space.

Theorem 33: A soft topological space X is a soft #$g\alpha_1$–space if and only if each singleton set is soft #gα-closed in X.

Proof:

Let x $\tilde{\in}$ X, since X is a soft #$g\alpha_1$-space for x ≠ y in X, we can choose a soft #gα-open nbhd (U,E) of y such that x $\tilde{\notin}$ (U,E). The union of all these soft #gα-open sets is soft #gα-open and it is the complement of {x}. Hence {x} is soft #gα-closed.

Conversely, suppose that each singleton set is soft #gα-closed in X. Let x,y $\tilde{\in}$ X with x ≠ y. Then X–{x} is a soft #gα-open set containing y but not x and X–{y} is a soft #gα-open set containing x but not y. Thus X is soft #$g\alpha_1$ –space.

V. SOFT #gα-CONTINUOUS AND SOFT #gα-IRRESOLUTE FUNCTIONS

Definition 34:

A soft mapping $f: X \to Y$ is said to be soft #gα-continuous if the inverse image of each soft open set in Y is a soft #gα-open set in X.

Theorem 35: Let $f: X \to Y$ be a mapping from soft space to X to soft space Y. Then, the following statements are equivalent

(1) f is soft #gα-continuous,

(2) the inverse image of each soft closed set in Y is soft #gα-closed in X.

Proof:

(1) ⇒ (2): Let (G,B) be a soft closed set in Y. Then $(G,B)^c$ is soft open. Since f is soft #gα-continuous, $f^{-1}((G,B)^c)$ $\tilde{\in}$ $\tilde{s}$#GαOS(X). Hence the inverse image of each soft closed set $f^{-1}((G,B))$ in Y is soft #gα-closed in X.

(2) ⇒ (1): Let (O,B) be a soft open set in Y. Then $(O,B)^c$ is soft closed. Since the inverse image of each soft closed set in Y is soft #gα-closed in X, we have $f^{-1}((O,B)^c)$ $\tilde{\in}$ $\tilde{s}$#GαCS(X). Hence $f^{-1}((O,B))$ is a soft #gα-open set in X. Therefore, f is soft #gα-continuous.

Theorem 36: Every soft continuous function is soft #gα-continuous.

Proof:

Let $f: X \to Y$ be a soft continuous function. Let (G,B) be a soft open set in Y. Since f is soft continuous, f^{-1}(G,B) is soft open in X and hence f^{-1}(G,B) is soft #gα-open in X since every soft open set is soft #gα-open. Therefore, f is soft #gα-continuous.

Converse need not be true as shown in the following example.

Example 37: Let X = {a,b}, Y = {m,n}, E = $\{e_1,e_2,e_3\}$, K = $\{k_1,k_2,k_3\}$, A($\subseteq E$) = $\{e_1,e_2\}$ and B($\subseteq K$) = $\{k_1,k_2\}$ and let (X,τ,E) and (Y,υ,K) be soft topological spaces.

Define u: $X \to Y$ and p: $E \to K$ as u(a)={m}, u(b)={n} and $p(e_1)=k_1$, $p(e_2)=k_2$, $p(e_3)=k_3$.

Let us consider the soft topology τ on X as τ = {$(F,A)_1$, $(F,A)_5$, $(F,A)_7$, $(F,A)_8$, $(F,A)_{16}$}. Now consider the soft topology υ on Y as υ ={$(G,B)_1$, $(G,B)_6$, $(G,B)_{16}$}.

Thus, the inverse images of soft open sets of Y are $(F,A)_1$, $(F,A)_6$, $(F,A)_{16}$. Soft #gα-open sets of X are $(F,A)_1$, $(F,A)_5$, $(F,A)_6$, $(F,A)_7$, $(F,A)_8$, $(F,A)_{13}$, $(F,A)_{14}$, $(F,A)_{15}$, $(F,A)_{16}$.

Let $f: (X,\tau,E) \to (Y,\upsilon,K)$ be a soft mapping. Then by the definition, the inverse image of each soft open set in Y is soft #gα-open in X but not soft open in X. Therefore, f is soft #gα-continuous but not soft continuous.

Theorem 38: Every soft α-continuous function is soft #gα-continuous.

Proof:

Let $f: X \to Y$ be a soft α-continuous function. Let (G,B) be a soft open set in Y. Since f is soft continuous, f^{-1}(G,B) is soft α-open in X and hence f^{-1}(G,B) is soft #gα-open in X as every soft α-open set is soft #gα-open. Therefore, f is soft #gα-continuous.

Converse need not be true as shown in the following example.

Example 39: Let X = {a,b}, Y = {m,n}, E = $\{e_1,e_2,e_3\}$, K = $\{k_1,k_2,k_3\}$, A($\subseteq E$) = $\{e_1,e_2\}$ and B($\subseteq K$) = $\{k_1,k_2\}$ and let (X,τ,E) and (Y,υ,K) be soft topological spaces.

Define u: X → Y and p: E → K as u(a)={m} , u(b)={n} and $p(e_1)=k_1$, $p(e_2)=k_2$.

Let us consider the soft topology τ on X as τ = {$(F,A)_1$, $(F,A)_5$, $(F,A)_7$, $(F,A)_8$, $(F,A)_{16}$}. Now consider the soft topology υ on Y as υ ={$(G,B)_1$, $(G,B)_6$, $(G,B)_{16}$}.

Thus, the inverse images of soft open sets of Y are $(F,A)_1$, $(F,A)_6$, $(F,A)_{16}$. Soft α-open sets of X are $(F,A)_1$, $(F,A)_5$, $(F,A)_6$, $(F,A)_7$, $(F,A)_8$, $(F,A)_{13}$, $(F,A)_{14}$, $(F,A)_{15}$, $(F,A)_{16}$.Soft #gα-open sets of X are $(F,A)_1$, $(F,A)_5$, $(F,A)_6$, $(F,A)_7$, $(F,A)_8$, $(F,A)_{13}$, $(F,A)_{14}$, $(F,A)_{15}$, $(F,A)_{16}$.

Let $f:(X,\tau,E)\rightarrow(Y,\upsilon,K)$ be a soft mapping. Then by the definition, the inverse image of each soft open set in Y is soft $^{\#}$gα-open in X but not soft α-open in X. Therefore, f is soft $^{\#}$gα-continuous but not soft α-continuous.

Definition 40:

A soft mapping $f:X\rightarrow Y$ is said to be soft $^{\#}$gα-irresolute mapping if the inverse image of every soft $^{\#}$gα-open set of Y is a soft $^{\#}$gα-open set in X.

Theorem 41: Every soft $^{\#}$gα-irresolute mapping is soft $^{\#}$gα-continuous mapping.

Proof:

Let $f:X\rightarrow Y$ be a soft $^{\#}$gα-irresolute mapping. Let (G,B) be a soft open set in Y, hence (G,B) is soft $^{\#}$gα-open set in Y. Since f is soft $^{\#}$gα-irresolute mapping, f^{-1}(G,B) is soft $^{\#}$gα-open in X. Therefore, f is soft $^{\#}$gα-continuous mapping.

Converse need not be true as shown in the following example.

Example 42: Let X = {a,b}, Y = {m,n}, E = {e_1,e_2,e_3}, K = {k_1,k_2,k_3}, A($\subseteq E$) = {e_1,e_2} and B($\subseteq K$) = {k_1,k_2} and let (X,τ,E) and (Y,υ,K) be soft topological spaces.

Define u: $X\rightarrow Y$ and p: $E\rightarrow K$ as u(a)={m}, u(b)={n} and p(e_1)=k_1, p(e_2)=k_2, p(e_3)=k_3.

Let us consider the soft topology τ on X as τ ={$(F,A)_1$, $(F,A)_3$, $(F,A)_5$, $(F,A)_7$, $(F,A)_{11}$, $(F,A)_{12}$, $(F,A)_{15}$, $(F,A)_{16}$}
Now consider the soft topology υ on Y as υ ={$(F,A)_1$,$(F,A)_5$, $(F,A)_7$, $(F,A)_8$, $(F,A)_{16}$}.

Thus, the inverse images of soft open sets in Y are $(F,A)_1$, $(F,A)_5$, $(F,A)_7$, $(F,A)_8$, $(F,A)_{16}$. Soft $^{\#}$gα-open sets in Y are $(F,A)_1$, $(F,A)_5$, $(F,A)_6$, $(F,A)_7$, $(F,A)_8$, $(F,A)_{13}$, $(F,A)_{14}$, $(F,A)_{15}$, $(F,A)_{16}$.

Let $f:(X,\tau,E)\rightarrow(Y,\upsilon,K)$ be a soft mapping. Then by the definition, the inverse image of each soft open set in Y is soft $^{\#}$gα-open in X which makes clear that f is soft $^{\#}$gα-continuous. But the inverse image of every soft $^{\#}$gα-open set in Y is not soft $^{\#}$gα-open in X. Hence f is not soft $^{\#}$gα – irresolute.

Theorem 43:

Let $f:(X,\tau,E)\rightarrow(Y,\upsilon,K)$, $g:(Y,\upsilon,K)\rightarrow(Z,\sigma,T)$ be two functions. Then

(1) $g\circ f:X\rightarrow Z$ is soft $^{\#}$gα-continuous, if f is soft $^{\#}$gα-continuous and g is soft continuous.
(2) $g\circ f:X\rightarrow Z$ is soft $^{\#}$gα-irresolute, if f and g are soft $^{\#}$gα-irresolute functions.
(3) $g\circ f:X\rightarrow Z$ is soft $^{\#}$gα-continuous, if f is soft $^{\#}$gα-irresolute and g is soft $^{\#}$gα continuous.

Proof:

(1) Let (H,T) be a soft closed set of Z. Since $g:Y\rightarrow Z$ is soft continuous, by the definition, g^{-1}((H,T)) is soft closed set of Y. Now $f:X\rightarrow Y$ is soft $^{\#}$gα-continuous and g^{-1}((H,T)) is soft closed set of Y, so by the definition, f^{-1}(g^{-1} ((H,T)) = $(g\circ f)^{-1}$((H,T)) is soft $^{\#}$gα-closed in X. Since the inverse image $(g\circ f)^{-1}$((H,T)) of soft closed set (H,T) in Z is soft $^{\#}$gα-closed in X, $g\circ f:X\rightarrow Z$ is soft $^{\#}$gα-continuous.

(2) Let $g:Y\rightarrow Z$ is soft $^{\#}$gα-irresolute and Let (H,T) be soft $^{\#}$gα-closed set of Z. Since $g:Y\rightarrow Z$ is soft $^{\#}$gα-irresolute then by the definition, g^{-1}((H,T)) is soft $^{\#}$gα-closed set of Y. Also $f:X\rightarrow Y$ is soft $^{\#}$gα-irresolute and g^{-1}((H,T)) is soft $^{\#}$gα-closed set of Y, so f^{-1}(g^{-1}((H,T)) = $(g\circ f)^{-1}$((H,T)) is soft $^{\#}$gα-closed in X. Since the inverse image $(g\circ f)^{-1}$((H,T)) of soft $^{\#}$gα-closed set (H,T) in Z is soft $^{\#}$gα-closed in X, $g\circ f:X\rightarrow Z$ is soft $^{\#}$gα-irresolute.

(3) Let (H,T) be a soft closed set of Z. Since $g:Y\rightarrow Z$ is soft $^{\#}$gα-continuous, then by the definition, g^{-1}((H,T)) is soft $^{\#}$gα-closed set of Y. Now $f:X\rightarrow Y$ is soft $^{\#}$gα-irresolute and g^{-1}((H,T)) is soft $^{\#}$gα-closed set of Y, so f^{-1}(g^{-1}((H,T)) = $(g\circ f)^{-1}$((H,T)) is soft $^{\#}$gα-closed in X. Since the inverse image $(g\circ f)^{-1}$ ((H,T)) of soft $^{\#}$gα-closed set (H,T) in Z is soft $^{\#}$gα-closed in X, $g\circ f:X\rightarrow Z$ is soft $^{\#}$gα-irresolute.

VI. CONCLUSIONS

In this paper, we introduced a new class of soft sets namely soft $^{\#}$generalized α-closed sets (soft $^{\#}$gα-closed) and some of the properties of soft $^{\#}$gα-closed sets in soft topological spaces were investigated. Moreover, using these sets we introduced a new type of space namely soft $T^{\#a}$ space. Also we introduced the concept of soft $^{\#}$gα-continuous and soft $^{\#}$gα-irresolute functions. We obtained some characterizations of these functions.

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