# QUEUEING CONCEPTS ON HEALTH CARE ANALYSIS

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**Abstract :- A multi-server queuing situation in health care center is considered. The arrival and service patterns of patients in two different units are discussed. The expected number of patients waiting for service and the probabilities of waiting time are computed numerically and the curves are exhibited.**

**Key words: Multi server, chi-square test, queue length, waiting time.**

## 1. Introduction

Queuing theory plays a vital role in different fields such as Telecommunication, Computer Networks, Industries, Medical and so on. Medical institutions and Hospitals mostly used statistical tools, particularly, the applications of queuing models for analyzing medical information. This analysis is used to appoint more number of medical staff or to reduce the number of staff. Generally single server Markovian queue is used for analyzing such data.

The applications of queuing theory are utilized in industrial sector, communication systems, and health care units and so on. For developing health care centers, the researchers analyzed the inflow of patients, provided services by doctors and other facilities. The major use of queuing theory in health care is to reduce service times of patients. Because of using queuing principles there are drastic changes occurring in the health care delivery, reimbursement methods, the changes in demographic disease profiles and highly fragmented delivery system. The application of queues is attempted to minimise the cost by minimising the inefficiencies and delays in the system. In this paper, the application of queuing theory in health care has been analyzed through multi server Markovian queue. The number of arrivals and services of patients are considered and estimated their parameters. The fitness of the corresponding distribution in justified by using chi-square statistic. The expected number of patients waiting for service as well as the probabilities of waiting times have been computed into different cases.

## 2. Review of Literature

The utilizations of queuing theory in Health care have been studied by many experts. Siddhartan et al (1996) have used the principles of the priority queues for reducing waiting times in single and multi-server models for general and emergency blocks. Gorunescu et al (2002) have studied the allocation of beds by using queuing model. Mackay and Loe (2005) have surveyed the facilities provided by the hospital and suggested the way of improving the hospital facilities. For improving the facilities and activities of staff members in an intensive care units have been analyzed by Griffiths et al (2006). Carter (2006) has discussed the duties and responsibilities of engineers in leading hospitals. Green (2002, 2006) has contributed many useful suggestions to improve the hospital facilities not only in wards but also in emergency department.

## 3. Model Description

The applications of queuing theory have been utilized in various service sectors. In particular, for improving the facilities of hospitals, the authorities are interested to monitor the nature of arrivals of patients and the services demanded by that patients. In this regard, the researchers used single and multi-server queuing models. In this study we have selected a leading hospital at Trivandrum. By the request of the hospital authorities, the name of the hospital is not mentioned here. The daily working hours of the hospital lie between 8 AM and 5 PM excluding 1 hour lunch break. The arrival instants and service times of out patients for the period of 15 working days are written from the medical records which are kept in the medical records department. The services of patients are performed in two different units such as the patients having health insurance belong to unit I and others belong to unit II. The service duration and the corresponding number of patients served based on unit I are given in the table (1).

**Table : 1**

**Service times of patients.**

| **Service time (minutes)** | **Number of patients served** |
|---|---|
| 0 - 10 | 87 |
| 10 - 20 | 125 |
| 20 - 30 | 91 |
| 30 - 40 | 54 |
| 40 - 50 | 33 |
| 50-60 | 20 |

The mean service time of patients is obtained from table (1) as 22 minutes and the service rate is $\mu = 0.045$ per minute. Based on the observations collected on 15 days, the average arrivals during average service period is obtained as $\lambda = 1.25$. On assuming the arrival pattern follows poisson distribution which is stated as

$$P(x) = \frac{M(1.25)^x e^{-1.25}}{x!};$$

$$x = 0, 1, 2, 3, 4, 5, 6 \quad \text{\_\_\_\_\_\_\_\_\_\_} \quad (1)$$

For different values of x, the expected frequencies are obtained by using the relation (1). Its corresponding distribution is given as

**Table 2**

**Distribution of arrivals**

| $\boldsymbol{x}$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $\boldsymbol{P(x)}$ | 116 | 137 | 80 | 42 | 8 | 5 | 2 |

Now, we state hypothesis as

$H_0$ : The given observations may fit for poisson distribution. The test statistic is given by

$$\chi^2 = \sum_{i=1}^{n} \frac{(0_i - E_i)^2}{E_i} \sim \chi^2_{(n-p-k-1)d.f}$$

……………(2)

By using the table (2) and equation (2), we have $\chi^2 = 1.7704$

The critical value of $\chi^2$ at 3 d.f at 5% level of significance is 7.015. This implies that the given observations are fit for poisson distribution.

## 4. Expected number of patients waiting for service

Under the multi-server Markovian queue, Gross and Harris (2010) have presented the expressions for steady state probability of no patients in the system and expected number of patients waiting for service are respectively as,

$$P_0 = \left[ \sum_{n=0}^{s-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!\,(1-\rho)} \right]^{-1}, \rho = \frac{\lambda}{\mu s}$$

…………….(3)

and

$$L_q = \frac{P_0 \left(\frac{\lambda}{\mu}\right)^s \rho}{s!\,(1-\rho)^2} = \frac{P_0 \left(\frac{\lambda}{\mu}\right)^s \left(\frac{\lambda}{\mu s}\right)}{s!\,(1-\frac{\lambda}{\mu s})^2}$$

…………….. (4)

 

Also, the probability of waiting time of patients is steaded as

$$P(>t) = \frac{\rho_0 \left(\frac{\lambda}{\mu}\right)^s}{s!\,(1-\rho)}\; e^{-s\mu t(1-\rho)} \;\ldots\ldots\ldots \quad (5)$$

**Unit I:**

By assuming different values of s with computed fixed $\lambda$ and $\mu$ in equations (3) and (4), the required mean number of patients are computed and given in table (3).

**Table : 3 Expected queue length**

| s | $P_0$ | $L_q$ |
|---|---|---|
| 2 | 0.2860 | 0.9852 |
| 3 | 0.2791 | 0.1105 |
| 4 | 0.2980 | 0.0199 |
| 5 | 0.2874 | 0.0032 |

The table (3) and its corresponding curve revealed that on increasing the number of servers the expected number of patients waiting are decreased.

For varying s and different times, the probabilities of waiting times are computed by using expression (5) and given in table (4).

**Table : 4 Probabilities of waiting times**

| s \ t | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 0.5932 | 0.4999 | 0.4214 | 0.3551 | 0.2993 | 0.2523 | 0.2126 | 0.17921 | 0.1510 |
| 3 | 0.1551 | 0.1041 | 0.0699 | 0.0469 | 0.0315 | 0.0212 | 0.0142 | 0.0095 | 0.0064 |
| 4 | 0.0439 | 0.0235 | 0.0126 | 0.0067 | 0.0036 | 0.0019 | 0.0010 | 0.0005 | 0.0002 |
| 5 | 0.0096 | 0.0038 | 0.0015 | 0.0005 | 0.0002 | 0.00008 | 0.00003 | 0.00001 | 0.000005 |

The table (4) and their corresponding curves showed that the probabilities of waiting times decrease when the number of servers increases for fixed time. On the other hand, the probabilities decrease for increasing time with fixed number of servers.

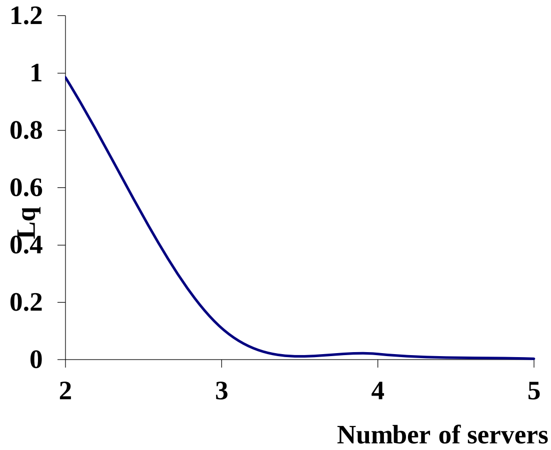


**Fig. (1): Expected queue length**

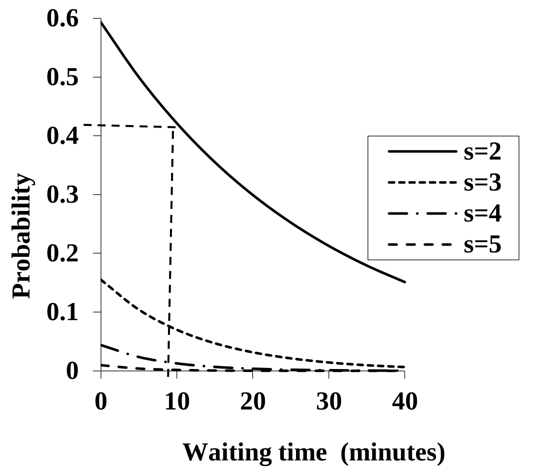


**Fig. (2): Probability of waiting time**

**Unit II :**

The researcher has attempted to study another medical unit and observed the number of served patients and the number of servers. The expected service time for a patient is computed as 12 minutes which gives $\lambda = 0.097$ and $\mu = 0.083$. The required $P_0$ and $L_q$ are given, for different s, in the following table through the expressions (3) and (4).

**Table : 5**

**Expected queue length**

| s | $P_0$ | $L_q$ |
|---|---|---|
| 2 | 0.2624 | 0.6059 |
| 3 | 0.3042 | 0.0846 |
| 4 | 0.3098 | 0.0140 |
| 5 | 0.3107 | 0.0022 |

The table (5) and its corresponding curve revealed that the mean number of patients decrease for increasing the number of servers.

Also the probabilities of waiting time of patients, under this case by using the expression (5), are given in the following table.

**Table : 6**

**Probabilities of waiting time**

| s \ t | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 0.43100103 | 0.30524370 | 0.21617980 | 0.15310294 | 0.10843062 | 0.07679277 | 0.05438620 | 0.03851742 | 0.02727882 |
| 3 | 0.13256907 | 0.06199810 | 0.02899443 | 0.01355972 | 0.00634143 | 0.00296567 | 0.00138695 | 0.00064863 | 0.00030334 |
| 4 | 0.03391753 | 0.01047438 | 0.00323469 | 0.00099893 | 0.00030849 | 0.00009527 | 0.00002942 | 0.00000909 | 0.00000281 |
| 5 | 0.00721237 | 0.00147079 | 0.00029993 | 0.00006116 | 0.00001247 | 0.00000254 | 0.00000052 | 0.00000011 | 0.00000002 |

The results, obtained from expression (5), in table (6) and their corresponding curves revealed that the probabilities of waiting time decrease for increasing the number of servers with fixed time and for increasing the time with fixed servers, the probabilities are decreased.

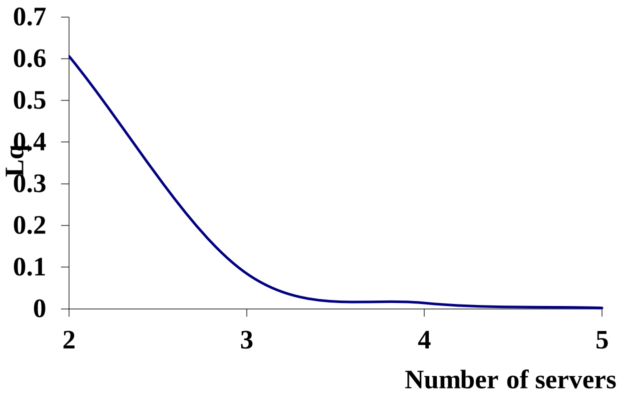


**Fig. (3): Expected queue length**

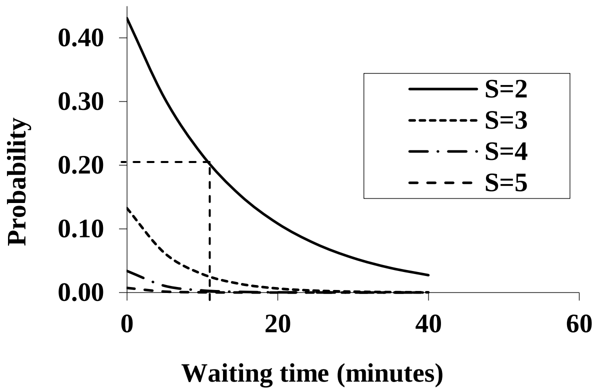


**Fig. (4): Probability of waiting time**

## 5. Conclusion

In this study, the arrival and service patterns of patients in multiple server queue are discussed. Based on the collected information from the hospital records, the average queue length and probabilities of waiting time of patients are computed and discussed. In addition that the corresponding curves are exhibited.

Fig (1) and (3) explained that the expected number of patients in the system decrease when the number of servers (doctors) increase. Figures (2) and (4) revealed that the probabilities of waiting time decrease when the number of servers increase at a particular time point. This study reveals that the patients may get services quickly when increasing the number of doctors. Similarly, for fixed number of servers when the time increases, the probabilities of waiting time decrease. On comparing both units, the mean number of patients decrease as well as the probabilities of waiting time decrease irrespective of time and number of servers for decreasing arrival rates. It is also observed that service rate of unit I is less than that of unit II and implied that doctors belong to unit I give much attention and spend more time for the patients having health insurance as compared with that of performance of doctors belong to unit II.

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