Code-Multiplexing

K.Varalakshmi[1] P. Prathusha[2]

varalakshmikarna1003@gmail.com

pratyu2mail@gmail.com

***Abstract*-Code multiplexed transmitted reference (CM-TR) and code-shifted reference (CSR) have newly drawn notice in the field of ultra-wideband communication mainly because they enable noncoherent detection without requiring either a delay component, as in transmitted reference, or an analog carrier, as in frequency-shifted reference, to separate the reference and data-modulated signal reference, to separate the reference and data-modulated signals at the receiver. We propose a generalized code-multiplexing (GCM) system based on the formulation of a constrained mixed-integer optimization problem. The GCM extends the concept of CM-TR and CSR while retaining their simple receiver structure, even offering better nit-error-rate performance and a higher data rate in the sense that more data symbols can be embedded in each transmitted block. The GCM framework is further extended to the cases when peak power constraint is considered and when inter-frame interference exists, as typically occurs in high data-rate transmissions. Numerical simulations performed over demanding wireless environments corroborate the effectiveness of the proposed approach.**

Index Terms - UWB communication, non coherent detectors, transmitted reference, code-multiplexing.

INTRODUCTION

In ultra-wideband (UWB) impulse radio (IR) signaling, information is conveyed by transmitting sequences of ultra short pulses at very low power spectrum density. After traveling through multipath channels, each transmitted pulse appears at the receiver as hundreds of echoes. To collect the energy through these multiple paths, Rake receivers are proposed, however, they exhibit high complexity due to a large number of fingers together with intensive computational cost and extremely high sampling rate required in estimating the amplitude and the delay of the channel paths. As a sub-optimal yet simple solution, transmitted reference (R) systems avoid channel estimation by transmitting each information symbol through two pulsed, namely the reference and the data pulses. Thus, the received reference pulses allow the recovery of the noisy channel template, which is then employed for data detection based on a correlation scheme. The TR concept enables simple receiver structures, but the delay component required by the correlation unit, amounting to tens or even hundreds of non negligible drawback in terms of hardware implementation. In both cases, the delay component is built via either analog circuitry or digital sampling. A viable alternative to the TR scheme for efficient energy capture is based on differential detection (DD), and in few improved multi-symbol differential detection (MSDD). These detectors are attractive in that they can efficiently gather energy from all the multiple paths however they still suffer from the need for accurate delay lines on the order of multiples of symbol intervals.

Purpose and contributions: The aim of this paper is to generalize the CM-TR and CSR concepts through a novel design we refer to as "generalized code-multiplexing" or GCM for short in the following. The rational of the proposed transmitter and receiver structure relies of the proposed transmitter and receiver structure relies on the formulation of a constrained optimization problem (OP), which maximizes the BER performance metric under a given set of constraints mainly adopted to keep complexity at affordable levels. Several features differentiate the proposed approach from previous work and define our contributions.

1) The GCM inherits the basic structure of the CM-TR and CSR systems based on a simple energy detector without any delay line components. As a further step, however, after solving offline a joint OP on the transmitted and decoding codes for a given frame size N_f and number of information symbols M conveyed within each block, improved BER link performance and higher spectral efficiency are enabled.

2) When the frame size $N_f > 2^M$, the non-deterministic polynomial hard (NP-hard) nature of the original constrained OP can circumvented by deriving the closed form optimal solution from an equivalent system with $N_f = 2^M$.

3) To take account of the emission power restriction imposed by the Federal Communications Commission (FCC) for UWB communication, we develop the GCM systems with peak power constraint, which can maintain the same error performance as the existing designs while enjoying lower peak power levels.

4) The GCM framework is then extended to the more general case when inter-frame interference (IFI) arises, as typically occurs in high data rate transmissions. Through the

formulation of an OP based on a properly modified signal model, the IFI effect can be mitigated, and thus obtaining a considerable performance improvement compared to some existing codes.

SYSTEM MODEL

Consider the GCM system depicted in fig.1. A sequence of M information symbols $a \triangleq [a_i, \ldots a_M]^T$, $a_i \in \{\pm 1\}$ are encoded at the transmitter into a block of N_f frame symbols $b \triangleq \left[b_i, \ldots b_{N_f - 1}\right]^T$ according to the rule $b = X(a)$, $X \triangleq \left[x_0, x_{1,} \ldots, X_{N_f - 1}\right]^T$. Thus, the transmitted signal corresponding to the data block a can be written as

$$x(t) = \sum_{j=0}^{N_f - 1} b_j p(t - jT_f), \quad (1)$$

Where $p(t)$ the Gaussian monocycle pulse with duration is T_p, N_f is the number of frames in the block, and T_f is the frame interval. Note that, for the time being, inter-frame interference (IFI) is avoided by choosing $T_f > T_m + T_p$, where T_m is defined as the maximum excess delay of the channel, however this assumption will be dropped in Sec.IV. For the sake of notational simplicity, we do not explicitly consider the typical frame structure for time hopping (TH) in that it can be removed at the receiver prior to further signal processing without incurring IFI under the condition of sufficiently long T_f.

The UWB propagation channel is assumed to be highly frequency-selective with the channel impulse response (CIR) modeled

$$h(t) = \sum_{n=0}^{N_p - 1} \alpha_n \delta(t - \tau_n), \quad (2)$$

where N_p is the total number of paths with amplitude α_n and delay τ_n. The channel coherence time, where in the CIR stays approximately constant, is assumed to be longer than the block transmission interval $T_b = T_f N_f$.

After processing the received signal with a low-pass filter having impulse response $h_{lp}(t)$, which eliminates the out-of-band (OOB) interference and noise, in correspondence of (1) we obtain

$$y(t) = \sum_{j=0}^{N_f - 1} b_j g(t - jT_f) + \omega(t) \quad (3)$$

Where $\omega(t)$ is a band-limited AWGN component with two-sided power spectrum density $N_0/2$, and the channel template $g(t) \triangleq p(t) * h(t) * h_{LP}(t)$ has frame energy

$$E_f \triangleq \int_0^{T_m + T_p} g^2(t) dt,$$

Under the assumption that timing has been acquired, energy integration is performed on the received signal

$$r_j = \int_{jT_f}^{(j+1)T_f} x^2(t) dt, \ \ j \in J, \quad (4)$$

With $J \triangleq \{0, \ldots, N_f - 1\}$. Then, the decision variable for the kth information symbol is obtained as

$$z_k = c_k^T r, \quad k \in K, \quad (5)$$

Where $c_k \triangleq \left[c_{k,0,} c_{k,1,} \ldots, c_k N_f - 1\right]^T$ is the decoding vector, $r \triangleq \left[r_{0,} r_{1,} \ldots, r_{N_f - 1,}\right]^T$ includes the outputs in (4) and $k \in \{1, \ldots, M\}$. As a final step, the estimate of the information symbol is given by

$$\hat{a}_k = sgn(z_k) k \in K, \quad (6)$$

The system model in Eqs. (1),(3)-(6) subsumes some existing code-multiplexed (CM) designs.

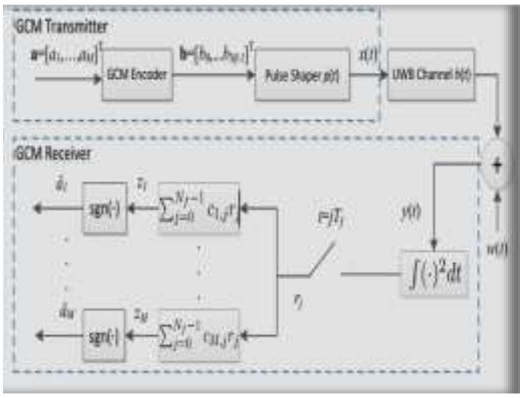

Figure : System diagram of a block transmission for GCM systems.

GCM OPTIMAL DESIGN

In this section, we formulate a constrained OP to design the GCM encoder $b = X(a)$ and decoding matrix $C \triangleq$

$[C_1, \ldots, C_M]$so that the link performance in terms of the BER metric is optimized under a given set of assumptions.

A. Formulation of GCM Systems

Let us first define the GCM system we are dealing with, which subsumes AM-TR and CSR as special cases.

Definition 1: A transmitter with encoder $b = X(a)$ and a receiver with decoding matrix C form a GCM system if the following assumptions are satisfied:

A1) $c_{k,j} \in \{\pm 1\}, \mathrm{k} \in K, \mathrm{j} \in \mathrm{J}$;

A2) $\sum_{j=0}^{N_f-1} c_{k,j} = 0, \mathrm{k} \in K$;

A3) The error probabilities on a_k, $k \in K$are equal.

Then, we derive an equivalent definition of the GCM system that will be particularly useful to formulate the GCM OP. To be specific, we take the conditions of both the absence of IFI and sufficiently large product BT_f, B being the bandwidth of the receiver low-pass filter$h_{LP}(t)$.

Proposition 1: A GCM system with encoder $b = X(a)$ and decoding matrix C holds of assumption A3) is replaced by A3a)-A3b) as:

A1) $c_{k,j} \in \{\pm 1\}, \mathrm{k} \in K, \mathrm{j} \in \mathrm{J}$;

A2) $\sum_{j=0}^{N_f-1} c_{k,j} = 0 , \mathrm{k} \in K$;

A3a) $\mathrm{c}_k^T(b_i \odot \mathrm{b_j}) = \Psi \mathrm{a_{i,k}}, \mathrm{i} \in \mathrm{I}, \mathrm{k} \in \mathrm{K}$;

A3b) $\|b_i\|_2^2 = E_b, i \in I$,

Where $a_i \triangleq [a_{i,1}, \ldots, a_{i,M}]^T$ and $b_i \triangleq [b_{i,1}, \ldots, b_{i,M}]^T$, with $\mathrm{i} \in \mathrm{I} \triangleq \{1, \ldots 2^{\mathrm{M}}\}$, denote the realizations of the information symbol a and the transmitted symbol b, respectively, with $b_i = X(a_i)$;E_b is the energy of the transmitted symbol b_i, assumed to be constant ,$\forall \mathrm{i} \in I$; and $\Psi$is a parameter that strictly depends on both the encoding rule $b = X(a)$and the decoding matrix C.

Now, a key result about the GCM system is ready to be derived, as stated in the sequel.

Proposition 2: Assuming a GCM system with encoder $X(a)$and decoding matrix C satisfying Proposition 1, the BER performance is asymptotically approximated in terms of the twice time-bandwidth product $L \triangleq [2\mathrm{BT_f}]$ when L is large as

$$BER_{GCM}(\Omega) = Q\left(\Omega\left[\frac{2\mathrm{M}}{\Upsilon} + \frac{\mathrm{N_f L}}{2\Upsilon^2}\right]^{-\frac{1}{2}}\right) \qquad (7)$$

Where$\Omega \triangleq M\Psi / \mathrm{E_b}$, $\Upsilon = \mathrm{E_{bit}} /$ N is the received-bit-energy-to-noise-spectral-density ratio, and

$$\mathrm{Q(x)} \triangleq \frac{1}{\sqrt{2}} \int_x^\infty \exp\left(-\frac{t^2}{2}\right) \mathrm{dt}.$$

Proof: See Appendix B.

Given Propositions 1-2, we are now ready to establish the relationship between our GCM systems and existing systems. The CSR system is equivalent to the GCM systems with $\Omega = \sqrt{\mathrm{M}}$.

B. Optimization Problem for GCM Systems

According to Proposition 2, it can be recognized that given $\mathrm{N_f}$, M, and L the BER performance metric is optimized whenever the encoder b = X(a) and the decoding matrix C are designed so that in (9) is maximized under assumptions A1)-A3b). Hence, is just the objective function of the OP we are addressing. As such, in light of A3a), it will be denoted in the sequel as $\Omega(C, X)$, namely depending on both the decoding matrix C and the $\mathrm{N_f}$ x 2^{M} matrix $X \triangleq [x_1, \ldots x_{2^M}]$ with $x_i \triangleq b_i \odot b_i \triangleq [x_{i,0}, \ldots x_i, N_{f-1}]^T, i \in I.$ Hence, after designating the $M \times 2^M$ matrix as $A \triangleq [a_1, \ldots a_{2^M}]$, we formulate the GCM joint constrained OP over C and X, or joint OP (J-OP) for short, as

$$\begin{cases} (\mathrm{C_0, X_0}) = \underset{c,x}{\arg\max}\{\Omega(\mathrm{C, X})\} \\ \mathrm{s.t.} \quad \mathrm{C^T X} = \Omega(\mathrm{C,X})\mathrm{A} \\ \quad 1_{N_f \times 1}^{\mathrm{T}} \mathrm{X} = \mathrm{M} 1_{2^M \times 1}^{\mathrm{T}} \\ \quad \mathrm{X} \geq 0_{\mathrm{N_f} \times 2^{\mathrm{M}}} \\ \quad \mathrm{C^T} 1_{N_f \times 1} = 0_{\mathrm{M} \times 1} \\ \quad \mathrm{C} \odot \mathrm{C} = 1_{N_f \times M} \end{cases}, \qquad (8)$$

where for convenience, we set $E_b = M; X \geq 0_{N_f \times 2^M}$ means that all entries of X are greater than or equal to 0; $\mathrm{C} \odot \mathrm{C} = 1_{\mathrm{N_f x M}}$ means that the entries of C take values in $\{\pm 1\}$; and the objective function is given by

$$\Omega(\mathrm{C,X}) = \frac{1}{M 2^M} \cdot 1_{M \times 1}^{\mathrm{T}} [(C^T X) \odot A] 1_{2^M \times 1}, \qquad (10)$$

which can be obtained from the first constraint of (8) originating from A3a). If the decoding matrix C is given, the J-OP in (8) is simplified to

$$\begin{cases} X_0 = \arg\max_{x} \{\Omega(X)\} \\ \text{s.t.} \ \ C^T X = \Omega(X) A \\ 1^T_{N_f \times 1} X = M 1^T_{2^M \times 1} \\ X \geq 0_{N_f \times 2^M} \end{cases} , \quad (11)$$

labeled as GCM encoder-based OP, or E-OP for short.

Now, the following remarks about the OPs (8)-(11) are of interest.

1. The J-OP in (8) is a mixed integer programming (MIP) problem since the optimization has to be performed over the matrices C and X, whose entries take integer and real values, respectively. As a result, it is generally NP-hard, and its computational complexity is really demanding even for small N_f and M. As will be shown in Sec. III-C, however, the optimal transmitted and decoding code matrices for $N_f \geq 2^M$ can be found by solving an equivalent problem for $N_f = 2^M$ with a closed-form optimal solution. In contrast, the (sub-optimal) E-OP in (11), which belongs to the class of linear programming (LP) OPs, can be solved by applying some well-known polynomial-complexity algorithms.

2. The optimal GCM design offers several advantages over the existing CM-TR and CSR: i) BER performance can be improved; ii) the system design does not rely on the properties of any code word set, such as the Walsh codes; iii) the number of symbols M that can be embedded into a single data block, can be greater than those of the CM-TR (M = 1) and the CSR ($M \leq N_f/2$), which results in a higher spectral efficiency.

3. The solutions to the J-OP or E-OP aim at optimizing the BER performance. The GCM framework gives the freedom to consider alternative optimization criteria as well. A viable option is to minimize the peak power of the transmitted signal (1) [32] under a predefined BER level determined by a value of Ω, say Ω_c , with $\Omega_c \leq \Omega_0$, Ω_0 denoting the optimal objective value of J-OP in (8). This means to constrain the entries of the matrix X to be below a threshold γ, or more formally $[X]_{i,j} \leq \gamma, \forall_j \in J, \forall_i \in I$, and to modify the first constraint of (8) into $C^T X = \Omega_c A$. Hence, the corresponding OP is to minimize the peak power γ while keeping the average power as $1^T_{N_f \times 1} X = M\, 1^T_{2M \times 1}$. Thus, this peak power based OP, or PP-OP for short, can be formulated as

$$\begin{cases} (C_0, X_0) = \arg\min_{c,x} \{\gamma(C, X)\} \\ \text{s.t.} \quad X \leq \gamma(C, X) 1_{N_f \times 2^M} \\ \qquad C^T X = \Omega_c A \\ \qquad 1^T_{N_f \times 1} X = M 1^T_{2^M \times 1} \\ \qquad X \geq 0_{N_f \times 2^M} \\ \qquad C^T 1_{N_f \times 1} = 0_{M \times 1} \\ \qquad C \odot C = 1_{N_f \times M} \end{cases} \quad (12)$$

4. For practical UWB communications with predetermined system parameters, i.e., N_f and M , the J-OP can be solved offline, and the optimized encoder X(a) and decoding matrix$C \triangleq [c_1, \ldots, c_M]$ can be stored locally as look-up tables at the transmitter and the receiver. When the system parameters are determined in the real-time communications, the transmitter can solve J-OP and then send the optimized decoding matrix to the receiver as preamble, or a central unit can optimize the J-OP and send the optimized results to both the transmitter and the receiver.

C. Optimal Codes for Large Frame Size N_f

The considerable complexity of the MIP constrained J-OP in (15) when $N_f > 2^M$ can be avoided by analytically solving an equivalent problem with$N_f = 2^M$. For the sake of convenience, the following two lemmas can help, where we designate the original J-OP in (8) with frame length$N_f > 2^M$ as “larger problem," or LJ-OP, and the corresponding equivalent J-OP with $N_f = 2^M$as “smaller problem," or SJ-OP.

Lemma 1: For any feasible solution to the LJ-OP, there exists a feasible solution to the SJ-OP such that the solutions provide the same objective value.

Proof. See Appendix C.

Lemma 2: Assume that the mappings $\Gamma : A_{2^M \times M} \rightarrow B_{N_f \times M}$ and $\Lambda : A_{2^M \times 2^M} \rightarrow B_{N_f \times 2^M}$ exist such that for any feasible solution (C, X) to the SJ-OP, $[\Gamma(C), \Lambda\Lambda(X)]$ is the feasible solution corresponding to the LJ-OP, both with the same objective value, i.e., $\Omega(\mathrm{C},\mathrm{X}) = \Omega[\Gamma(C),\Lambda\Lambda(\mathrm{X})]$. Then, $[\Gamma(C_0),\Lambda\Lambda(\mathrm{x}_0)]$ is the optimal solution to the LJ-OP, when (C_0, X_0) is the optimal solution of the SJ-OP.

Proof: Corresponding to the optimal solution (C_0, X_0) for the SJ-OP, there exists a feasible solution $[\Gamma(C_0),\Lambda\Lambda(\mathrm{x}_0)]$ for the LJ-OP such that $\Omega(C_0, X_0) = [\Gamma(C_0),\Lambda\Lambda(\mathrm{x}_0)] = \Omega_0]$. Then, $[\Gamma(C_0), \Lambda(X_0)]$ must also be optimal since if there exists a solution (C_0,X_0) to the LJ-OP which is better than $[\Gamma(C_o), \Lambda(X_0)]$, i.e., with $\Omega(C_0,X_0) > \Omega[\Gamma(C_0), \Lambda(X_0)]$, according to Lemma 1, there would exist a feasible solution for the SJ-OP with objective value equal to $\Omega(C_0,X_0)$ greater than $\Omega(C_0, X_0)$, which results in a contradiction.

Lemmas 1 and 2 allow us to establish a one-to-one relationship between the optimal solutions of the GCM J-OPs with $\mathrm{N_f} > 2^{\mathrm{M}}$ and those with $\mathrm{N_f} = 2^{\mathrm{M}}$. Thus, the problem is how to find the mappings Γ and Λ. A simple option is to apply the zero padding method, which gives

$$\mathrm{C}_{\mathrm{N_f}\times\mathrm{M}} = \Gamma_{\mathrm{zp}}\ (\mathrm{C}'_{2^{\mathrm{M}}\times\mathrm{M}})\begin{bmatrix}\mathrm{C}'_{2^{\mathrm{M}}\times\mathrm{M}}\\ 1_{((\mathrm{N_f}-2^{\mathrm{M}})/2)\times\mathrm{M}}\\ -1_{((\mathrm{N_f}-2^{\mathrm{M}})/2)\times\mathrm{M}}\end{bmatrix}, \tag{13}$$

$$\mathrm{X}_{\mathrm{N_f}\times2^{\mathrm{M}}} = \Lambda_{\mathrm{zp}}\ (\mathrm{X}'_{2^{\mathrm{M}}\times2^{\mathrm{M}}}) = \begin{bmatrix}\mathrm{X}'_{2^{\mathrm{M}}\times2^{\mathrm{M}}}\\ 0_{(\mathrm{N_f}-2^{\mathrm{M}})/2^{\mathrm{M}}}\end{bmatrix} \tag{14}$$

or alternatively, the repetition codes with $\mathrm{N_f} = \mathrm{P}2^{N,}$ P being a positive integer, as

$$\mathrm{C}_{\mathrm{N_f}\times\mathrm{M}} = \Gamma_{\mathrm{zp}}\left(\mathrm{C}'_{2^{\mathrm{M}}\times\mathrm{M}}\right) = 1_{p\times1}\otimes\mathrm{C}'_{2^{\mathrm{M}}\times\mathrm{M}}, \tag{15}$$

$$\mathrm{X}_{\mathrm{N_f}\times2^{\mathrm{M}}} = \Lambda_{\mathrm{zp}}\ \mathrm{X}'_{2^{\mathrm{M}}\times2^{\mathrm{M}}}) = \frac{1}{p}\ 1_{p\times1}\otimes\mathrm{X}'_{2^{\mathrm{M}}\times\,2^{\mathrm{M}}} \tag{16}$$

It can be easily verified that $[\Gamma(C),\Lambda\Lambda(\mathrm{X})]$ in (13)-(14) is the feasible solution for LJ-OP, given the feasible solution (C, X) for the SJ-OP, and the solutions provide the same objective value Ω.

Now, the next step is to show that the optimal encoding and decoding matrices solving the SJ-OP can be analytically found, as stated in the sequel.

*Proposition 3***.** Considering the GCM system with $\mathrm{N_f} = 2^{\mathrm{M}}$, the optimal decoding matrix C_o is the $2^M \times M$ matrix

$$C_0 = [z_1, \ldots, z_{2^M}]^{\mathrm{T}}, \tag{17}$$

Where the vectors z_i , $i \in I$, are all the 2^{M} realizations of length M with entries ±1.

In addition, the optimal encoder for the information symbols a is given by

$$b_j = X_j(a) = \begin{cases}\pm\sqrt{\mathrm{M}}, & \mathrm{Z_j} = \mathrm{a}\\ 0, & \text{otherwise}\end{cases},\quad j \in J, \tag{18}$$

with the optimal objective value $\Omega_o = M$.

As a further result, Lemmas 1-2 can be exploited together with Proposition 8 to derive the optimal performance of the GCM system with $\mathrm{N_f} \geq 2^{\mathrm{M}}$, as summarized in the following proposition.

Proposition 4 **:** For a GCM system with $\mathrm{N_f} \geq 2^{\mathrm{M}}$, the optimal BER performance can be asymptotically approximated as a function of the received-bit-energy-to-noise spectral-density ratio by

$$\mathrm{BER_{GCM}}\,|_{\mathrm{N_f} \geq 2^{\mathrm{M}}} = Q\left[\left(\frac{2}{\Upsilon\mathrm{M}} + \frac{\mathrm{N_f}\mathrm{L}}{2\Upsilon^2M^2}\right)^{-1/2}\right] \tag{19}$$

Proof: This follows from Lemmas 1-2 and Proposition 3 by plugging $\Omega_0 = \mathrm{M}$ into (9).

The following remark about the optimal codes of GCM systems is now of interest.

When $\mathrm{N_f} = 2^{\mathrm{M}}$, the optimal GCM system derived in Proposition 3 is essentially an M -PPM, and when $\mathrm{N_f} \geq 2^{\mathrm{M}}$, the optimal GCM system can be treated as a generalized M -PPM (e.g., PPM with zero padding or repetition in Eqs. (13) -(14)). However, different from the conventional PPM, where data symbols are carried via different delays of the transmitted pulse, the GCM systems convey the data symbols via the amplitude values of frame symbols b, thus allowing higher data rate communications by embedding more symbols in one block, i.e., $M > \log_2(\mathrm{N_f})$, than the M -PPM, and enabling the system optimization with emission power constraint.

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$$C_0 = [z_1, \ldots, z_{2^M}]^{\mathrm{T}}, \tag{20}$$

Where the vectors z_i , $i \in I$, are all the 2^{M} realizations of length M with entries ±1.In addition, the optimal encoder for the information symbols a is given by

$$b_j = X_j(a) = \begin{cases}\pm\sqrt{\mathrm{M}}, & \mathrm{Z_j} = \mathrm{a}\\ 0, & \text{otherwise}\end{cases},\ j \in J, \tag{21}$$

with the optimal objective value $\Omega_o = M$.

As a further result, Lemmas 1-2 can be exploited together with Proposition 8 to derive the optimal performance of the GCM system with $N_f \geq 2^M$, as summarized in the following proposition.

Proposition 4: For a GCM system with $N_f \geq 2^M$, the optimal BER performance can be asymptotically approximated as a function of the received-bit-energy-to-noise spectral-density ratio (defined after Eq. (7)) by

$$\mathrm{BER}_{\mathrm{GCM}}|_{N_f \geq 2^M} = Q\left[\left(\frac{2}{\gamma M} + \frac{N_f L}{2\gamma^2 M^2}\right)^{-1/2}\right] \qquad (22)$$

Proof: This follows from Lemmas 1-2 and Proposition 3 by plugging $\Omega_0 = M$ into (7).

The following remark about the optimal codes of GCM systems is now of interest.

When $N_f = 2^M$, the optimal GCM system derived in Proposition 3 is essentially an M -PPM, and when $N_f \geq 2^M$, the optimal GCM system can be treated as a generalized M -PPM. However, different from the conventional PPM, where data symbols are carried via different delays of the transmitted pulse, the GCM systems convey the data symbols via the amplitude values of frame symbols b, thus allowing higher data rate communications by embedding more symbols in one block, i.e., $M > \log_2(N_f)$, than the M-PPM, and enabling the system optimization with emission power constraint.

V. Numerical Results

In this section, we illustrate the optimal solutions of the proposed OPs for some values of the number of frames N_f and the number of symbols M per block. Then, the performance of the proposed optimal GCM systems is quantified through numerical simulations, taking as benchmark the existing CSR design in [16] using Walsh codes. We do not consider the FSR system, which shows identical performance to the CMTR systems in the absence of IFI and inferior performance in the presence of IFI [15],[16]. The transmitted pulse p(t) is the second derivative of a Gaussian function with width T_p= 1:0 ns. We use the channel models described in [17] for random channel realizations. The one-sided bandwidth of the low-pass filter at the receiver is B = 2:5 GHz. In this section, all OPs are solved using the general solver in [27].

A. Optimal Codes for GCM Systems

Table 1 summarizes the optimization results of J-OP (15) corresponding to the number of frames N_f = 2, 4, 6, 8 and a few values of the number of symbols M conveyed by each block. When (N_f = 2, M = 1), (N_f = 4; M = 1), (N_f = 8, M = 1), and (N_f = 8, M = 4), the proposed codes offer the same performance as the CSR systems using Walsh codes, which means that Walsh codes are optimal for these cases. On the other hand, when (N_f = 4, M = 2), (N_f = 8, M = 2), and (N_f = 8, M = 3), since the CSR systems using Walsh codes yield sub-optimal solutions to the OP in (15), i.e., the CSR systems are not optimized in the view of power efficiency, the proposed codes achieve significant improvement compared to the CSR systems. Additionally, the optimization performed on (N_f = 4; M = 3) and N_f = 6, where Walsh codes do not exist, gives us the flexibility to design GCM systems with different N_f and M. Finally, the results for $N_f \geq 2^M$ corroborate Proposition 9, where Ωo = M .

Frame length	Number of symbols	Walsh codes	Optimal codes
$N_f = 2$	M = 1	1	1
$N_f = 4$	M = 1 M = 2 M = 3	1 $\sqrt{2}$ N/A	1 2 1
$N_f = 6$	M = 1 M = 2 M = 3	N/A N/A N/A	1 2 1
$N_f = 8$	M = 1 M = 2 M = 3 M = 4	1 $\sqrt{2}$ $\sqrt{3}$ 2	1 2 3 2
$N_f = 16$	M = 1 M = 2 M = 3 M = 4 M = 5	1 $\sqrt{2}$ $\sqrt{3}$ 2 $\sqrt{5}$	1 2 3 2 2

Table 1: Objective value for the CSR with Walsh codes and the GCM with optimal codes.

B. Performance of Optimal Codes for GCM Systems

Fig. 2 displays the BER performance of the proposed GCM systems for N_f = 4, 8 and different numbers of information symbols per block M. We adopt CM1 channel model with T_f = 80 ns to avoid IFI and L = $2BT_f$ = 400. Given N_f and M, it is worth noting that the theoretical BERs in (9) overlap with the simulated curves. This result validates the accuracy of the Gaussian approximation whenever L is large which we assumed in the proof of Proposition 1. In all, the system with (N_f = 8, M = 3) achieves the best BER performance and gains about 1:8 dB over the (N_f = 8, M = 2) one at BER = 10^{-5}. When N_f = 4, the optimal system with M = 2 is close to that with (N_f = 8, M = 3), while outperforms the (N_f = 4, M = 1) one by about 3 dB at BER= 10^{-5}.

C. Performance comparison of Optimal GCM Systems with Existing Designs

In this subsection, we compare the performance of the GCM system with optimal codes to the CSR system in [16] and simple TR (STR) system in [11] with CM1 channel model and T_f = 80 ns. Fig.3 verifies the BER improvement of the proposed GCM over the existing designs. at BER= 10^{-5}, indeed, for the cases of (N_f = 4; M = 2) and (N_f = 8; M = 2) the proposed GCM design outperforms the CSR by about 1.8 dB, whereas for (N_f= 8; M = 3) case, the advantage of the optimal system increases to about 2.7 dB.

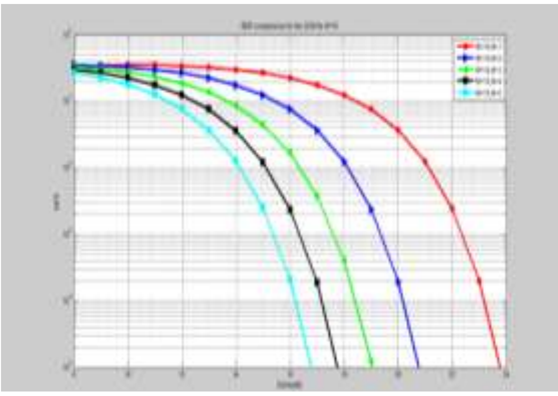

Figure 1: BER performance of the optimal GCM with different frame sizes N_f and numbers of symbols M.

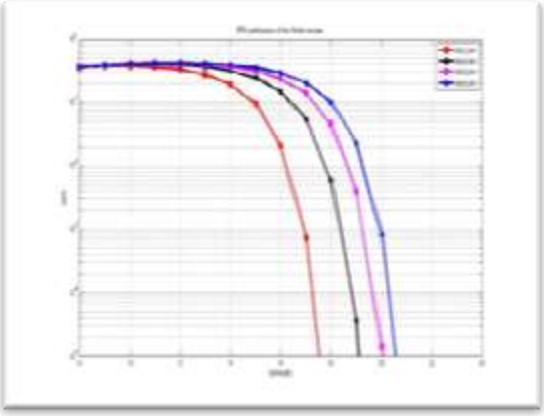

Fig:2 BER Performance of the STR Receiver

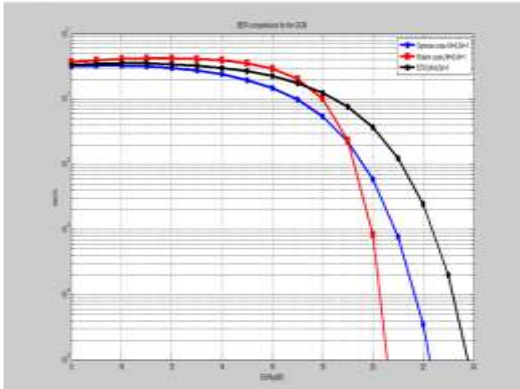

Fig:3Performance Comparisons of Optimal GCM Systems with Existing Design

In this subsection, we compare the performance of the GCM systems with optimal codes for simple TR (STR) system in [11] with CM1channel model and T_f = 80 ns. Fig.3 verifies the BER improvement of the proposed GCM over the existing designs. at BER= 10^{-5}, indeed, for the cases of (N_f = 4; M = 2) and (N_f = 8; M = 2) the proposed GCM design outperforms the CSR by about 1.8 dB, whereas for (N_f= 8; M = 3) case, the advantage of the optimal .

VI. CONCLUSION

In this paper, we have proposed a GCM system that extends and outperforms the existing CM-TR and CSR schemes. We have formulated a constrained optimization problem with the aim of maximizing the BER link performance, shoes complexity can be mitigated by considering an equivalent system with a shorter frame size.

Simulations over demanding propagation environments corroborate the competitiveness of the proposed approach. Furthermore, we have studied the GCM optimization with emission power constraint and generalized the GCM optimization problem to situations when moderate IFI may occur, with the result of gaining improved BER performance over the existing Walsh-code based design.

PROOF OF PROPOSITION

Due to assumption A3b), i.e., $||b_i||_2^2 = E_b$, i ∈ I, the received bit energy results in

$$E_{bit} = \frac{E_f}{M}\sum_{j=0}^{N_{f-1}} b_j^2 = \frac{E_f}{M} E_b. \quad (23)$$

Hence, from (38)-(39), the error probabilities on the information symbols a_k become identical over k and are expressed by

$$BER_{GCM} \mathrm{E}\{z_k|\mathbf{a}\}(\Omega) = \mathrm{Q}\left[\frac{\mathrm{E}\{z_k|\mathbf{a}\}^2}{\mathrm{Var}\,\{z_k|\mathbf{a}\}}\right]^{1/2}$$

$$= \mathrm{Q}\left[\left(\frac{E_f^2\Psi^2}{2E_f N_0 E_b + N_f L {N_0}^2/2}\right)^{1/2}\right]$$

$$= \mathrm{Q}\left[\left(\frac{2E_f N_0 E_b + N_f L {N_0}^2/2}{E_f^2\Psi^2}\right)^{-1/2}\right]$$

$$= \mathrm{Q}\left[\left(\frac{E_b^2}{\Psi^2 M^2}\left(\frac{2MN_0}{E_{bit}} + \frac{N_f L {N_0}^2}{2E_{bit}^2}\right)\right)^{-1/2}\right]$$

$$= \mathrm{Q}\left[\Omega\left(\frac{2M}{\gamma} + \frac{\mathrm{N_f} L}{2\gamma^2}\right)^{-1/2}\right] \quad (24)$$

Where $\gamma \triangleq E_{bit}/N_0$ and $\Omega \triangleq \mathrm{M}\Psi/E_b$.

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Author's profile

K.Varalakshmi received the B.Tech degree in Electronic instrumentation enginerring from St.Ann's college of engineering , JNTUK, AP India in 2012.currently doing M.Tech in DECS in St.Ann's college of engineering& Technology, JNTUK, chirala India Her research interests include wireless communication.

P.prathyusha received the B.Tech degree in Electronic Communication Engineering from St.Ann's college of engineering and Technology, JNTUK, AP India in 2008 and received M.Tech in VLSI Designfrom St.mary's college of engineering &Technology JNTUH, AP India Her research interests include VLSI.