Process of sustainable project development with Rubik's Cube using Game Theory interpretations

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***Abstract*— It is very difficult to calculate in advance the positive and negative long-term impacts of an sustainable investment, or a development venture. A serious global problem arises from the fact that numerous environmental-protection oriented private and government ventures are implemented in an incorrect manner significantly impair the conditions of both the environment and the economy (market). There is a high number of innovative energy related investments, waste and water management projects, etc. in Europe, which cause more harmful effects then was earlier.**

Various sustainability logics can be synchronised with the 3×3×3 Rubik's Cube's solution algorithms, and the relations of the cube's sides define a planning strategy that provides a new scientific approach for investment planning. We theoretically evaluated the various solution processes, and paralell investment planning levels following the solution levels and stages of the cube. After these various level-evaluations, we made „low-carbon interpretation" summaries. According to the hypothesis on the solution algorithms of the Rubik's Cube, the parts rotated next to each other, meaning the project attributes which have an impact on each other, have a relation system which can be defined in mathematical terms, therefore, their point of balance (e.g. Nash's) can also be determined by Game Theory models (games of finite kind, zero sum games, oligopolistic games, etc.). In this paper, we would also like to prove that the hypothesis which states that the solution algorithm of Rubik's Cube, namely the "Layer by layer" method, can be used to model the process of project development. Also, the correspondence system of project attributes can be represented by the proper Game Theory models. This way, the various enviro- and climate-friendly investments can be realized in a well-plannable, low-risk economic environment regarding both human resource planning and the preserving and advancement of environmental criteria.

***Keywords*— Game Theory interpretations, Rubik's Cube method, Sustainable planning and practice, Environmental modelling, Solution algorithms, Low-carbon interpretation, Introduction**

I. Introduction

The low-carbon project planning (1) and project development using Rubik's Cube is a specially constructed (2) planning concept which – as of now – is a one of a kind concept that can interpret factors with an impact on processes in 3D (3). For "setting" the equilibrium point of the economical or resource-usage of input and output sides, and to describe the relation between them, we used Game Theory solutions which weren't used for this purpose during scientific research before.

Used before the process of modeling, the evaluation of the process of tolerance in the sense of engineering means the determination of the allowed maximum differentiation from the determined sizes, quantities or qualities. In the case of Game Theory algorithms, we researched the following: which method is the same as the solution process model of Rubik's Cube in terms of its attributes, and in what scale does it differ from it while still remaining representative. For the Game Theory algorithms we were searching for, we used the process of tolerance, meaning we was researching the admissible differences between the attributes of the cube and the parameterization of the Game Theory functions.

During the complex modeling, we analyzed the Game Theory models one by one, and through the process of modeling We assigned the relevant models to the various rotation algorithms (interpretations). we separated the attribute groups of the cube to three different aggregations, which are INPUT side attributes, MIDDLE CUBE side attributes, and OUTPUT side attributes. We used Game Theory methods to determine the points of equilibrium between the three attribute groups. The gist of this was that where the attribute elements were tagged with a "not allowed difference" by the SMART (Simple Multi Attribute Ranking Technique) analysis, we listed parameters which lead to the points of equilibrium (Nash equilibrium) through strategic models. Both the analyses and the modeling were conducted via a three-stage system; therefore we also conducted the Game Theory modeling of the entire process on three levels, meaning the matching of three different types of Game Theory models (or three different cost-functions).

The Game Theory payoff functions referring to the various modeling levels were made by analyzing the attributes of the Input side, the middle cube, and the Output side, which were

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tagged with a "not allowed difference" by the SMART (Simple Multi Attribute Ranking Technique) analysis in their respective attribute groups, which we took and optimized as sustainability strategies interpreted in a business environment.

II. MATERIALS AND METHODS

Imaging algorithms of Input-side, the project begins in this phase. We can find the answer to the following question: what do we have to keep in mind when starting a project? The incorrect rotation of the first layer, or row of cubes, results in incorrect continuation, therefore, we can't approach the next layer. We can easily explain this with a simple energy-transaction. If we change our initial energy-supply system in a way that the old one still has a life expectation of 20-40% of its estimated use duration, then we may end up with a considerable financial loss if we intervene. To avoid ending up in such a situation, we can use e.g. a Nash equilibrium to calculate the optimal intervention time.

III. RESULTS

A. Game Theory modeling of input side (Level 1)

Enviro-orientated developments are fundamentally against the economic development priority system (e.g. the program for lowering greenhouse gases and for the use of fossilized energy sources contradict each other, since the former promotes the minimization of energy consumption, while the latter promotes the increased use of pollutants). When planning the first layer, this can be used in the process of project planning in terms of regulation policy and financing policy (Figure 1). We also have the same situation concerning the water base defense and the rising requirements of favored water-dependant energy plants. In case of various projects, we have to include the criteria of non-cooperative competitors as well for the sake of realizing clear business regulations and sustainable business strategies. In this situation, it is incredibly hard to find the Nash equilibrium, but it is imperative nevertheless since the project can't be further developed in a controversy.

Definition:

By the definition for the Nash equilibrium:

At the equilibrium point of a $J = (n, S, (\varphi_i)_{i=1}^{n})$ *n*-member game or strategy, we classify a point (strategic n), where

$$\varphi_i(x_1^*, \ldots, x_{i-1}^*, x_i^*, x_{i+1}^*) \geq \varphi_i(x_1^*, \ldots, x_{i-1}^*, x_i, x_{i+1}^*)$$

holds true not strictly for every i=1,…..,n player. Therefore, the point of equilibrium is called a Nash equilibrium.

Thesis:

Following the completion of the first layer, only the connection with a Nash equilibrium can be further developed, meaning that we can only rotate the cube further from this position. The first layer always correlates with the second layer's middle cube, and can only be the same color.

Figure 1: Equilibrium point for the first row or layer (circled), where the middle cube is always the same colour (illustrated by the lines).

Source: Fogarassy, 2014 (4)

Proof:

Let $x^* = (x_1^* \ldots, x_n^*)$ be one point of equilibrium for the game. In this instance, in case of any given $y = (y_1 \ldots \ldots y_n) \in S$:

$$\varphi_k(x_1^*, \ldots \ldots, x_k^*, \ldots \ldots, x_n^*) \geq \varphi_k(x_1^* \ldots \ldots, y_k, \ldots \ldots x_n^*) \quad (k = 1,$$

from where, through simple addition, it is obvious that φ(x^*,x^*)≥φ(x^*,y). Based on this, a well-performing algorithm can be provided to define the points of equilibrium which have an impact on planning and to solve the fixed problems of the aggregations.

Example:

During the planning of biomass-based renewable energy production, whether the high amount of water consumed can have a detrimental effect on the project's profitability and can become the criteria for use of the most effective technology is a critical point (5). Therefore, the question and the criteria is viewed as strictly technological in nature, and we try to match the strategy and Game Theory optimum with the corner cube which has 3D attributes (colors are red-green-white), where white means input, red means regulation criteria, and green means technological solutions, which we handle collectively (Figure 2).

Luckily, solving water distribution problems plays a major role in Game Theory solutions, but we can usually reach the points of equilibrium that provide criteria for the outlines of an assured system usage only through defining many intricate function-correspondences, calculating mathematic correlations for which is quite difficult. Multi-purpose water usage and the interests and cost-functions of those connected to it offer different optimums, which usually suppose a game of multi-player and nonlinear nature, and yet which is somehow still a non-cooperative game based on some kind of Nash equilibrium.

To define the problem – according to the low-carbon developments using Rubik's Cube – I made a three player optimization regarding water usage for the process of strategic planning using Rubik's Cube, based on the guide by Molnár (1994) (6).

Multi-purpose water usage as a decision-method task has been a problem for decades, and one with many solution options. In our case, we're searching for one on a non-cooperative three-player (agricultural consumer (for

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irrigation), industrial consumer (for cooling), and household consumer (for functional uses)) Nash equilibrium (7). The central element of the low-carbon strategy problem is how the agricultural (biomass producer) water usage project developer will decide whether the project has enough water out of the resources at hand.

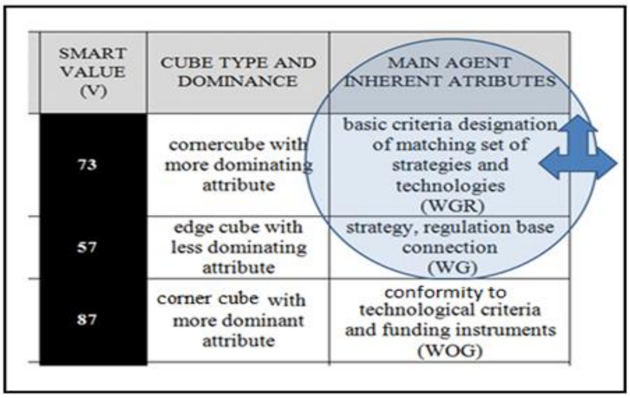

Figure 2: 3D attributes of "white-green-red" corner cubes (with White-Green-Red/WGR colors), the technological solution that assures payoff (optimized for three-person water usage) (Dimensions from left to right: SMART value, Cube type and dominance, Main agent inherent attributes)

Source: Fogarassy, 2014 (4)

The problem has three dimensions, where the Rubik solution is the issue of the input side. The basis of water usage can be water, underground water, and purified wastewater. Let k = 1,2,3 be the three players who can follow variations of decision during their decision phase as follows:

The strategy for each player can be described by a five variable vector:

$$x_k = (f_k, t_k, k_k, f_k^*, t_k^*).$$

where

f_k = *local water*, t_k = *local underground water*, k_k = *purified wastewater*

f_k^* = *import water*, t_k^* = *import underground water*

The payoff function for the total amount of water used for each player is as follows en:

$$\phi_k = f_k + t_k + k_k + f_k^* + t_k^*$$

All players have two common complicating criteria, one of which states that the amount of used water may not be less than the minimal requirement *D_k^min*, while the other states that it may not be more than the maximum requirement of technology (*D_k*), either. (These sustainability criteria are to avoid wasting water.)

$$f_k + t_k + k_k + f_k^* + t_k^* \geq D_k^{min}$$

$$f_k + t_k + k_k + f_k^* + t_k^* \leq D_k$$

In addition, the agricultural player (k=1) has to introduce two additional criteria for water usage, which have the following variables:

G = *group of plants exclusive to underground water*
ai = *rate of plants (i) by entire agricultural area*
wi = *water-dependence of plants (i) by hectare*
T = *group of plants which can be watered with purified wastewater*

$$W = \sum_i a_i w_i = total\ water - dependence\ for\ all\ plants\ by\ hectare$$

We know that the underground water supply offers the best quality water while purified wastewater offers the worst, so we have to define the volume of plants (sensitive) in the agricultural portfolio which can't be watered with purified wastewater. The water requirement which draws solely from the underground water sources may not exceed the water-dependence of the plants which are exclusive to clean, quality underground water:

$$\frac{t_1 + t_1^*}{f_1 + t_1 + k_1 + f_1^* + t_1^*} \geq \frac{\sum_{i \in G} a_i w_i}{W}$$

the equation converted to linear form:

$$a_1 f_1 + (\alpha_1 - 1)t_1 + a_1 k_1 + \alpha_1 f_1^*(\alpha_1 - 1)t_1^* \leq 0$$

where $\alpha_1 = \frac{\sum_{i \in G} a_i w_i}{W}$

Similarly, the rate of use and availability of purified water can also be modeled. The water requirement for purified wastewater may not exceed the total available amount, either. This correspondence gives the volume of plants that can either only or also be watered thus (e.g. plants for energy use).

$$\frac{t_1}{f_1 + t_1 + k_1 + f_1^* + t_1^*} \geq \frac{\sum_{i \in T} a_i w_i}{W}$$

equation converted to linear form:

$$-\beta_1 f_1 - \beta_1 t_1 + (1 - \beta_1)k_1 - \beta_1 f_1^* - \beta_1 t_1^* \leq 0$$

where $\beta_1 = \frac{\sum_{i \in T} a_i w_i}{W}$

For the other players, we similarly have to define the correspondences of the functions defined by complications, for which the system can be found in the cited publications, before adding numeric data.

In light of the above facts, it can be stated that if we design our agricultural systems for the use of biomass as energy by allocating the complicating energy source (in this case, water) into an equilibrium state right at the beginning with Game

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Theory methods, then the planning process is also applicable to the sustainability criteria system. The actual result of the entire analysis can be one of the following: either we won't over-calculate water usage (over-calculate, as in the allocation won't be disproportionate), or we will discard the project entirely because it doesn't abide by the sustainability criteria, since if it's clear at this point that the amount of water at hand is insufficient to reach the Pareto optimal production state, then the shortage of water causes a water-deficit in the analyzed system.

B) Defining input and output connections with Game Theory correlations

Game Theory modeling of middle cube connections (level 2) - keeping the middle cube in position and solving the row or layer imitates the zero sum game, since the position of the middle cube cannot be changed, so it serves as a fix point for the rotation of the other cubes. Their position is fixed (meaning they can't be rotated out of their position, or correspondence systems) and their defined value elements can be considered constant (Figure 3).

Figure 3: Zero sum games are always illustrated with the fixed middle cube (circled), which serve as criteria for the optimization of edge cubes (two colours).

Source: Fogarassy, 2014 (4)

<u>Definition:</u>

A J game with $n-$ players is called a constant sum game, if the sum of the wins and losses of the player is a constant c, regardless of strategy.

Formula:

$$\sum_{i=0}^{n} \varphi_i(x) = c\,(x \in S).$$

Where $c = 0$, the game is *zero sum*.

<u>Thesis:</u>

With the zero sum game, we do a constant sum optimization because the resource has a limited sum due to the fixed point trait; therefore, the goal is to harmonically divide the resources at hand, and we search for the point of equilibrium of the attribute group (Figure 4). During the SMART analysis, we verified that the orange middle cube of Rubik's Cube shows a "not allowed difference" attribute. Currently, the inherent attribute group of the orange side is the monetary value of the project, and the time needed for payoff. The analysis of this trait with Game Theory optimization methods shows us how the fixed resources of the low-carbon project will optimize themselves into a Nash equilibrium.

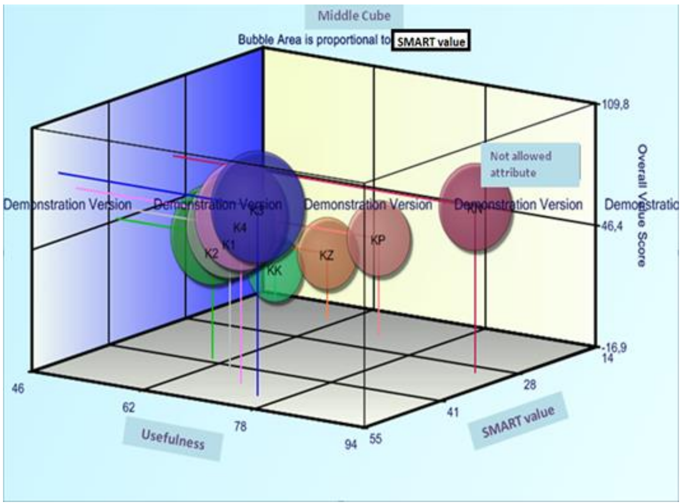

Figure 4: Prohibited attribute of the SMART analysis (time for payoff, value of project is not in equilibrium with the other attributes)

(Title: Middle Cube, Dimensions from left to right: Usefulness, SMART value, Value score)

Source: Fogarassy, 2014 (4)

The imbalance on Figure 4. can be ascribed to the insufficiency of the stability of external factors which have an impact on the payoff of the investment. We have to analyze the circumstances of market entry of the newcomer.

It isn't easy to solve the problem if there are attributes in the group which are non-market elements (externals) but nevertheless have an impact on the time required for payoff (e.g. tax- and regulation policy, pollution control, foreign currency policy, etc.).

Proof:

I defined the points of Nash equilibrium for the middle cubes of Rubik's Cube (four different fixed attributes) by searching for the attributes which aren't part of the Pareto optimal state.

$n-$ player constant sum games can be used to demonstrate the points of equilibrium for the four different attributes.

If we take a $(x_1^*, x_2^*, x_3^*, x_4^*) \epsilon S$ point of equilibrium, we can define that

$\varphi_1(x_1^*, x_2^*, x_3^*, x_4^*) \geq \varphi_1(x_1, x_2^* x_3^*, x_4^*)$ for every $x_1 \in S_1$.

and

$\varphi_2(x_1^*, x_2^*, x_3^*, x_4^*) \geq \varphi_2(x_1^*, x_2, x_3^*, x_4^*)$ for every $x_2 \in S_2$.

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and

$\varphi_3(x_1^*,x_2^*,x_3^*,x_4^*) \geq \varphi_3(x_1^*,x_2^*,x_3,x_4^*)$ for every $x_3 \in S_3$.

and

$\varphi_4(x_1^*,x_2^*,x_3^*,x_4^*) \geq \varphi_4(x_1^*,x_2^*,x_3^*,x_4)$ for every $x_4 \in S_3$.

The game is zero sum, therefore

$$\varphi_1(x_1,x_2,x_3,x_4)+\varphi_2(x_1,x_2,x_3,x_4)+\varphi_3(x_1,x_2,x_3,x_4)+\varphi_4(x_1,x_2,x_3,x_4)=0,$$

The second equality goes as follows

$$\varphi_1(x_1^*,x_2^*,x_3^*,x_4^*)+\varphi_2(x_1^*,x_2^*,x_3^*,x_4^*)+\varphi_3(x_1^*,x_2^*,x_3^*,x_4^*)+\varphi_4(x_1^*,x_2^*,x_3^*,x_4^*)=0$$

For either attribute to get a "not allowed difference" tag, as $\varphi_1(x_1,x_2^*x_3^*,x_4^*)$

$$\varphi_1(x_1^*,x_2^*,x_3^*,x_4^*) \geq \varphi_1(x_1,x_2^*x_3^*,x_4^*)$$

Prevalent for a constant sum game's every strategy as follows:

$$\varphi_1(x_1^*,x_2^*,x_3^*,x_4^*)+\varphi_2(x_1^*,x_2^*,x_3^*,x_4^*)+\varphi_3(x_1^*,x_2^*,x_3^*,x_4^*)+\varphi_4(x_1^*,x_2^*,x_3^*,x_4^*)$$
$$\geq \varphi_1(x_1,x_2^*x_3^*,x_4^*)+\varphi_2(x_1^*,x_2^*,x_3^*,x_4^*)+\varphi_3(x_1^*,x_2^*,x_3^*,x_4^*)+\varphi_4(x_1^*,x_2^*,x_3^*,x_4^*)$$

The point of equilibrium of the four player constant sum game ceases, if a shift in strategy happens for either of the factors:

$$(x_1^*,x_2^*,x_3^*,x_4^*) \longrightarrow (x_1,x_2^*,x_3^*,x_4^*)$$

thus the shift in strategy (the change of any element of strategies) leads to inequality,

$$\varphi_1(x_1^*,x_2^*,x_3^*,x_4^*) \geq \varphi_1(x_1,x_2^*,x_3^*,x_4^*)$$

This inequality-system states that if player one chooses a strategy different from x_i^* and thus leaves the $(x_1^*,x_2^*,x_3^*,x_4^*)$ equilibrium and the game itself, his payoff-function can only be either equal to or lower than that of the others. If the fourth player differs in a not allowed manner but the others don't change their strategies, then his payoff-function will also be equal to or lower compared to the $\varphi_{1,2,3}(x_1^*,x_2^*,x_3^*,x_4^*)$ of the others.

$$\varphi_{1,2,3}(x_1^*,x_2^*,x_3^*,x_4^*) \geq \varphi_4(x_1^*,x_2,x_3^*,x_4^*)$$

Since this is a zero sum game, meaning the total payment can neither get higher or lower, the payoff-function of the $\varphi_{1,2,3}(x_1^*,x_2^*,x_3^*,x_4^*)$ factors will either be equal to or greater as well.

Imaging algorithms of Output-side

One of the popular types of non-cooperative Game Theory solutions is conflict alleviation methods. From these, we can highlight the axiomatic solution system of Nash, which creates axiom aggregations in order to assure the solution always places on the Pareto-line. The Kálai-Smorodinsky solution defines the minimum reachable or the last available point (meaning worst acceptable) to the solution of the conflict by defining the worst possible leaving point of the conflict.

C) Game Theory modeling of output side (level 3)

The phase of setting the final equilibrium state by the corner switch on the leaving side, the equilibrium search, and the finalization of the sustainability criteria can usually only be done with cooperative strategy.

Definition:

Cooperative games can be defined by the following concepts. $\mathrm{N} = \{1, \ldots\ldots, \mathrm{n}\}$ as in aggregation of players, where the S subset is known as a coalition: $\mathrm{S} \subseteq \mathrm{N}$. Let S be an aggregation of the subsets, meaning the aggregation of possible coalitions. The N main aggregation is called coalition total.

Thesis:

In low-carbon investment concepts, the project generates energy drawn from renewable sources, but the produced electricity can only reach the consumer if the owners of both the green electricity producer (Investor/B) and the electricity system (System/H) agree with each other that the product reaches the consumer through the system. A criterion of cooperation is that the investor pays a usage/transport fee to the owner of the system, and the owner acknowledges that instead of the previous (fossilized) product, he transports a private product via the system, and in a lower volume. As compensation, the system gets the pay from the investor. This compromise, in essence, means that there has to be a valid agreement on provisioning conditions on the market. We tried to match the "green-yellow-orange" attribute cube of the previously established Rubik's Cube project planning method with the model, and to assign the proper strategy to the cooperation.

Proof:

We can introduce our conflict-alleviation method with a two-player game. In the example, let the players' strategies be represented by S_1 and S_2, and the two payoff-functions by φ_1 and φ_2. The aggregation of possible payoffs will therefore be 2D, and can be shown as follows:

$$H = \{\varphi_1(x,y), \varphi_2(x,y) \,|\, (x,y) \in S_1 \times S_2\}$$

In this case, as always, the payoff of both players aims at maximization, but naturally the various payoffs of one player depend on that of the other and the fact that raising one player's payoff will lower the other's stands as a rule. Therefore, the objective is to find a solution that is acceptable to both the investor and the system owner, meaning both parties simultaneously. We also have to state that in case of

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the agreement not being "signed," both parties get a lower payoff, or a punishment.

Standard representations:

$$f_* = (f_{1*}, f_{2*})$$

this will be our standard payoff vector, where we assume that there is a $(f_1, f_2) \in H$ where $f_1 > f_{1*}$, and $f_2 > f_{2*}$. The problem is defined mathematically with the pair. This pair was defined in Figure 5. We also assume that aggregation H is not open, convex, or bounded, so in the case of:

$$(f_1, f_2) \in H \; and \; \bar{f_1} \leq f_1, \bar{f_2} \leq f_2$$

$(\bar{f_1}, \bar{f_2}) \in H$ and bounded in both coordinates, meaning

$$sup\{f_i | (f_1, f_2) \in H\} < \infty$$

in case of $i = 1{,}2$.

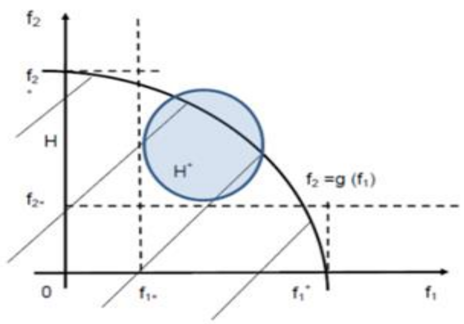

Figure 5: Figure of conflict state with the position of the payoff-function

Source: Fogarassy, 2014 (4)

We also assume that the borderline of H is the graph of a $f_2 = g(f_x)$ function, which is strictly falling in f_1 and is concave. The graph of function $g-$ is usually called the Pareto line; therefore, the conditions of satisfying the optimum criteria of sustainability can be met here. We must also take into consideration with the game and solution criteria that no rational player will accept a compromise that means a worse payoff than the payoff without agreement.

This way, we can tighten the payoff aggregation as follows:

$$H^* = \{f_1, f_2 | f_1 \geq f_{1*}, f_2 \geq f_{2*}, (f_1, f_2 \in H\}$$

IV. CONCLUSIONS

We concluded an unorthodox Game Theory optimum search on the different (cube) levels for the low-carbon planning of the project development process. During the Game Theory optimum search, we defined a theoretic model structure, which means fundamentally placing three different types of Game Theory solutions after each other, while keeping tabs on which Game Theory method is most efficient for featuring the various economic criteria systems:

1. Cube level one: non-cooperative three player game (for the correction of not allowed differences on Input side),
2. Cube level two: non-cooperative zero sum game (for the correction of not allowed differences of middle cube connections),
3. Cube level three: conflict alleviation method with two player game (for the correction of not allowed differences on Output side).

The three different Game Theory models can together define the states of Nash equilibrium required during project development, which help achieve sustainability during the realization of the project. The sufficient selection of Nash equilibrium is possible through the SMART value definition based on the correspondence system of the cubes. An introduction to this will be given later in this document. However, we must stress that the Game Theory row that we selected (three person cooperative game, non-cooperative zero sum game, conflict-alleviation method) is applicable mainly for typified energetic development, and a strictly defined economic environment (Hungary and Central Eastern Europe). Therefore, we can say that economic externals or development goals that differ from these can allow different Game Theory sequences to be used as well.

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