Performance Evaluation Of Different Control Schemes For Stabilization Of a Driven Pendulum System

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Abstract- This paper presents four different control methods to stabilize a driven pendulum system based on linear quadratic regulator (LQR) controller, lead-lag controller, simplified intelligent Controller and Propotional integral derivative (PID) controller methods. Driven pendulum is a suspended pendulum, which has a motorized propeller at the end of the stick. So it can be controlled with controlling the voltage given to direct current (DC) motor. The characteristics of the transient response of this system such as overshoots and settling time are not acceptable. For example the settling time of this system is upwards of 10 seconds. The discussed methods can improve the time domain performance of the linearized system. Performances of the controllers are examined in terms of the desire angle and angular velocity of pendulum. The simulation result shows that these methods enhance stability as well as ease of tuning. The presented controllers are designed and evaluated using MATLAB®/Simulink®. Finally, a comparative assessment of the impact of each controller on the system performance is presented and discussed.

Keywords – *Compound pendulum; Driven pendulum; PID controller; LQR controller; Lead-lag controller ; Simplified intelligent controller;*

I. INTRODUCTION

A simple pendulum system is a mechanical system that exhibits periodic motion. It consists of particle like bob of mass suspended by a light string of length that is fixed at the upper [1]. A Compound Pendulum is a standard topic in most physics courses because it includes some physical subjects such as the simple harmonic motion, the period of oscillation, the acceleration of gravity, the center of mass, the moment of the inertia, momentum, etc. [2].

This type of pendulum described in this paper has a motorized propeller at the end of the pendulum so it can lift the pendulum after given voltage. This concept of pendulum system is useful and can be applied in real life.This system has many applications such as measurement, scholar tuning, coupled pendulum, entertainment etc. The control problem in driven pendulum system is controlling the pendulum behavior, such as the stability, rise time, overshoots etc. with adjusting the given voltage[3]-[5].

Proportional-integral-derivative (PID) controller is a common feedback loop component used for control system. The controller takes a measured value from a process or other apparatus and compares it with a reference set point value. The difference (or error signal) is then used to adjust some input to the process in order to bring the process measured value back to its desired set point [6].

In this paper a PID controller was used so the pendulum reaches a steady-state angle with desired transient response and the presented method is based on the time domain performance of the system. The simulation results of PID control method proven that it is an easy-tuning, simple and effective way to control a driven pendulum.

Linear quadratic regulator (LQR) controller is linear-quadratic state feedback regulator for state-space system, appropriate matrix Q and R to enable system to achieve optimal stability by modifying weights in the matrix Q and R corresponding variables. The theory of optimal control is concerned with operating a dynamic system at minimum cost. The pendulum with this controller reaches a steady-state angle with desired transient response [7].

Phase-lead control generally improves rise time and damping but increases the natural frequency of the closed-loop system. However, phase-lag control when applied properly improves damping but usually results in a longer rise time and settling time. Therefore, each of these control schemes has its advantages, disadvantages, and limitations, and there are many systems that cannot be satisfactorily compensated by either scheme acting alone. It is natural, therefore, whenever necessary, to consider using a combination of the lead and lag controllers, so that the advantages of both schemes are utilized [8].

Fuzzy logic controller (FLC) performance is greatly dependent on its inference rules. In most cases, the more rules being applied to a FLC, the accuracy of the control action is enhanced. Nevertheless, a large set of rules requires more computation time. As a result, FLC implementation requires fast and high performance processors. In this paper, it is shown that the inference rule table of a two-input FLCs used to control a pendulum system can be reduced to form a single input fuzzy logic controller (SIFLC), which can be easily implemented using a lookup table[9]-[10].

The contents are organized as follows. Section II describes the driven pendulum and system modeling. Section III proposes a different control method for this system. In section IV, results of simulation of system with proposed controller. Section V gives the conclusion of this paper.

II. DRIVEN PENDULUM SYSTEM

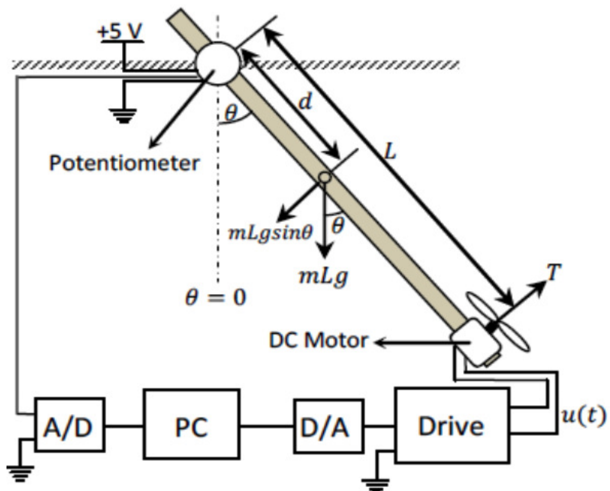

Fig. 1 Systematic diagram of a driven pendulum control systems

The systematic diagram of the driven pendulum system is given in Fig.1. This pendulum is driven by DC motor. It has a motorized propeller at the end of the stick as was shown in the figure. After applied voltage, the propeller spins and generates torque T to pull up the pendulum. It is clear that in order to analysis and control of a physical system, it is necessary to be known the mathematical model of it. Here, $u(t)$ is the control input and the angle which is between pendulum arm and vertical axis is the control variable.

By applying voltage, the propeller derives and generates torque T to move the pendulum. The control problem is to move the pendulum to a desired angle. The suspended point is connected to an encoder to give the measurements of angle and angular velocity of pendulum. It is the most benefits of driven pendulum that enables us controlling its operation with changing the applied voltage. Therefore, the controlled variable for this system is the angle of the pendulum settled and the manipulated variable is the voltage fed to the motorized-propeller [3]-[5].

According to Newton's laws and angular momentum, the motion equation of driven pendulum is derived as

$$j.\ddot{\theta} + c.\dot{\theta} + m_L.g.d.\sin\theta = T \qquad (1)$$

where,

θ = angular position of the pendulum

d = the distance between center of mass and pivot point

c = viscous damping coefficient

T = the trust which is provided by DC motar

l = length of pendulum

J = inertia moment

g = acceleration of gravity

m_l = weight of the pendulum

By considering Sin θ ≈ θ, the linearized motion equation can be written as follows

$$j.\ddot{\theta} + c.\dot{\theta} + m_l.g.d.\theta = T \qquad (2)$$

A. Transfer Function

Eq.(2) gives the transfer function of driven pendulum.

$$\frac{\theta(s)}{T(s)} = \frac{1}{j.s^2 + c.s + m_l.g.d} \qquad (3)$$

And the standard representation is

$$\frac{\theta(s)}{T(s)} = \frac{\frac{1}{j}}{s^2 + \frac{c}{j}s + \frac{m_l.g.d}{j}} \qquad (4)$$

The generated trust T in above equations is not manipulated variable for control system since the pendulum is adjusted by applied voltage.

The transfer function of motorized propeller can be modeled as shown in Fig.2

Fig.2 Block diagram of Motorized Propeller V(s)

The method to obtain this gain was shown in [6] as

$$k_m = \frac{m_l.g.d.\theta}{v(s)} \qquad (5)$$

The block diagram of driven pendulum is given in Fig.3

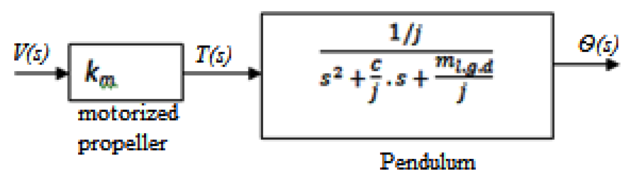

Fig.3 Block diagram of driven pendulum

Moreover, the transfer function is obtained by

$$\frac{\theta(s)}{V(s)} = \frac{Km/j}{s2 + \frac{c}{j}s + \frac{ml.g.d}{j}} \qquad (6)$$

B. State space

Consider this system represented in state space by

$$X_1 = \theta,\ X_2 = \dot{\theta},\quad X = \dot{X}_1 \qquad (7)$$

is written as

$$\begin{bmatrix} \dot{X}1 \\ \dot{X}2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{ml.g.d}{j} & -\frac{c}{j} \end{bmatrix} . \begin{bmatrix} X1 \\ X2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{Km}{j} \end{bmatrix} .u \qquad (8)$$

$$Y = \begin{bmatrix} 1 & 0 \end{bmatrix} . \begin{bmatrix} X1 \\ X2 \end{bmatrix} + 0$$

III.CONTROL METHODS

1 .PID Control method

When the characteristics of a plant are not suitable, they can be changed by adding a compensator in the control system. One of the simple and useful compensators feedback control design is described in this section. In this paper, the control method is designed based on the time dimension performance specifications of the system, such as settling time rise time, peak overshoot, and steady state error and so on.

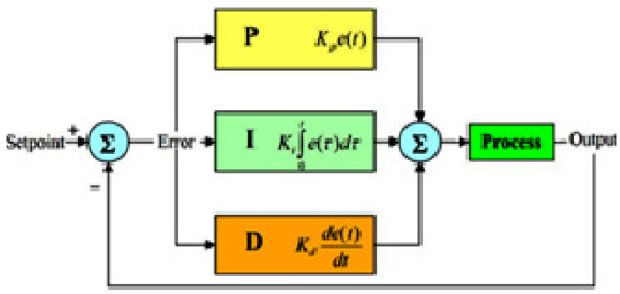

Fig 4. Block diagram of a PID controller [10]

A PID controller calculates an "error" value as the difference between a measured process variable and a desired set point. The controller attempts to minimize the error by adjusting the process control inputs [10].

$$u(t) = k_p e(t) + k_i \int_0^{\infty} e(t)\, dt + k_d \frac{de(t)}{dt} \quad (9)$$

Three parameters must be adjusted in the PID controller k_p, k_i, and k_d. In guaranteeing stability and performance and shaping the closed-loop response, it is important to select a suitable compensator.

(A) Proportional gain k_p : Large proportional control can increase response speed and reduce the steady state error, but will lead to oscillation of the system.

(B) Derivative gain k_d : The derivative term slows the rate of change of the controller output and this effect is most noticeable close to the controller set point. Hence, derivative control is used to reduce the magnitude of the overshoot produced by the integral component and improve the combined controller-process stability.

(C) Integral gain k_I: Integral control is favorable for diminishing the steady state error but it will lengthen the transient response.

This paper attempts to design optimal values for controller parameters. Then it obtained the value of PID gains by Ziegler and Nichols method [11]. Ziegler and Nichols provided a technique for selecting thePID gains that works for a large class of industrial Systems.These equations can be written as

$$k_p = 0.6\, k_m \;,\; k_d = \frac{k_p \pi}{4 w_m} \quad k_i = \frac{k_p w_m}{\pi} \quad (10)$$

2 . LQR Controller design

Here we see the linear quadratic regulator (LQR) control technique. This technique uses a state-space approach to analyze a system. This method provides a systematic way of computing the state feedback control gain matrix. In optimal control one attempts to find a controller that provides the best possible performance with respect to some given measure of performance. In general, optimality with respect to some criterion is not the only desirable property for a controller. One would also like stability of the closed-loop system [7].

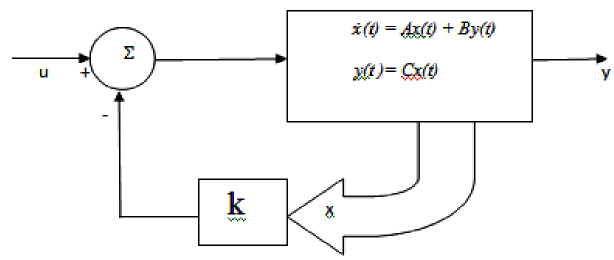

Fig 5 : Block diagram for optimal configuration.

We shall now consider the optimal regulator problem that, given the system equation

$$x = Ax + Bu \quad (11)$$

Determines the matrix K of the optimal control vector

$$u(t) = -k\, x(t) \quad (12)$$

So as to minimize the performance index

$$J = \int_0^{\infty} (x * qx + u * Ru)\, dt \quad (13)$$

where, Q is a positive-semi definite and R is a positive-definite matrix. The matrices Q and R determine the relative importance of the error. Here the elements of the matrix K are determined so as to minimize the performance index, then u(t) = - k x(t) is optimal for any initial state x(0).

The eq.(13), can be further simplified to,

$$A^*P + PA - PBR^{-1}B^*P + Q = 0 \quad (14)$$

where, P is a positive-definite Hermitian or real symmetric matrix. If the system is stable, there always exists one positive-definite matrix P to satisfy this equation. Equation (14) is called the reduced-matrix Riccati equation. The design steps may be stated as follows.

1. Solve equation (14), the reduced-matrix Riccati equation, for the matrix P.
2. Substitute the matrix P in equation $K=R^{-1}B^*P$ The resulting matrix K is the optimal matrix.

Another option is to use the LQR function in matlab to obtain the optimal controller. By using LQR function in matlab, two matrices i.e. Q and R are to be chosen which will balance the relative importance of the input and state of the function, for achieving optimization

3. Lead-Lag Method

Phase-lead control generally improves rise time and damping but increases the natural frequency of the closed-loop system. However, phase-lag control when applied properly improves damping but usually results in a longer rise time and settling time. Therefore, each of thesAe control schemes has its advantages, disadvantages, and limitations, and there are many systems that cannot be satisfactorily compensated by either scheme acting alone. It is natural, therefore, whenever necessary, to consider using a combination of the lead and lag controllers, so that the advantages of both schemes are utilized [8].
The transfer function of a simple lag-lead or lead-lag controller can be written as

$$G_c(s) = G_{c1}(s)\, G_{c2}(s) = \left(\frac{1+a1T1s}{1+T1s}\right)\left(\frac{1+a2T2s}{1+T2s}\right) \qquad (15)$$

4. Simplified Intelligent Controller

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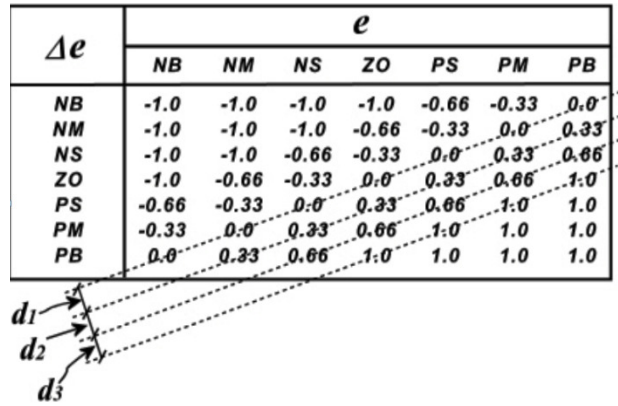
Table I Rule base with 49 rules

Δe \ e	NB	NM	NS	ZO	PS	PM	PB
NB	-1.0	-1.0	-1.0	-1.0	-0.66	-0.33	0.0
NM	-1.0	-1.0	-1.0	-0.66	-0.33	0.0	0.33
NS	-1.0	-1.0	-0.66	-0.33	0.0	0.33	0.66
ZO	-1.0	-0.66	-0.33	0.0	0.33	0.66	1.0
PS	-0.66	-0.33	0.0	0.33	0.66	1.0	1.0
PM	-0.33	0.0	0.33	0.66	1.0	1.0	1.0
PB	0.0	0.33	0.66	1.0	1.0	1.0	1.0

Complex fuzzy rule bases usually suffer from large number of parameters, heavy computation load, and large memory space, which usually affects the design and application of FLCs. A reduction or simplification of a given fuzzy rule base to obtain a simple controller is therefore a desirable process.
In particular, the symmetry of the rules in Table I can be exploited to reduce the set of rules and develop an equivalent single-input single-output FLC. It is seen that TableI establishes a control action magnitude proportional to the perpendicular distance from a given consequence in the table to the line of zero consequence (switching line), as shown in Fig. 7, where *d1*, *d2* and *d3* are correspondingly the same for the negative control action area. A better picture of this is shown through the phase-plane, as in Fig. 8. From the switching line in Fig. 2, points in the upper half-plane are associated with positive control actions and those in the lower half-plane demand negative control signals [9].



Δe \ e	NB	NM	NS	ZO	PS	PM	PB
NB	-1.0	-1.0	-1.0	-1.0	-0.66	-0.33	0.0
NM	-1.0	-1.0	-1.0	-0.66	-0.33	0.0	0.33
NS	-1.0	-1.0	-0.66	-0.33	0.0	0.33	0.66
ZO	-1.0	-0.66	-0.33	0.0	0.33	0.66	1.0
PS	-0.66	-0.33	0.0	0.33	0.66	1.0	1.0
PM	-0.33	0.0	0.33	0.66	1.0	1.0	1.0
PB	0.0	0.33	0.66	1.0	1.0	1.0	1.0

Fig 6. Illustration of *d1*, *d2* and *d3* in the rule table.

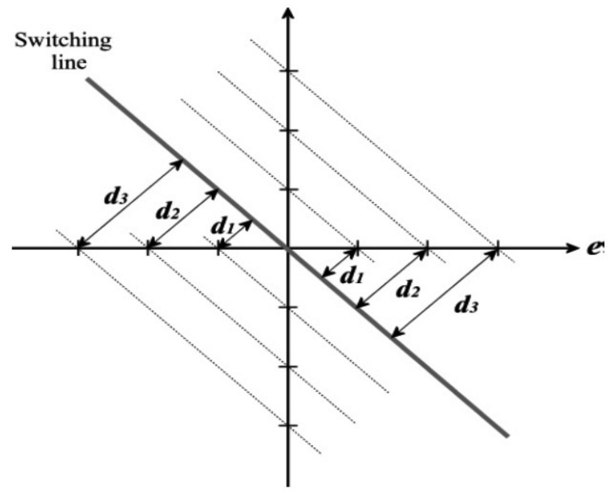


Fig 7. Illustration of *d1*, *d2* and *d3* in the phase-plane.

If the switching line in Fig. 8 is expressed in the general form for the equation of a line as

$$Ax + By + C = 0 \qquad (16)$$

then the following expression is obtained as

$$Ae + B\Delta e + C = 0 \qquad (17)$$

where $A=-1$, $B=-1$ and $C=0$.
Now, for any point $Po(eo\ ,\Delta eo)$ in the phase-plane, the perpendicular distance d illustrated in Fig.9 from this point to the line defined by is given by [9]

$$d = \frac{Ae+B\Delta e+C}{\sqrt{A^2+B^2}} \qquad (18)$$

and applying the particular values for A, B and C, then

$$d = f_1(e,\Delta e) = \frac{-e-\Delta e}{\sqrt{2}} = \frac{-(e+\Delta e)}{\sqrt{2}} \qquad (19)$$

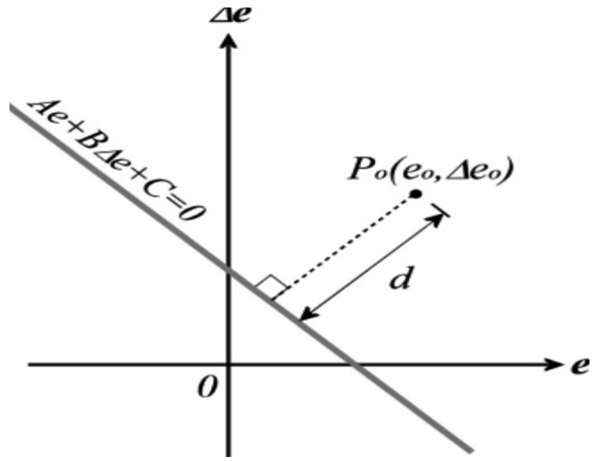


Fig 8. Distance *d* from *Po(eo ,Δeo)*.

Look at Table I, and consider that triangular membership functions, evenly distributed in the range {-1,1}, are used for the inputs. Now, if *Δe* is 0.0 and *e* takes as values the center points of the fuzzy sets ZO, PS, PM,PB,NS,NM,and NB, then the distance *d* computed from will be 0.0, 0.2357, 0.4714 and 0.7071, respectively. When normalizing these values in the range {0,1}, the following quantities will be obtained, respectively: 0.0, 0.3333, 0.6666 and 1.0.

The same results are found when *e* is 0.0 and *Δe* varies from ZO to PB through the corresponding center points, and *d* is computed and normalized in a similar way. From this, three parallel lines with normalized *d* equal to 0.3333, 0.6666 and 1.0, respectively from the switching line, are found in the upper half-plane, as in Fig. 8. Since the negative control action area in Table I is symmetrical to its positive counterpart, the situation is similar in the lower half-plane [9]. These values of *d* can be associated with the center points of triangular membership functions specified in a normalized universe of discourse. Therefore, a one-dimensional fuzzy rule table with the following fuzzy terms for the normalized distance *d* and fuzzy singletons for the control action $\dot{u}$ can be shown in table II.

Table II. Reduced rule base

d	NB	NM	NS	ZO	PS	PM	PB
$\dot{u}$	-1.0	- .66	- .33	0	0.33	0.66	1.0

Where ZO,PS,PO,PM,NS,NM,NB are zero, positive small, positive medium, positive big, negative small, negative medium ,negative big respectively. It is clear that the size of the original rule base in Table I can be significantly reduced (see Table II) by taking advantage of its symmetrical form.

The main advantage of SIFLC is the significant reduction of the rules that needs to be inferred. The reduction in the number of rules results in faster Calculation [10].

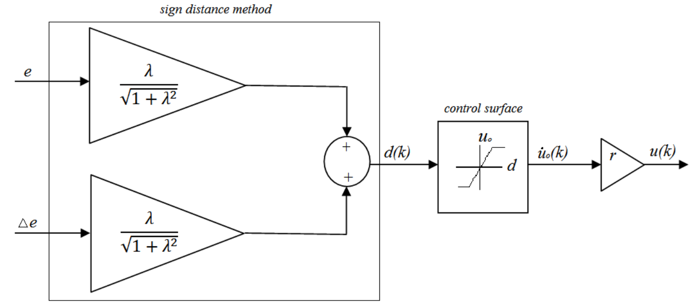


Fig 9. Simplified intelligent fuzzy logic controller

IV SIMULINK MODELS AND REULTS

After the mathematical model of system obtained, control system designed. Then MATLAB®/ Simulink® was used to done the simulation of system behavior.

The model parameters are based on an experimental set up previously by [4].

where,

$d = 0.03\ m,\ \ m_{pl} = 0.36\ kg,\ g = 9.8\ m/sn^2,$

$J = 0.0106\ Kgm^2,\ c = 0.0076\ Nms/rad,\ Km = 0.0296$

Therefore, the transfer function is given by

$$\frac{\theta(s)}{V(s)} = \frac{2.7922}{s2 + 0.7191s + 9.9989}$$

From eq.(10) pid gains are computed as follows

$$K_p = 2.2,\ \ K_d = 1.1,\ K_i = 0.1$$

The open loop response is given in Fig 11. It illustrates the need for control. The settling time of the system is upwards of 10 seconds. The reason for the persistent oscillations is a lack of damping in the system.

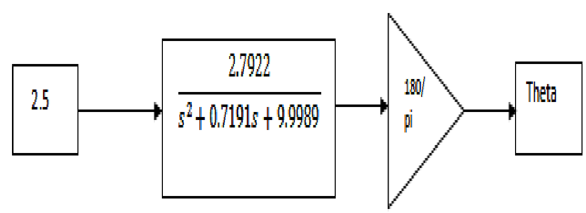


Fig. 10 Open Loop System Modeling in Matlab/Simulink

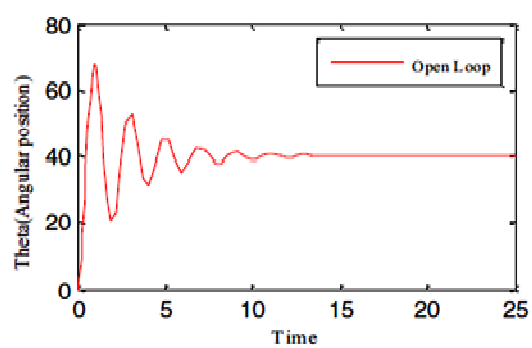


Fig.11 Responce of Open Loop System

1 .PID Control Method

The response of driven pendulum with PID controller is shown in Fig.13. From figure the rise time and settling time decreased with control system. In addition, output of the system reaches the desired value without overshoot. Results illustrate the fact that simulation response curves are in good agreement.

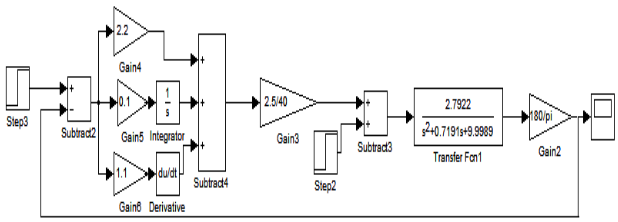


Fig.12 MATLAB®/Simulink® model of driven Pendulum with PID controller

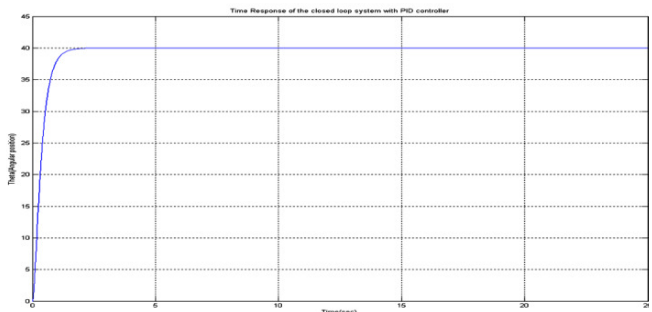


Fig.13 Time response of the closed loop System with PID Controller Theta (Angular Position) Vs time

2 . LQR Controller Design

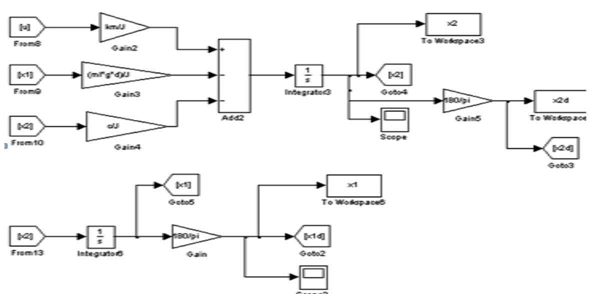


Fig.14 MATLAB®/Simulink® model of driven Pendulum with LQR controller

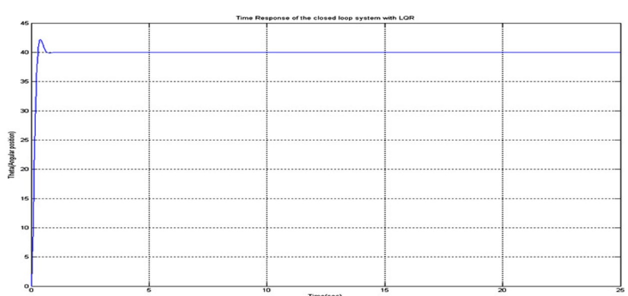


Fig .15 Time Response of the driven Pendulum system with LQR controller Theta (Angular Position) Vs time

3. Lead-Lag Method

Based on the settling time and velocity error constant the transfer function of the lag-lead compensator by trial and error as follows

:

$$\left(\frac{11S+22}{S+10}\right)*\left(\frac{0.837S+0.1}{S+0.1}\right)$$

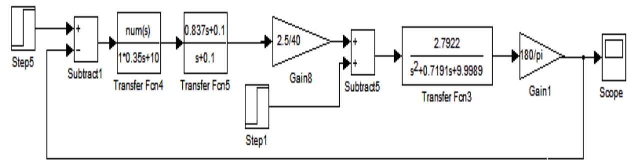


Fig.16 MATLAB®/Simulink® model of driven Pendulum with Lag-Lead compensator

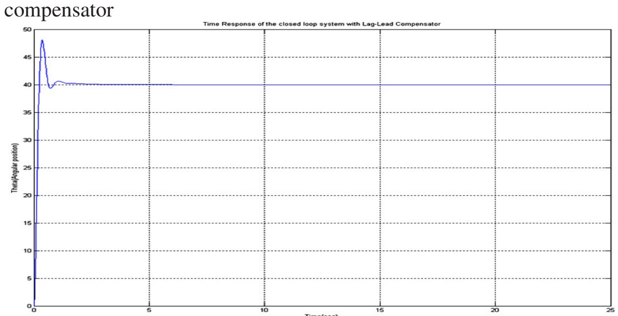


Fig. 17 Time Response of the driven pendulum System with Lead Lag Controller Theta (Angular Position) Vs time

4. Simplified Intelligent Controller

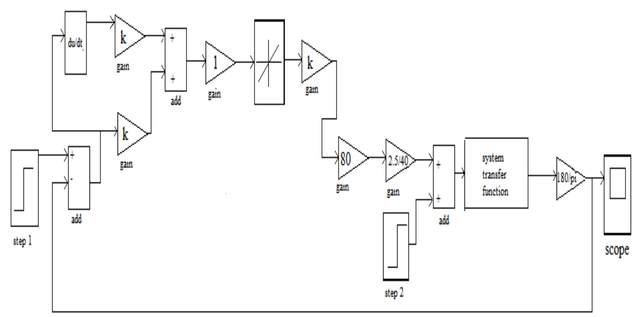


Fig.18 MATLAB®/Simulink® model of driven Pendulum with Simplified intelligent fuzzy logic controller

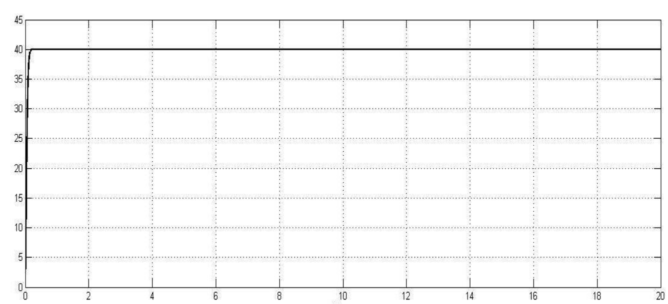


Fig.19 Time Response of the driven Pendulum system with SIFLC Theta (Angular Position) Vs time

Comparing results of control mtehods

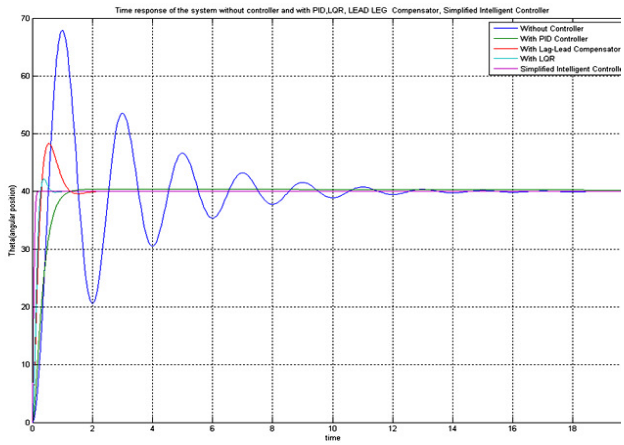


Fig. 20 Comparison of Time Responses of the driven Pendulum system with different controllers Theta (Angular Position) Vs time.

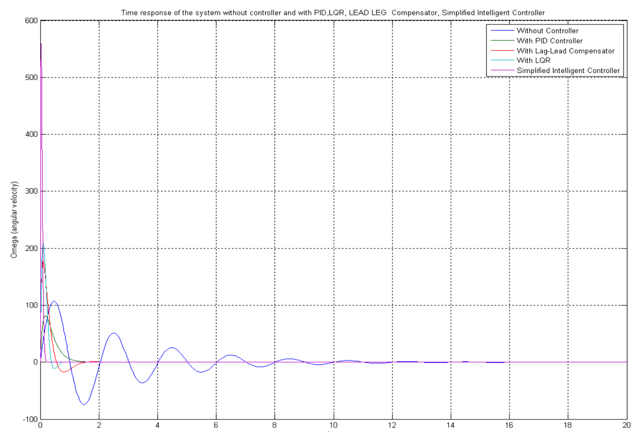


Fig. 21 Comparison of Time Responses of the driven pendulum System with different controllers omega (Angular Position) Vs time.

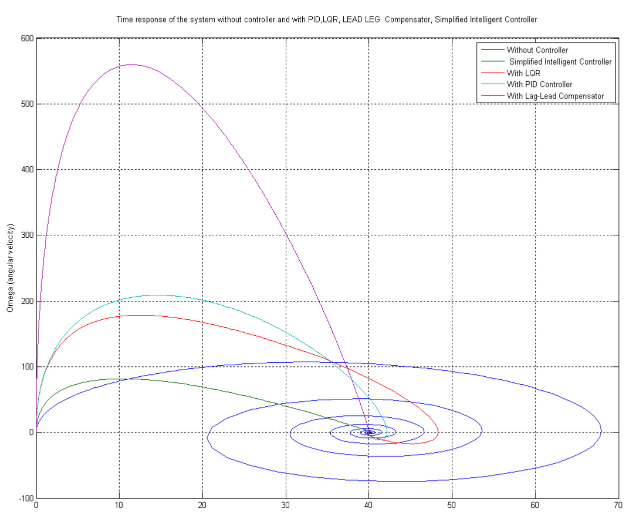


Fig.22 Comparison of stability of the driven Pendulum System with different controller in phase –plane.

Comparison of different control schemes for system shown in table III

Table III Comparison of parameters

Controlling method	Rise time(t_s)	Settling time(t_s)	Over shoot(θ)
Without controller	1.5	14	30
PID controller	0.6	1.2	2
LQR controller	0.2	0.6	4
Lead-lag controller	0.3	0.2	9
SIFLC controller	0	0.1	0

V CONCLUSION

In this paper, Simplified Intelligent Controller, PID controller, LQR controller and Lag-Lead compensator are utilized to stabilize and control the behavior of a driven pendulum.

The simulation was developed for the system with and without controllers in MATLAB®/Simulink®. The time response characteristics of driven pendulum improved with LQR control. LQR controller is a simple and effective way to stabilize a driven pendulum.PID controller is designed for a linearized driven pendulum system. The Response characteristics of driven pendulum improved with PID control. The PID control method is an easy-tuning and more effective way to enhance stability of time domain performance of the driven pendulum system. The time response characteristics of driven pendulum improved with Lag-Lead compensator.Simplified Intelligent Controller can be easily implemented using a lookup table. It can be observe that the simplified intelligent controller offers better dynamic and steady state response.Compared to all methods simplified intelligent controller shows no overshoot and good time response characteristics.

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