TIME STEP STRESS ACCELERATED LIFE TEST MODEL FOR PARETO DISTRIBUTION

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Abstract: **The step-stress accelerated life tests allow the experimenter to increase the stress levels at fixed times during the experiment. This paper discusses the design of the optimal SSALT plan using type-I censoring for Pareto distribution. The scale parameter of the distribution is assumed to be a log-linear function of the stress and a cumulative exposure model holds. Point estimates as well as the interval estimates of the model parameters are obtained. Optimal step stress ALT plan is proposed by minimizing the asymptotic variance of the MLE of the percentile of the lifetime distribution at normal stress condition. A simulation study is also performed to analyze the performance of parameter estimates.**

Keywords: **Cumulative Exposure Model; Type-I Censoring; Maximum Likelihood Estimation Method; Fisher Information Matrix; Asymptotic Confidence Intervals; Simulation Study.**

1. INTRODUCTION

In traditional life tests, products are tested under normal operating conditions to infer the parameters of the life distribution so that the life, reliability and other quality associated measures of the product can be expected. These days' products have great reliability because of their good manufacturing designs, quality material used in it and use of advanced technologies for quality improvements. Therefore, traditional life tests have become very time consuming and costly which renders them of no practical use. One way to overcome this problem is the use accelerated life tests (ALTs) in which the products are tested at more severe than operating conditions to induce early failures. The test data obtained at these severe (accelerated) conditions is then analyzed and extrapolated by using a suitable physical model to obtain the life characteristics of the product at use stress level. Interested readers may refer to Meeker and Escobar [9] and Nelson [11].

In Step-stress ALT all test items are first tested at a specified constant stress for a specified period of time and then Items which are not failed will be tested at next higher level of stress for another specified time and so on until all items have failed or the test stops for other reasons. The step-stress scheme allows the stress setting of a product to be changed at pre-specified times or upon the occurrence of a fixed number of failures. The former is called time-step-stress ALT and the latter failure-step-stress ALT. The step-stress ALT has been studied by several authors. Miller and Nelson [10] obtained the optimal simple step-stress ALT plans for the case where test products have exponentially distributed lives and are observed continuously until all test products fail; Bai et al. [1] extended their results to the case of censoring. The optimal step-stress test under progressive type-I censoring, assuming exponential lifetime distribution was considered by Gouno et al. [5]. Gouno [4] studied optimum step-stress for temperature ALT models. Balakrishnan et al. [2] considered the simple step-stress ALT under type II censoring, assuming a cumulative exposure model with lifetimes being exponentially distributed. They have obtained distributions of the MLEs of the parameters using exact distributions. For more recent research on step-stress ALTs, see Xu and Fei [14], Li and Fard [7], Fan et al. [3], Nelson [12], Ma and Meeker [8], Wu et al. [13] and Kamal et al. [6].

In this paper the problem of simple-step-stress ALT with type-I censoring for Pareto distribution is considered. The estimates of parameters of model are obtained by using maximum likelihood estimation method. Fisher information matrix is constructed to obtain the asymptotic variance of the parameters. The asymptotic confidence intervals for model parameters are also obtained. An optimal step stress ALT plan is also proposed. To examine the performance and statistical properties of the estimates obtained here in this study are evaluated by a simulation study with different pre-fixed values of parameters.

2. TEST PLAN AND MODEL DESCRIPTION

2.1 *The Pareto Distribution*

The concept of this distribution was first introduced by Vilfredo Pareto (1897) in his well known economics text "Cours d'Economie Politique".

The two parameter form of Pareto probability density function (pdf), cumulative distribution function (CDF), the reliability function (RF) and the hazard rate (HR) with shape parameter and scale parameter given respectively by

$$f(t;\theta,\alpha)=\frac{\alpha\theta^{\alpha}}{(\theta+t)^{\alpha+1}},\ t>0,\theta>0,\alpha>0 \tag{1}$$

$$F(t;\theta,\alpha)=1-\frac{\theta^{\alpha}}{(\theta+t)^{\alpha}},\ t>0,\theta>0,\alpha>0 \tag{2}$$

$$R(t)=\frac{\theta^{\alpha}}{(\theta+t)^{\alpha}} \tag{3}$$

$$h(t)=\frac{\alpha}{(\theta+t)} \tag{4}$$

The HR is a decreasing function as $t>0$ and an increasing function as $t<0$.

2.2 *Pareto Cumulative Exposure Model*

According to cumulative exposure model, the remaining life of test items depends only on the current cumulative fraction failed and current stress regardless of how the fraction accumulated. Moreover, if held at the current stress, items will fail according to the CDF of stress, but starting at the previously accumulated fraction failed. Numerically CE model the is given by

$$F(t)=\begin{cases}F_1(t) & 0\le t<\tau\\ F_2(t-\tau+\tau') & \tau\le t<\infty\end{cases} \tag{5}$$

where τ', is obtained by solving $F_1(\tau)=F_2(\tau')$ for τ' that is $\tau'=(\theta_2/\theta_1)\,\tau$.

Now using the value of τ' in (5), the corresponding CDF and PDF for the simple SSALT model is given by

$$F(t)=\begin{cases}F_1(t), & 0<t<\tau\\ F_2\left(\frac{\theta_2}{\theta_1}\tau+t-\tau\right), & \tau\le t<\infty\end{cases} \tag{6}$$

$$f(t)=\begin{cases}f_1(t), & 0<t<\tau\\ f_2\left(\frac{\theta_2}{\theta_1}\tau+t-\tau\right), & \tau\le t<\infty\end{cases} \tag{7}$$

From the equations (6) and (7), the CDF and PDF of a test product failing according to Pareto distribution under simple SSALT are given respectively by

$$F(t)=\begin{cases}1-\frac{\theta_1^{\alpha}}{(\theta_1+t)^{\alpha}}, & 0<t<\tau\\ 1-\frac{\theta_2^{\alpha}}{\left[\theta_2+\tau\left(\frac{\theta_2}{\theta_1}-1\right)+t\right]^{\alpha}} & \tau\le t<\infty\end{cases} \tag{8}$$

$$f(t)=\begin{cases}\frac{\alpha\theta_1^{\alpha}}{(\theta_1+t)^{\alpha+1}}, & 0<t<\tau\\ \frac{\alpha\theta_2^{\alpha}}{\left[\theta_2+\tau\left(\frac{\theta_2}{\theta_1}-1\right)+t\right]^{\alpha+1}} & \tau\le t<\infty\end{cases} \tag{9}$$

2.3 *Assumptions*

i. Two stress levels S_1 and S_2 $S_1<S_2$ are used.

ii. A random sample of n identical products are placed on test initially at stress level S_1 and run until time τ, then the stress is changed to S_2 and the test is continued until a pre-determined censoring time η (type-I censoring).

iii. For any level of stress, the product failure times follow Pareto distribution.

iv. The scale parameter θ is a log-linear function of stress i.e. $\log(\theta_i)=\beta_0+\beta_1 S_i$, where β_0 and β_1 are unknown parameters depending on the nature of the product and the test method.

v. A cumulative exposure model holds

Now we have first to estimate the parameters β_0, β_1 and α in a time step stress ALT and second is to obtain the optimal stress changing time τ which minimizes the AV of the ML estimate of the P^{th} percentile $t_p(S_0)$ of the lifetime distribution at normal stress condition.

3 ESTIMATION OF MODEL PARAMETERS

Maximum likelihood (ML) method of parameter estimation is used to estimate the model parameters because it is not only very robust but gives the estimates of parameter with good statistical properties. However, it is very simple for one parameter distributions but its implementation in ALT is mathematically more complicated and does not give the estimates of parameters in closed form, therefore, numerical techniques such as Newton-Raphson method and some computer programs are used to compute them.

Let n_1 the number of failures that occurs before τ at stress level S_1 and n_2 denote the number of failures that occur before η at stress level S_2, and N denote the total number of failures observed before termination (i.e. $N=n_1+n_2$). Failures occur in order statistic $0<t_{1:n}<\ldots<t_{n_1:n}<\tau<t_{n_1+1:n}<\ldots<t_{N:n}<\eta$ and the likelihood function in SSALT with type-I censoring for Pareto distribution can be written as

$$L(t;\theta,\alpha)=\frac{n!}{n_\eta!}\left[\prod_{i=1}^{n_1} f_1(t_{i:n})\right]\left[\prod_{i=n_1+1}^{N} f_2\left(\frac{\theta_2}{\theta_1}\tau+t_{i:n}-\tau\right)\right]\left[1-F_2\left(\frac{\theta_2}{\theta_1}\tau+\eta-\tau\right)\right]^{n_\eta} \tag{10}$$

Where $n_\eta=n-N$ and $t\equiv(t_{1:n},\ldots,t_{N:n})$

The log-likelihood function $\log L(t;\theta,\alpha)$, denoted by $l(t;\theta,\alpha)$ takes the form

$$l(t;\theta,\alpha)=\log\left(\frac{n!}{n_\eta!}\right)+\sum_{i=1}^{n_1}\log f_1(t_{i:n})$$
$$+\sum_{i=n_1+1}^{N}\log f_2\left(\frac{\theta_2}{\theta_1}\tau+t_{i:n}-\tau\right)+n_\eta\log\left[1-F_2\left(\frac{\theta_2}{\theta_1}\tau+\eta-\tau\right)\right] \quad (11)$$

Now the parameters of the Pareto distribution can be estimated by substituting the relevant values in the above mentioned form of likelihood function. Hence, the log likelihood function of a two parameter Pareto distribution for time step stress ALT takes the form

$$l(t;\theta,\alpha)=\log\left(\frac{n!}{n_\eta!}\right)+\sum_{i=1}^{n_1}\left[\log\alpha+\alpha\log\theta_1-(\alpha+1)\log(\theta_1+t_{i:n})\right]$$
$$+\sum_{i=n_1+1}^{N}\left[\log\alpha+\alpha\log\theta_2-(\alpha+1)\log\left(\theta_2+\left(\frac{\theta_2}{\theta_1}-1\right)\tau+t_{i:n}\right)\right] \quad (12)$$
$$+n_\eta\left[\alpha\log\theta_2-\alpha\log\left(\theta_2+\left(\frac{\theta_2}{\theta_1}-1\right)\tau+\eta\right)\right]$$

By using assumption (iv) in equation (12), the log likelihood function of for Pareto distribution in time step stress ALT takes the form

$$l(t;\theta,\alpha)=\log\left(\frac{n!}{n_\eta!}\right)+\sum_{i=1}^{n_1}\left[\log\alpha+\alpha(\beta_0+\beta_1S_1)-(\alpha+1)\log(e^{(\beta_0+\beta_1S_1)}+t_{i:n})\right]$$
$$+\sum_{i=n_1+1}^{N}\left[\log\alpha+\alpha(\beta_0+\beta_1S_2)-(\alpha+1)\log\left(e^{(\beta_0+\beta_1S_2)}+\left(e^{\beta_1(S_2-S_1)}-1\right)\tau+t_{i:n}\right)\right]$$
$$+n_\eta\left[\alpha(\beta_0+\beta_1S_2)-\alpha\log\left(e^{(\beta_0+\beta_1S_2)}+\left(e^{\beta_1(S_2-S_1)}-1\right)\tau+\eta\right)\right] \quad (13)$$

Differentiating (13) partially with respect to α,β_0 and β_1, we get

$$\frac{\partial l}{\partial\alpha}=\frac{N}{\alpha}+(N+n_\eta)\beta_0+(n_1S_1+n_2S_2+n_\eta S_2)\beta_1$$
$$-\sum_{i=1}^{n_1}\log A-\sum_{i=n_1+1}^{N}\log B-n_\eta\log C \quad (14)$$

$$\frac{\partial l}{\partial\beta_0}=(N+n_\eta)\alpha-(\alpha+1)\left[\sum_{i=1}^{n_1}\frac{e^{(\beta_0+\beta_1S_1)}}{A}+\sum_{i=n_1+1}^{N}\frac{e^{(\beta_0+\beta_1S_2)}}{B}\right] \quad (15)$$
$$-\frac{n_\eta\alpha}{C}e^{(\beta_0+\beta_1S_2)}$$

$$\frac{\partial l}{\partial\beta_1}=\alpha(n_1S_1+n_2S_2+n_\eta S_2)$$
$$-(\alpha+1)\left[\sum_{i=1}^{n_1}\frac{S_1e^{(\beta_0+\beta_1S_1)}}{A}+\sum_{i=n_1+1}^{N}\frac{\left\{S_2e^{(\beta_0+\beta_1S_2)}+\tau(S_2-S_1)e^{\beta_1(S_2-S_1)}\right\}}{B}\right]$$
$$-\frac{n_\eta\alpha}{C}\left\{S_2e^{(\beta_0+\beta_1S_2)}+\tau(S_2-S_1)e^{\beta_1(S_2-S_1)}\right\} \quad (16)$$

where

$$A=(e^{(\beta_0+\beta_1S_1)}+t_{i:n})$$
$$B=\left(e^{(\beta_0+\beta_1S_2)}+\left(e^{\beta_1(S_2-S_1)}-1\right)\tau+t_{i:n}\right)$$
$$C=\left(e^{(\beta_0+\beta_1S_2)}+\left(e^{\beta_1(S_2-S_1)}-1\right)\tau+\eta\right)$$

4. FISHER-INFORMATION MATRIX

The Fisher-information matrix composed of the negative second partial derivatives of log likelihood function can be written as

$$F=\begin{bmatrix}-\frac{\partial^2 l}{\partial\alpha^2} & -\frac{\partial^2 l}{\partial\alpha\partial\beta_0} & -\frac{\partial^2 l}{\partial\alpha\partial\beta_1}\\ -\frac{\partial^2 l}{\partial\beta_0\partial\alpha} & -\frac{\partial^2 l}{\partial\beta_0^{\,2}} & -\frac{\partial^2 l}{\partial\beta_0\partial\beta_1}\\ -\frac{\partial^2 l}{\partial\beta_1\partial\alpha} & -\frac{\partial^2 l}{\partial\beta_1\partial\beta_0} & -\frac{\partial^2 l}{\partial\beta_1^{\,2}}\end{bmatrix}$$

Where the elements of matrix are given as

$$\frac{\partial^2 l}{\partial\alpha^2}=-\frac{N}{\alpha^2}$$

$$\frac{\partial^2 l}{\partial\beta_0^{\,2}}=-(\alpha+1)\left[\sum_{i=1}^{n_1}\frac{t_{i:n}e^{(\beta_0+\beta_1S_1)}}{A^2}+\sum_{i=n_1+1}^{N}\frac{e^{(\beta_0+\beta_1S_2)}\left\{\left(e^{\beta_1(S_2-S_1)}-1\right)\tau+t_{i:n}\right\}}{B^2}\right]$$
$$-\frac{n_\eta\alpha}{C^2}e^{(\beta_0+\beta_1S_2)}\left\{\left(e^{\beta_1(S_2-S_1)}-1\right)\tau+\eta\right\}$$

$$\frac{\partial^2 l}{\partial\beta_1^{\,2}}=-(\alpha+1)\left[\sum_{i=1}^{n_1}\frac{t_{i:n}S_1^2e^{(\beta_0+\beta_1S_1)}}{A^2}+\sum_{i=n_1+1}^{N}\frac{t_{i:n}\left\{S_2^2e^{(\beta_0+\beta_1S_2)}+\tau(S_2-S_1)^2e^{\beta_1(S_2-S_1)}\right\}+S_1^2\tau e^{\{\beta_0+\beta_1(2S_2-S_1)\}}}{B^2}\right]$$
$$-\frac{n_\eta\alpha\eta}{C^2}\left\{S_2^2e^{(\beta_0+\beta_1S_2)}+\tau(S_2-S_1)^2e^{\beta_1(S_2-S_1)}\right\}+S_1^2\tau e^{\{\beta_0+\beta_1(2S_2-S_1)\}}$$

$$\frac{\partial^2 l}{\partial\alpha\partial\beta_0}=\frac{\partial^2 l}{\partial\beta_0\partial\alpha}=(N+n_\eta)-e^{(\beta_0+\beta_1S_1)}\sum_{i=1}^{n_1}\frac{1}{A}-e^{(\beta_0+\beta_1S_2)}\left[\sum_{i=n_1+1}^{N}\frac{1}{B}+\frac{n_\eta}{C}\right]$$

$$\frac{\partial^2 l}{\partial\alpha\partial\beta_1}=\frac{\partial^2 l}{\partial\beta_1\partial\alpha}=(n_1S_1+n_2S_2+n_\eta S_2)-\sum_{i=1}^{n_1}\frac{S_1e^{(\beta_0+\beta_1S_1)}}{A}$$
$$-\sum_{i=n_1+1}^{N}\frac{\left\{S_2e^{(\beta_0+\beta_1S_2)}+\tau(S_2-S_1)e^{\beta_1(S_2-S_1)}\right\}}{B}$$
$$-\frac{n_\eta}{C}\left\{S_2e^{(\beta_0+\beta_1S_2)}+\tau(S_2-S_1)e^{\beta_1(S_2-S_1)}\right\}$$

$$\frac{\partial^2 l}{\partial\beta_0\partial\beta_1}=\frac{\partial^2 l}{\partial\beta_1\partial\beta_0}$$
$$=-(\alpha+1)\left[\sum_{i=1}^{n_1}\frac{S_1t_{i:n}e^{(\beta_0+\beta_1S_1)}}{A^2}+e^{(\beta_0+\beta_1S_2)}\sum_{i=n_1+1}^{N}\frac{\left\{S_2t_{i:n}-\tau(S_2-S_1)\right\}}{B^2}\right]$$
$$-\frac{n_\eta\alpha}{C^2}e^{(\beta_0+\beta_1S_2)}\left\{\eta S_2+\tau(S_2-S_1)\right\}$$

5. INTERVAL ESTIMATES

According to large sample theory, the ML estimators, under some appropriate regularity conditions, are consistent and normally distributed. Since ML estimates of parameters are not in closed form, therefore, it is impossible to obtain the exact confidence intervals, so asymptotic confidence intervals based on the asymptotic normal distribution of ML estimators instead of exact confidence intervals are obtained here.

The asymptotic variance-covariance matrix of $\hat{\alpha}$, $\hat{\beta}_0$ and $\hat{\beta}_1$ is obtained by inverting the Fisher-information matrix and given by

$$\Sigma = F^{-1} = \begin{bmatrix} -\frac{\partial^2 l}{\partial \alpha^2} & -\frac{\partial^2 l}{\partial \alpha \partial \beta_0} & -\frac{\partial^2 l}{\partial \alpha \partial \beta_1} \\ -\frac{\partial^2 l}{\partial \beta_0 \partial \alpha} & -\frac{\partial^2 l}{\partial \beta_0^2} & -\frac{\partial^2 l}{\partial \beta_0 \partial \beta_1} \\ -\frac{\partial^2 l}{\partial \beta_1 \partial \alpha} & -\frac{\partial^2 l}{\partial \beta_1 \partial \beta_0} & -\frac{\partial^2 l}{\partial \beta_1^2} \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} AVar(\hat{\alpha}) & ACov(\hat{\alpha}\hat{\beta}_0) & ACov(\hat{\alpha}\hat{\beta}_1) \\ ACov(\hat{\beta}_0\hat{\alpha}) & AVar(\hat{\beta}_0) & ACov(\hat{\beta}_0\hat{\beta}_1) \\ ACov(\hat{\beta}_1\hat{\alpha}) & ACov(\hat{\beta}_1\hat{\beta}_0) & AVar(\hat{\beta}_1) \end{bmatrix}$$

Now, the two-sided approximate $100\lambda\%$ confidence limits for population parameters α, β_0 and β_1 can be constructed as

$$\left[\hat{\alpha} \pm Z_\lambda \sqrt{AVar(\hat{\alpha})}\right], \left[\hat{\beta}_0 \pm Z_\lambda \sqrt{AVar(\hat{\beta}_0)}\right]$$

$$and \left[\hat{\beta}_1 \pm Z_\lambda \sqrt{AVar(\hat{\beta}_1)}\right]$$

6. OPTIMAL TEST PLAN FOR TIME STEP STRESS ALT

The optimum criterion here is to find the optimum stress change time τ. Since the accuracy of ML method is measured by the asymptotic variance of the MLE of the 100 P^{th} percentile of the lifetime distribution at normal stress condition $t_p(S_0)$, therefore the optimum value of the stress change time will the value which minimizes the AV of the MLE of $t_p(S_0)$. The 100 P^{th} percentile of a distribution $F()$ is the age t_p by which a proportion of population fails, Nelson [11]. It is a solution of the equation $P = F(t_p)$, therefore the 100 P^{th} percentile for Pareto distribution is

$$t_p = \frac{\theta\left\{1-(1-P)^{1/\alpha}\right\}}{(1-P)^{1/\alpha}}$$

The 100 P^{th} percentile for Pareto distribution at use condition is

$$t_p(S_0) = \frac{\exp(\beta_0 + \beta_1 S_0)\left\{1-(1-P)^{1/\alpha}\right\}}{(1-P)^{1/\alpha}}$$

Now the AV of MLE of the 100 P^{th} percentile at normal operating conditions is given by

$$AVar(t_p(\hat{S}_0)) = \left[\frac{\partial t_p(\hat{S}_0)}{\partial \hat{\alpha}}, \frac{\partial t_p(\hat{S}_0)}{\partial \hat{\beta}_0}, \frac{\partial t_p(\hat{S}_0)}{\partial \hat{\beta}_1}\right] \Sigma \left[\frac{\partial t_p(\hat{S}_0)}{\partial \hat{\alpha}}, \frac{\partial t_p(\hat{S}_0)}{\partial \hat{\beta}_0}, \frac{\partial t_p(\hat{S}_0)}{\partial \hat{\beta}_1}\right]^{-1}$$

The optimum stress change time τ will be the value which minimizes $AVar(t_p(\hat{S}_0))$.

7. SIMULATION STUDY

To assess the performance of the method described in present study a simulation study is performed in which a number of data sets with sample sizes $n = 100, 200, ..., 500$ are generated from Pareto distribution which are censored at $\eta = 50, 75$. The values for true parameters and stress levels are chosen to be $\alpha = 0.80$, $\beta_0 = 1.50$, $\beta_1 = 2.50$ and $s = 3$ *and* 5. The estimates and the corresponding statistical values are obtained by using the present SSALT model. For different given samples, stress levels and censoring times with $\alpha = 0.80$, $\beta_0 = 1.50$, $\beta_1 = 2.50$, the ML estimates, Mean squared errors (MSEs), absolute relative biases (RBias), relative error (RE), and the 95% and 99% asymptotic confidence intervals for α, β_0 and β_1 are obtained. The results of the estimates for α, β_0 and β_1 based on 750 simulation replications are summarized in Table 1 and 2 while the confidence intervals and interval coverage are shown in Table 3 and 4 respectively.

Table 1: Simulation Study Results with $\alpha=0.80,\ \beta_0=1.50,\ \beta_1=2.50,\ S_1=3, S_2=5$ and $\eta=50$

n	$\hat{\alpha}$ $\hat{\beta}_0$ $\hat{\beta}_1$	MSE($\hat{\alpha}$) MSE($\hat{\beta}_0$) MSE ($\hat{\beta}_1$)	RAB($\hat{\alpha}$) RAB($\hat{\beta}_0$) RAB($\hat{\beta}_1$)	RE($\hat{\alpha}$) RE($\hat{\beta}_0$) RE($\hat{\beta}_1$)	Var($\hat{\alpha}$) Var($\hat{\beta}_0$) Var($\hat{\beta}_1$)
100	0.846 1.439 2.534	0.0203 0.0076 0.0295	0.0575 0.0407 0.0136	0.1782 0.0582 0.0687	0.0182 0.0039 0.0283
200	0.839 1.446 2.529	0.0194 0.0085 0.0210	0.0488 0.0360 0.0116	0.1742 0.0615 0.0580	0.0179 0.0056 0.0202
300	0.810 1.512 2.511	0.0133 0.0079 0.0170	0.0125 0.0080 0.0044	0.1442 0.0594 0.0522	0.0132 0.0078 0.0169
400	0.792 1.520 2.502	0.0115 0.0096 0.0157	0.0100 0.0133 0.0008	0.1338 0.0653 0.5010	0.0114 0.0092 0.0157
500	0.784 1.534 2.498	0.0105 0.0175 0.0121	0.0200 0.0227 0.0008	0.1278 0.0881 0.0440	0.0102 0.0163 0.0121

Table 2: Simulation Study Results with $\alpha=0.80,\ \beta_0=1.50,\ \beta_1=2.50,\ S_1=3, S_2=5$ and $\eta=75$

n	$\hat{\alpha}$ $\hat{\beta}_0$ $\hat{\beta}_1$	MSE($\hat{\alpha}$) MSE($\hat{\beta}_0$) MSE ($\hat{\beta}_1$)	RAB($\hat{\alpha}$) RAB($\hat{\beta}_0$) RAB($\hat{\beta}_1$)	RE($\hat{\alpha}$) RE($\hat{\beta}_0$) RE($\hat{\beta}_1$)	Var($\hat{\alpha}$) Var($\hat{\beta}_0$) Var($\hat{\beta}_1$)
100	0.893 1.594 2.587	0.0225 0.0130 0.0237	0.1163 0.0627 0.0348	0.1877 0.0761 0.0615	0.0139 0.0042 0.0161
200	0.889 1.568 2.542	0.0204 0.0113 0.0201	0.1113 0.0453 0.0168	0.1786 0.0709 0.0567	0.0125 0.0067 0.0183
300	0.874 1.502 2.499	0.0313 0.0083 0.0204	0.0925 0.0013 0.0004	0.2211 0.0608 0.0571	0.0258 0.0083 0.0204
400	0.832 1.483 2.474	0.0178 0.0054 0.0199	0.0400 0.0113 0.0104	0.1669 0.0489 0.0564	0.0168 0.0051 0.0192
500	0.804 1.492 2.482	0.0087 0.0122 0.0231	0.0050 0.0053 0.0072	0.1167 0.0735 0.0608	0.0087 0.0121 0.0228

Table 3: Confidence intervals with $\alpha=0.80,\ \beta_0=1.50,\ \beta_1=2.50,$ $S_1=3, S_2=5$ and $\eta=50$

n	$\hat{\alpha}$ $\hat{\beta}_0$ $\hat{\beta}_1$	95 % Confidence Interval		99 % Confidence Interval	
		LCL	UCL	LCL	UCL
100	0.846 1.439 2.534	0.5816 1.3166 2.2043	1.1104 1.5614 2.8637	0.4979 1.2779 2.0999	1.1941 1.6001 2.9680
200	0.839 1.446 2.529	0.5768 1.2993 2.2504	1.1012 1.5927 2.8076	0.4938 1.3539 2.2504	1.1842 1.6391 2.8957
300	0.810 1.512 2.511	0.5848 1.3389 2.2562	1.0352 1.6851 2.7658	0.5136 1.2841 2.1756	1.1064 1.7399 2.8464
400	0.792 1.520 2.502	0.5827 1.3320 2.2564	1.0013 1.7080 2.7476	0.7626 1.2725 2.1787	1.0675 1.7675 2.8253
500	0.784 1.534 2.498	0.5860 1.2838 2.2824	0.9820 1.7842 2.7136	0.5234 1.4919 2.1962	1.0446 1.8634 2.7818

Table 4: Confidence intervals with $\alpha=0.80,\ \beta_0=1.50,\ \beta_1=2.50,\ S_1=3, S_2=5$ and $\eta=75$

n	$\hat{\alpha}$ $\hat{\beta}_0$ $\hat{\beta}_1$	95 % Confidence Interval		99 % Confidence Interval	
		LCL	UCL	LCL	UCL
100	0.893 1.594 2.587	0.6619 1.4669 2.3383	1.1241 1.7210 2.8357	0.5888 1.4268 2.2596	1.1972 1.7612 2.9144
200	0.889 1.568 2.542	0.6699 1.4076 2.2769	1.1081 1.7284 2.8071	0.6005 1.3568 2.1929	1.1775 1.7792 2.8910
300	0.874 1.502 2.499	0.5592 1.3234 2.2191	1.1888 1.6806 2.7789	0.4596 1.2669 2.1305	1.2884 1.7370 2.8675
400	0.832 1.483 2.474	0.5779 1.3430 2.2024	1.0860 1.6229 2.7456	0.4976 1.2988 2.1165	1.1664 1.6672 2.8315
500	0.804 1.492 2.482	0.6212 1.4799 2.1860	0.9868 1.7076 2.7780	0.5634 1.2082 2.0924	1.0446 1.7758 2.8716

From the results obtained in table 1, 2, 3 and 4, the following observations are made

i. For the first set of values the ML estimators have good statistical properties (as the parameter estimates are close to their true values) than the second.

ii. As the sample size increase the estimates have smaller MSEs, RABs and REs. This indicates that the ML estimates provide asymptotically normally distributed and consistent estimator for the parameters.

iii. It is also found that confidence intervals are getting narrower as the sample size increases.

8. DISCUSSION AND CONCLUSION

In this paper SSALT model for Pareto distribution with type-I censored data has been considered. It is found from the results that the present model works well with good statistical properties. Hence, it can be said that the proposed model can be used in the analysis of ALT. For future research in SSALT one can choose another optimization criterion, different censoring schemes and test settings for other lifetime distributions.

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