

Linear State Estimator for Phasor Measurement Unit

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Abstract— The present paper deals with the ability of the Phasor Measurement Unit (PMU) which directly measure the system state and the increasing implementation of PMUs across the electric power industry. A natural expansion of state estimation techniques would be one that employed the exclusive use of PMU data. My research objective is to implement a three phase linear tracking state estimator on network that would use only PMU measurements to compute the system state. This paper presented the initial development and testing of two applications: the three phase linear state estimator and the topology processor. Also presented is a brief history of state estimation and PMUs, traditional state estimation techniques and techniques with mixed phasor data, a development of the linear state estimation algorithms and a discussion of the future work associate with this research work. System states are evaluated using the chosen measurements by applying linear State Estimation and are validated through the simulations carried out on standard IEEE.

Index Terms— Linear State Estimation, SCADA, Synchronous Phase Measurement Units, Binary Linear Integer Programming, Measurement Jacobian Matrix.

I. INTRODUCTION

An Energy Management System (EMS) provides a variety of measured data and computer applications for monitoring and control of the power network. When we refer to computer applications we mean the following two:

- State estimator (an on-line application)
- Contingency constrained OPF (an off-line application)

Started as engineering tool, the power system state estimator became the key data processing tool in modern EMS systems, and evolved in today's industry as a very important application for Locational Marginal Pricing algorithms for charging congestion in power networks. Monitoring and control of power system assets is conducted through the supervisory control and data acquisition (SCADA) system. In the early days, it was believed that the real-time data base provided by SCADA could provide an operator with an accurate system view. Very soon, the deficiencies of SCADA [2] were realized. To mention a few: hard to assure availability of all measurements at all times, measurements prone to errors, etc. A more powerful tool was needed to process collected measurements and to filter bad ones. A

central master station, located at the control centre, gathers information through the SCADA system. The SCADA system collects measurement data in real time from remote terminal units (RTUs) installed in substations across the power system. Typical RTU [2] measurements include power flows (both active and reactive), power injections, voltage magnitude, phase angles and current magnitude.

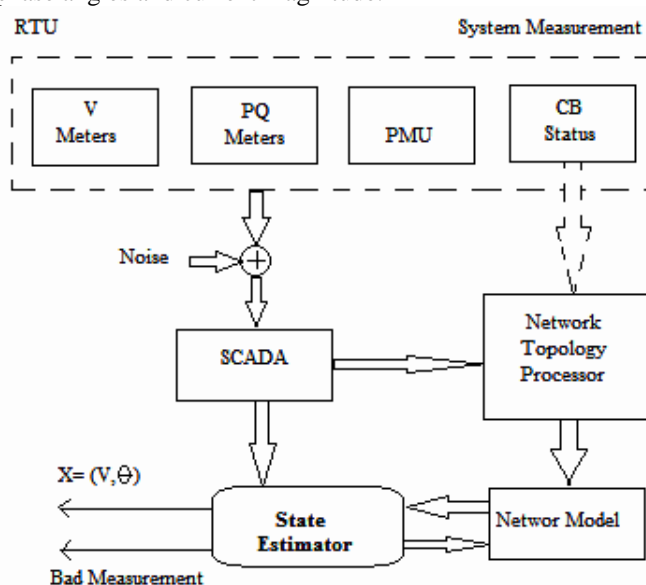


Figure. 1 State Estimation block diagram

While there is not much to be said that is not already known about active and reactive power and voltage magnitude measurements, voltage angle measurements are relatively new in practice. Direct measurement of voltage phase angle was impossible for a long time. In order to be valid, those measurements should be synchronized, i.e. a time reference should be provided. The global positioning system (GPS) signal made synchronization possible with accuracy better than $1 \mu s$. A phasor measurement unit (PMU) equipped with a GPS receiver allows for synchronization of measurements, yielding accurately measured and time-stamped voltage phase angles. A study of impact of PMU measurements on state estimation and optimal placement of PMUs is given in [2]. The general conclusion is that PMUs have greatly improved observability and accuracy of voltage angle estimates. Despite some opinions to the contrary, PMUs will not make state

estimation obsolete even if they are available at every bus in the system. As we know, measurements are not perfect; thus a redundant set of measurements will still be needed in order to identify bad data.

All of these measurements can be considered dynamic since snapshots are performed every few seconds. The status of the assets (line status, breaker status etc.) as well as network parameters can be considered as static measurements. The network topology processor determines the topology of the network from the telemeter status of circuit breakers. Having an observable set of measurements is a necessary, although not sufficient condition, for EMS computer applications. While it is desired, coordination across the network quite often does not happen in real-time. The reasons for not having real time-model are varied. While many control and monitoring functions are computer based, there are still functions handled by telephone calls between the system operator and utility control centers. These improvements in power system monitoring and control are motivated by

- Economics of the new market
- Blackout prevention
- Reliability improvement

The point of our research is not to give an optimal recommendation regarding tree trimming but to try to explore the ways of improving reliability of monitoring tools, particularly state estimator software. The state estimator (SE) computes the static state of the system (voltage magnitude and phase angle) by monitoring available measurements. The SE has to be modeled in such a way so as to ensure that the system is monitored reliably not only in day-to-day operations, but also under the most likely conditions of system stress. The question is how to improve SE and make it more reliable, so that is more likely to capture situations like blackout and identify critical nodes in the network. A more robust state estimator is an essential need in the years to come. Successful SE solution relies heavily on the numerical technique used to perform the estimation. Current numerical algorithms too frequently fail to provide a successful solution. In our research was to apply linear state estimation technique that are more reliable and have a quick response.

Recent developments of phasor measurement technologies provide high-speed sensor data (typically 30 samples/second) with precise time synchronization [1], [2]. Synchronized phasor measurements are commonly referred to as synchrophasors. This is in comparison with traditional Supervisory Control and Data Acquisition (SCADA) RTU measurements, which have cycle times of five seconds or longer and are not time synchronized.

PMUs are becoming increasingly attractive in various power system applications such as system monitoring, protection, control, and stability assessment. PMUs can provide real-time synchrophasor data to the SCADA system to capture the

dynamic characteristics of the power system, and hence facilitate time-critical control. Compared to estimating relatively stationary state elements such as bus voltage magnitudes and phase angles, dynamic state estimation seeks to estimate more transient states of a power system.

II. POWER SYSTEM STATE ESTIMATION

In this section we review the current state estimation formulation and solution methods and provide motivation for further improvement. Several excellent review papers [1], [4] cover this topic in detail. When we say power system state estimation we mean the original and most widely used problem definition in practice. That is, an over determined system of nonlinear equations solved as an unconstrained weighted least-squares (WLS) problem. The WLS estimator minimizes the weighted sum of the squares of the residuals.

$$\min_{x \in R^n} J(x) = \frac{1}{2} (z - h(x))^T R^{-1} (z - h(x))$$

where: x is the state vector; z is the measurement vector and $h(x)$ is the nonlinear vector function relating measurements to states and R is a diagonal matrix whose elements are the variances of the measurement error.

The following iterative process is obtained

$$\begin{aligned} (H^T R^{-1} H) \Delta x &= H^T R^{-1} [z - h(x)] \\ x^{k+1} &= x^k + \Delta x \end{aligned}$$

The symmetric matrix $H^T R^{-1} H$ $2 R n_{\times n}$ is called the gain or information matrix. Above equations are the so-called normal equations of the least-squares method and the iteration step Δx can be found only when the gain matrix is non-singular. An extensive bibliography of the first two decades (1968-1989) of power system state estimation was prepared by Coutto, Silva and Falcao [6]. Comprehensive treatment of modern power system state estimation can be found in books first by Monticelli [7] in 1999 and then by Abur and Gómez Expósito in 2004 [1]. Beginning with the role of the state estimator in a security framework as one of the key modern Energy Management System (EMS) applications, they covers all parts of the state estimation process starting with power flow, problem formulation, basic solution techniques, observability, detection and identification of bad data, and robust state estimation procedures. An overview paper by K. Jamuna and K.S. Swarup [2] covers the overall role of the SE in the power system control centers starting from topology processing, then goes through an overview of state estimation numerical algorithms, network observability, and bad data detection.

In this paper, we focus on bridging these gaps. Specifically our contributions include: We develop a stochastic model of

linear state estimation, to getting the state vector for reliable operation of network.

III. MATHEMATICAL MODELING

The proposed state estimation model is based on a linear measurement model composed of classical SE state estimate augmented by PMU measurements

$$[z] = \begin{bmatrix} E \\ I \end{bmatrix} = \begin{bmatrix} II \\ yA + y_s \end{bmatrix} [x] + [e]$$

It can be seen that the set of measurements, [z] is a vertical concatenation of the set of voltage and current phasor measurements, respectively. The system state, [x] is then related to the set of measurements by a vertical concatenation of the voltage measurement-bus incidence matrix and a system matrix composed of the series and shunt admittance matrices and the current measurement-bus incidence matrix. M relates the system state to the set of line flow phasor measurements.

Consider the fictitious 5 bus system.

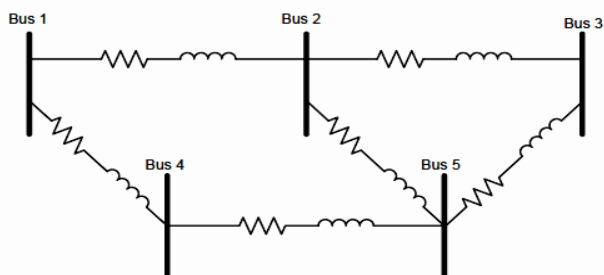


Figure.2 Fictitious 5 Bus System

The system matrix, M will take the following form

$$M = yA + y_s = \begin{bmatrix} y_1 + y_{10} & -y_1 & 0 & 0 & 0 \\ y_3 + y_{30} & 0 & 0 & -y_3 & 0 \\ 0 & -y_2 & y_2 + y_{20} & 0 & 0 \\ 0 & 0 & y_5 + y_{50} & 0 & -y_5 \\ 0 & -y_4 & 0 & 0 & y_4 + y_{40} \\ 0 & 0 & -y_5 & 0 & y_5 + y_{50} \\ 0 & 0 & 0 & -y_6 & y_6 + y_{60} \end{bmatrix}$$

Then the two matrices are vertically concatenated. In the following equation, the full system state as well as each of the separate voltage and current measurements has been represented as they were in the diagram of the 5 bus system.

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ y_1 + y_{10} & -y_1 & 0 & 0 & 0 \\ y_3 + y_{30} & 0 & 0 & -y_3 & 0 \\ 0 & -y_2 & y_2 + y_{20} & 0 & 0 \\ 0 & 0 & y_5 + y_{50} & 0 & -y_5 \\ 0 & -y_4 & 0 & 0 & y_4 + y_{40} \\ 0 & 0 & -y_5 & 0 & y_5 + y_{50} \\ 0 & 0 & 0 & -y_6 & y_6 + y_{60} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 \\ e_8 \\ e_9 \\ e_{10} \\ e_{11} \\ e_{12} \end{bmatrix}$$

$$[B] = \begin{bmatrix} II \\ yA + y_s \end{bmatrix}$$

The three phase representation of the voltage measurement bus incidence matrix,

$$[II] = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}$$

Then the state equation can be rewritten as follows.

$$[Z] = [B] [x]$$

It desired to solve for the system state [x]. Because [B] is taller than it is wide, a direct inverse is not achievable. The pseudo-inverse must be determined to solve for the system state

$$[x] = [(B^T B)^{-1} B^T] [Z] = [H] [Z]$$

This is the solution for an error-free measurement set. If the measurements contain errors then the covariance matrix appears in the solution. The solution then takes the following form

$$[x] = [(B^T W^{-1} B)^{-1} B^T W^{-1}] [Z] = [H] [Z]$$

Branch impedances of example network

Table 1. Data of the buses

From Bus	To Bus	R (p. u.)	X (p. u.)
1	2	0.02	0.06
1	4	0.01	0.08
2	3	0.05	0.10
2	5	0.02	0.07
3	5	0.01	0.07
4	5	0.03	0.09

The network admittance matrix Y,

$$Y = \begin{bmatrix} 6.5 - 27.3i & -5.0 + 15.0i & 0 & -1.5 + 12.3i & 0 \\ -5.0 + 15.0i & 12.7 - 36.2i & -4.0 + 8.0i & 0 & -3.7 + 13.2i \\ 0 & -4.0 + 8.0i & 6.0 - 22.0i & 0 & -2.0 + 14.0i \\ -1.5 + 12.3i & 0 & 0 & 4.8 - 22.3i & -3.3 + 10.0i \\ 0 & -3.7 + 13.2i & -2.0 + 14.0i & -3.3 + 10.0i & 9.1 - 37.2i \end{bmatrix}$$

IV. STATE ESTIMATOR SIMULATION RESULTS

The desired function of any state estimator is to filter out the measurement error using redundancy and knowledge about the measurement errors. The testing script used to test the MATLAB application of the three phase linear state estimator using the data created by the test to demonstrate

- 1) The true value and the estimated value of a single state variable are plotted on the complex plane for each iteration of the testing procedure. The raw voltage measurements corresponding to that particular state variable are also included on the same plot. This type of plot shows the ability of the state estimator to filter out measurement error and hone in on the true value of the state variable.
- 2) A different plot which shows the estimators ability to filter out measurement error from the entire state vector. These plots only show results from a single iteration. Since the true value of the system state is known from the load flow, it is compared to the corresponding voltage measurements and estimated state vector and plotted to show the effects. This is done in four different plots: a plot showing the effects on the real part of the state variable, a plot showing the effects on the imaginary part of the state variable, a plot showing the effects on the magnitude of the state variable, and a plot showing the effects on the angle of the state variable.

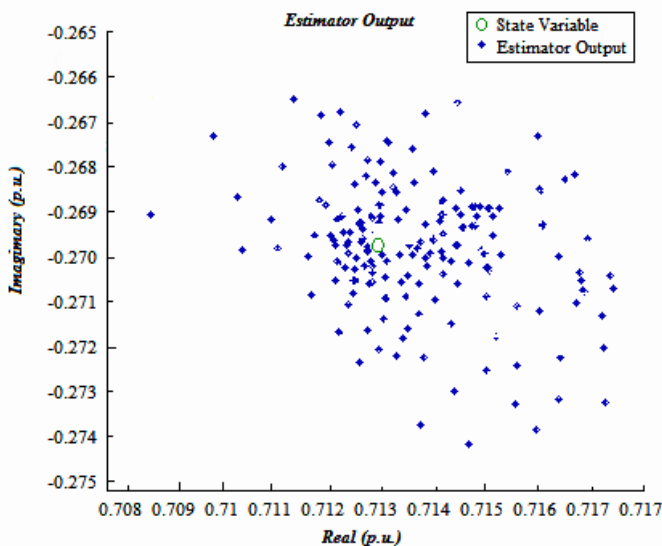


Figure 3: State Estimator Output for 100 iterations

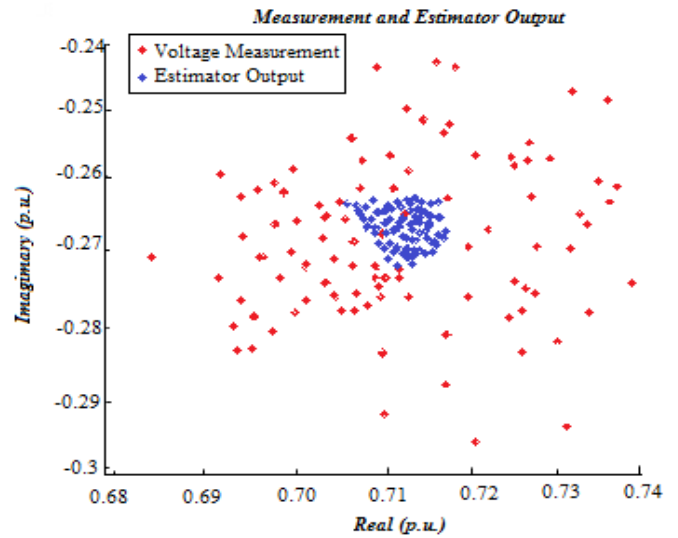


Figure 4: State Estimator Output for 100 iterations

Figure 3 and Figure 4 are two plots showing a single state variable over 100 iterations of the testing script. This particular state variable corresponds to a substation which is monitored by a PMU. Figure 5-3 shows in green the true value of the state variable and in blue the estimated values of the state variable for 100 iterations. Figure 5-4 shows in blue the estimated values of the state variable and in red the voltage measurements that correspond to this particular state variable. Both of these plots are from the same set of iterations so they can be visually compared to each other. These plots show very clearly the ability of the estimator to filter out measurement errors and determine the best estimate of the system state. The next set of plots show the effects of the estimation process on the entire state vector for a single iteration of the testing script. The absolute value of the difference between the voltage measurements and the actual state vector are shown in red. The absolute value of the difference between the estimated state vector and the actual state vector are shown in blue. Comparison of Real Part of Output & Measurements.

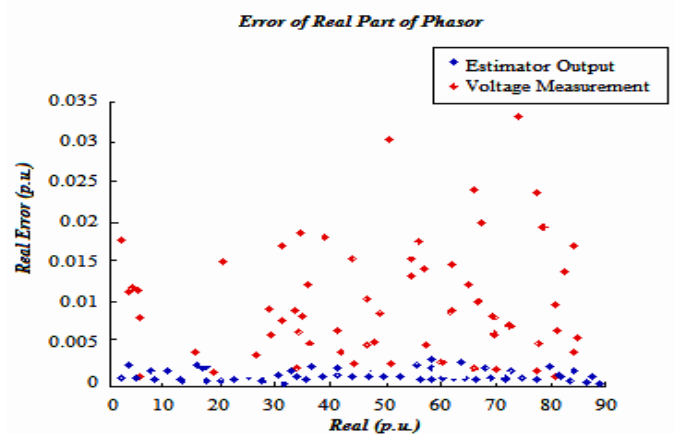


Figure 5: Comparison of Real Part of Output & Measurements

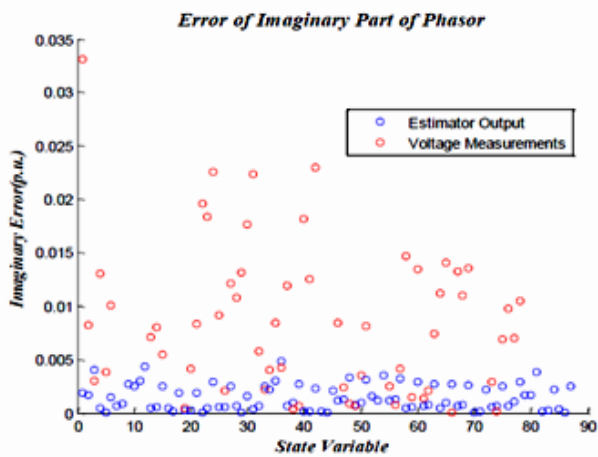


Figure 6: Comparison of Imaginary Part of Output & Measurements

It can be seen from these plots that the state estimator can effectively filter out measurement normally distributed random additive errors and estimate with a good degree of precision the system state. In the case of above two comparisons, the estimator has the ability to come within approximately 0.002 of the true value of each of the parameters.

V. PROPOSED SYNCHROPHASOR SUPERVISORY SYSTEM

- PMU reporting rate - 20ms -0.1 sec or 10-50 Hz; Nyquist criteria satisfied
- Communication latency _ 100 ms;

Syn SE is a faster application while backup protection is a slower application

- AND logic improves security
- Dependability depends upon accuracy of fault detection logic

In case of failure of communication system, the scheme degenerates to existing backup scheme.

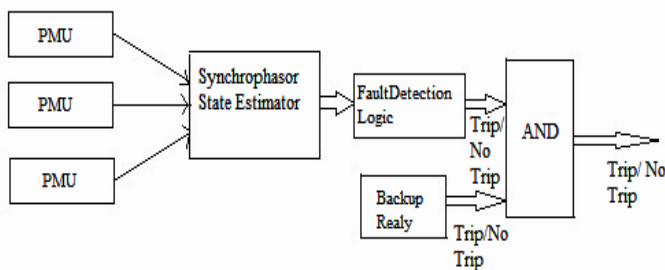


Figure 7: Proposed Synchrophasor Supervisory System

VI. CONCLUSION

This paper proposes a generalized integer linear programming formulation for state estimator fully observable busses by

PMU . From the simulation study performed on standard IEEE systems, it is observed that this algorithm provides the best state estimates with less computation time. Simulation results show that the proposed algorithm is computationally efficient and can be used in practice. An important advantage of this method is non-iterative which reduces the computation time and suitable to implement in the near future. For future work, Synchrophasor state Estimator can be developed for detection of transmission line faults.

Synchrophasor Standards:

- IEEE 1344-1995, R2001
- IEEE C37.118, 2005
- IEEE C37.118-1, 2011 - Standard for Synchrophasor Measurements for Power Systems:
- IEEE C37.118-2, 2011 - Standard for Synchrophasor Data Transfer for Power Systems
- IEEE PC37.242™

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