

# PERFORMANCE OF SYNCHRONOUS MACHINE DYNAMICS USING A NOVEL EQUIVALENT CIRCUIT MODEL

Rajnish Kumar<sup>1</sup>, Er.Pratibha Tiwari<sup>2</sup>,

Department of Electrical and Electronics Engg.(SSET), SHIATS, Allahabad, India

<sup>1</sup> rajnishkumarnishad@gmail.com, <sup>2</sup>p-tiwari28@rediffmail.com

**Abstract**— This paper proposes dynamic modelling simulation for performance of synchronous machines using a novel Magnetic Equivalent Circuit (MEC) model with the aid of MATLAB-Simulink environment. The proposed model offers sufficient detail richness for design calculations, while still keeping the simulation time acceptably short. Different modelling methods and circuit alternatives are considered. The selected approach is a combination of several previous methods added with some new features. A detailed description of the new model is given. The flux derivative is chosen as the magnetic flow variable which enables a description with standard circuit elements. The model is implemented in dq-coordinates to reduce complexity and simulation time. A new method to reflect winding harmonics is introduced. Extensive measurements have been made to estimate the traditional dq-model parameters. These in combination with analytical calculations are used to determine the parameters for the new MEC model.

**Keywords**—Synchronous machine, Equivalent circuit, Magnetic equivalent circuit model, Simulation model, Parameter determination, MATLAB

## I. INTRODUCTION

A synchronous generator is a machine for converting mechanical power from a prime mover to a.c.(alternating current) electric power at a specific voltage and frequency. A synchronous machine rotates at a constant speed called synchronous speed. A synchronous motor is a machine that converts electric power to mechanical power. Synchronous machines are usually of 3-phase type because of the several advantages of 3-phase generation, transmission and distribution. Large synchronous machines are used to generate bulk power at thermal, hydro and nuclear power stations. A synchronous machine with power ratings of several hundred MVA are very commonly used in generating stations. The biggest size used in India has a rating of 500 MVA used in super-power thermal stations. Synchronous machines are the primary sources of world's electric power systems nowadays. Induction machines have taken over, partly due to cost reasons and partly because they need no excitation equipment. The synchronous machines have still a number of important

advantages which makes them very interesting. To this counts high efficiency, robustness and good controllability. In the upper power range they are the only option. It can therefore be expected that the synchronous machines will continue to play an important role, also in the future. There are two basic constructions: machines with a cylindrical rotor and machines with a salient pole rotor. For mechanical reasons, the cylindrical rotor is preferred for two pole machines because of the large centrifugal forces that arise. The salient pole rotor is usually the more efficient solution for machines with four poles and upwards, both for cost reasons and for performance reasons. The rotor can be made of either laminated steel or solid iron. The solid iron rotor is the dominating solution for machines with low pole numbers, partly because of its robust mechanical properties, but also because of the good starting properties for direct on-line connection. This is the machine type that is studied in this work. For machines higher pole number, a laminated core is often used. Traditional applications for the salient pole motor are pump systems, paper mills, ship propulsion and other applications with moderate dynamic requirements. Since energy cost has become more and more important, variable speed operation with inverter supply is gaining market share. Traditionally, thyristor based solutions such as the Cyclo Converter and the Load Commutated Inverter have been the only options. These solutions have however a number of drawbacks such as high engineering cost and poor starting properties. The Voltage Source Inverter based on turn-off capable semiconductors is therefore taking over the market. This technique shift took place a couple of decades ago in the low voltage range, but has just recently also reached the medium voltage range. This brings up a number of new issues for the design engineers, especially for cases with a solid rotor. The machine must now be designed for a very wide speed range. Further, harmonics will create extra heating in different machine parts and there are also a number of other parasitic effects related to the inverter supply, such as noise and bearing currents. This has brought forth a need for new and improved calculation methods, which was the original motivation for this work.

**II. FLUX DISTRIBUTION IN A REAL MACHINE**

The starting point for construction of an effective MEC model is to understand how the flux flows in the machine under different operation conditions. The goal is to find the natural “flux tubes”, and to use the corresponding magnetic areas to define the equivalent circuit elements. Since it is not practically possible to measure the flux inside the machine, the best option is to use FEM simulations. A FEM model of the GA84 machine has therefore been developed in the software FLUX2D. Three different operation cases have been studied. The first case is synchronous operation at rated speed, both with and without load. The main goal was to find out whether there are parts that experience considerable magnetic saturation, and in which cases this must be considered in the MEC model. The flux density at no-load operation. As indicated by the yellow color, it is mainly the stator teeth that are significantly saturated. The corresponding situation but at full load torque. It can be seen that in this case also the pole edges are heavily saturated. The conclusion is thus that at least in the stator teeth and pole edges, saturation needs to be considered. the stator winding is supplied with a 100A 50Hz 3-phase current. Such a situation arises during a direct on-line start, and it is important to map how the generated flux links with the electric circuits and generates starting torque. A snapshot of the flux distribution when the current vector points in the d direction. The flux density is highest at the mid of the poles: The flux flows through the air gap, then circumferentially along the pole surface, further radially along the pole core boundary and then finally either directly over the pole gap or via the pole core to the adjacent pole.

**Basic equivalent circuit relations**

The electromagnetic field can be described by the well-known Maxwell’s equations, see Table 2-1. Gauss’s equations (2.2) and (2.4) describe the effect of point sources and they will in an electric circuit correspond to a battery or another voltage source. There are however corresponding point sources in the magnetic case, see (2.4). Faraday’s law and Ampere’s law describe the consequence of a vortex source, which in the electric case is the magnetic induction and in the magnetic case the magneto motive force from the electric current.

Faraday’s law:  $\oint_c \vec{E} \cdot d\vec{r} = -\frac{d\phi}{dt}$  (Vortex source strength) (2.1)

Gauss’s law:  $\oint_v \vec{D} \cdot d\vec{S} = Q$  (Point source strength) (2.2)

Ampere’s law:  $\oint_c \vec{H} \cdot d\vec{r} = I + \left( \int_s \frac{d\vec{D}}{dt} \cdot d\vec{S} \right)$  (Vortex source strength) (2.3)

Gauss’s law:  $\oint_v \vec{B} \cdot d\vec{S} = 0$  (Point source strength) (2.4)

It is very difficult to solve Maxwell’s equations directly, at least for complex geometries; therefore we will use equivalent circuits instead. One prerequisite for this to be possible is that the flow follows well defined paths, so called flux tubes in the magnetic case and conductors in the electric case. It is then possible to treat the corresponding geometrical section as one equivalent “lumped component”, and then to describe its external properties as a relation between an effort quantity (voltage in the electric case) and a flow quantity (current in the electric case). This relation is called the constitutional relation and can in the electric case be for instance Ohm’s law. How these flow paths should be selected is usually obvious for electrical fields, but it is not always so for the magnetic fields. The difference in magnetic permeance is much smaller than the difference in electric permittivity which makes the magnetic flow less confined than the electric one.

Quantity	Electrical domain			Magnetic domain		
Effort	Voltage	U	[V]	mmf	$U_m$	[A]
Flow	Current	$I=dQ/dt$	[A]	Flux rate	$I_m$	[V]
Displacement	Charge	Q	[As]	Flux	$\Phi$	[Vs]
Momentum	Flux linkage	$\psi$	[Vs]	Charge linkage	$\Gamma_m$	[As]
Resistance	Resistance	R	[V/A]	Dampance	$R_m$	[A/V]
Capacitance	Capacitance	C	[F]	Permeance	$C_m$	[H]
Inductance	Inductance	L	[F]	-	$L_m$	[As/V]

Table 2.1: Equivalent circuit analogies

The two basic circuit quantities are thus an effort and a flow. By combining these two quantities and the time, it is possible to derive a number of new quantities as shown in Table 2-2. With this choice of circuit variables, the interaction between the electrical and magnetic circuits is described by a gyrator:

$$u_e = N \frac{d\phi}{dt} = Ni_m \tag{2.5}$$

$$i_e = \frac{1}{N} u_m \tag{2.6}$$

The magnetic correspondence to a resistor is a dampance element. The electric resistance is related to a flow of electric charges. There are no magnetic charges; instead the flow is constituted of the flux derivative. It is therefore believed that the magnetic dampance element has little relevance, at least for the magnetic circuit which is used here. The electric capacitance is the quotient between the charge and the voltage, which corresponds to the basic quantities displacement and effort. The magnetic correspondence is thus the quotient between flux and mmf.

$$C_m = \frac{\phi}{u_m} = \Lambda \frac{\phi}{\phi} = \Lambda \tag{2.7}$$

$$i_m = \frac{d\phi_m}{dt} = \Lambda \frac{du_m}{dt} \tag{2.8}$$

The capacitor voltage times its capacitance equals to the flux in the circuit, and the flux in turn relates to the stored magnetic energy. One can therefore say that the equivalent circuit describes the magnetic energy distribution.

**III. Analytical parameter determination**

There are two types of parameters: resistances for the electrical circuits and permeances for the magnetic circuits.

**1. Resistive parameters**

The stator winding resistance can easily be calculated or even measured, while the equivalent pole surface resistance is more difficult to obtain. Pole surface currents are induced by an AC magnetic field through the machine. These currents will flow in a thin layer in the pole shoes, and the thickness of the layer is characterized by the skin depth. The flux concentration can be quite high, which results in local saturation of the iron. This saturation will make the flux to choose a different path, which in turn, decreases the induction of pole currents. The flux can then penetrate deeper into the pole until a balance arises. This process is complex to treat analytically, and only a very approximate calculation is made here. Neglecting saturation, the skin depth (for a plane wave) is given by:

$$\delta = \sqrt{\frac{2}{\omega \sigma \mu}}$$

The axial equivalent resistance can then be written:

$$R_z = \frac{2L\rho}{\theta_z (r^2 - (r - \delta)^2)}$$

A typical value for the pole resistivity is 0.4-0.5 μΩm, and assuming a constant relative permeability ur=700 gives the pole surface resistance according to Figure.

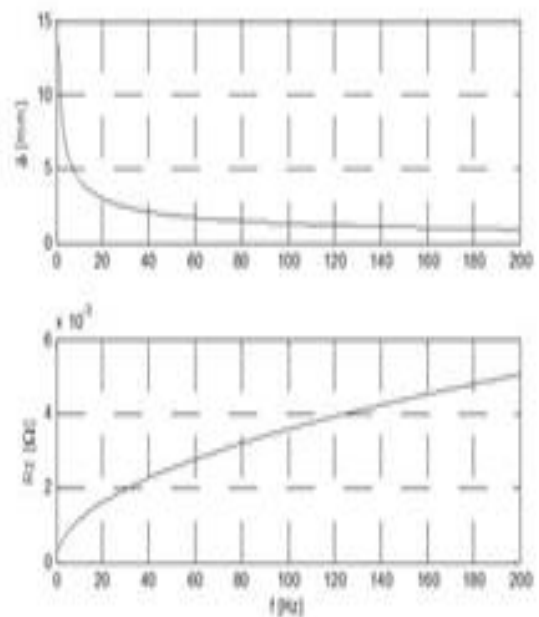
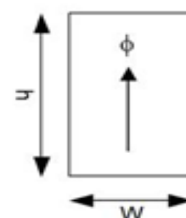


Figure : Skin depth as a function of frequency. The figure shows that the surface resistance is highly frequency dependent and it drops rapidly below 10 Hz.

**2. Permeance parameters**

The value of the capacitor elements in the magnetic circuit equals to the value of the permeance of the corresponding flux path in the real machine. This can be approximated by different geometrical elements. The permeance for a rectangle can be defined by

using Ampere’s law (2.3) (L is the axial length):

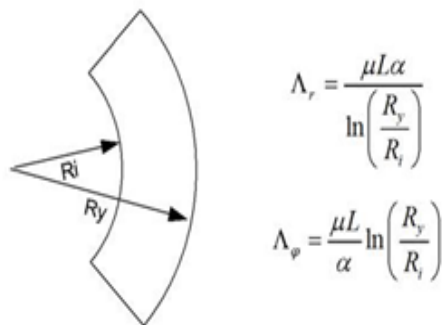


$$\Phi \frac{h}{\mu \cdot L \cdot w} = I = V_m$$

$$\Lambda_y = \frac{\phi}{V_m} = \mu \frac{L \cdot w}{h}$$

This relation can be directly used to find the parameter values for Cst, and Cpcr. However, because of the high permeances of iron, the effect of the Cpcr on the total flux level is

negligible. This permeance for more complex geometries can be found by dividing them into infinitesimal rectangles and integrating these over an appropriate area. Several geometries have been treated and the results for the used geometries are repeated here for convenience. The air gap permeances under the pole can be approximated by a circle segment with an angular width  $\alpha$



Equation (Ar) is used to determine the parameter Cagr, and (A) gives the Cagc. There is however a longer air gap at the pole edges. The extra permeance associated to their gap can be approximated by a trapezoid:

**IV. Stator winding and stator core**

To avoid excessive calculations time the whole stator part is transformed into a synchronously rotating frame. The sacrifice is then that the effect of stator slotting must be disregarded. Other spatial effects can however be preserved as shown below.

The stator voltage vector is first transformed into its 2-phase equivalent (that is, with the so-called Clarke's transformation). It is here assumed that the negative sequence voltage has no impact on the machine behavior, which is normally true. Using amplitude invariant scaling, the stator voltage equation is:

$$\vec{u}_s^{abc} = \vec{u}_s^{\alpha\beta} T_{\alpha\beta} = R_s \vec{i}_s^{abc} + \frac{d\vec{\psi}_s^{abc}}{dt} = R_s T_{\alpha\beta} \vec{i}_s^{\alpha\beta} + T_{\alpha\beta} \frac{d\vec{\psi}_s^{\alpha\beta}}{dt} \tag{4.3}$$

$$T_{\alpha\beta} = \frac{1}{2} \begin{bmatrix} 2 & 0 \\ -1 & \sqrt{3} \\ -1 & -\sqrt{3} \end{bmatrix} \tag{4.4}$$

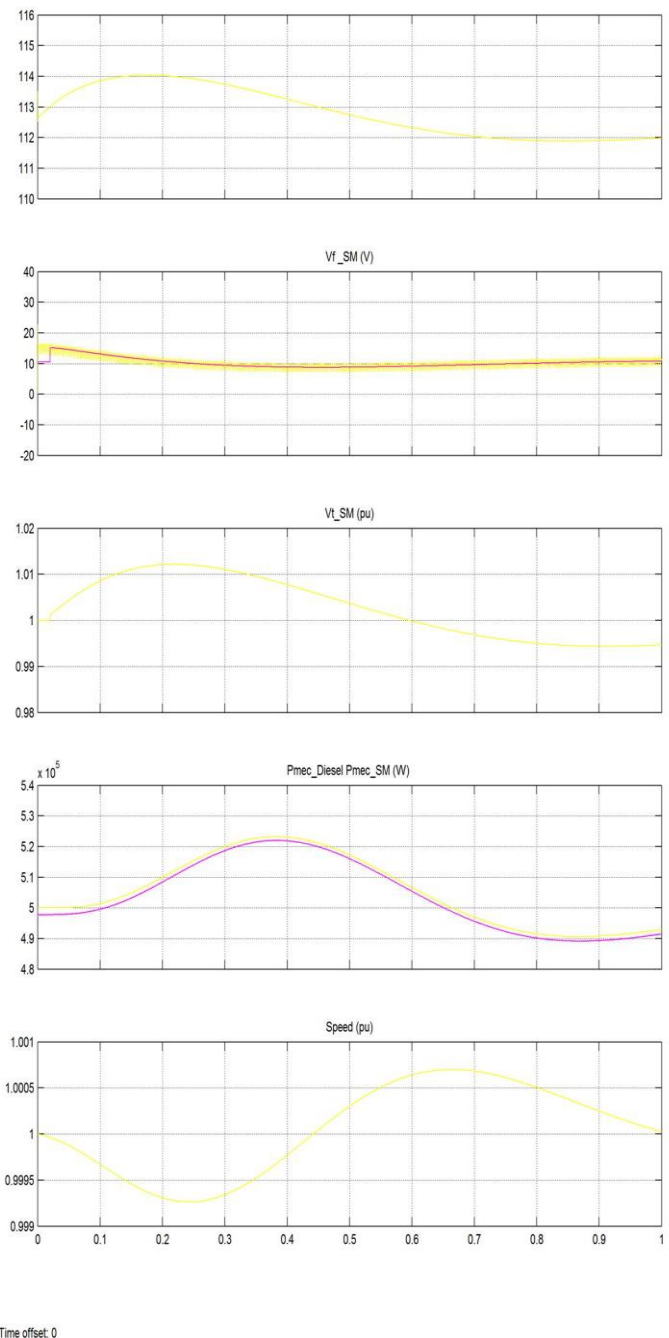
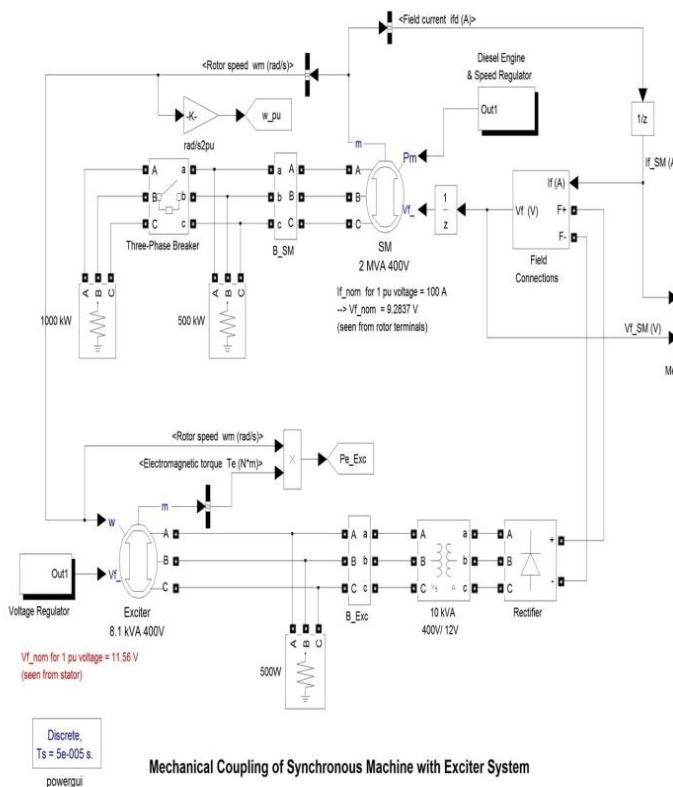
The equation (4.3) can now be further transformed into the rotor synchronous frame:

$$\frac{d\vec{\psi}_s^{\alpha\beta}}{dt} = \frac{d(T_{\alpha\beta} \vec{\psi}_s^{\alpha\beta})}{dt} = \frac{dT_{\alpha\beta}}{dt} \vec{\psi}_s^{\alpha\beta} + T_{\alpha\beta} \frac{d\vec{\psi}_s^{\alpha\beta}}{dt} = \frac{dT_{\alpha\beta}}{dt} \vec{u}_s^{\alpha\beta} + T_{\alpha\beta} \vec{u}_s^{\alpha\beta} \tag{4.5}$$

$$T_{\alpha\beta} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \tag{4.6}$$

The dynamic stator voltage  $u_r$  expresses the induced voltage relative the rotor frame and it is obtained from the synchronous frame MEC model. This gives a convenient structure of the model. All coordinate transformations are thus performed in the electrical circuit outside the MEC-model and they can be implemented in a separate block. It should however be observed that the stator resistance must be placed on the 3-phase side in the Dymola implementation. Otherwise the flux estimation in (4.5) will be wrong (that is, the integral term).

**V. SYNCHRONOUS MACHINE WITH EXCITER SYSTEM SIMULATION MODEL**



**VI. SIMULATION RESULTS**

The synchronous machine delivers 500 kW (25 % of rating). At  $t=3$  sec, an additional 1000 kW is switched on by closing the circuit breaker. In order to keep voltage at 1 pu, the excitation current increases from 112 A to 185 A (trace 1). The field voltage (trace 2) contains a 300 Hz ripple, but this ripple does not appear in the field current because of the large field inductance. Trace 3 shows that after load switching the synchronous machine terminal voltage resumes to its nominal value after a 3 second transient.

Traces 4 and 5 show the mechanical output power of the diesel engine and the speed of the engine-generator set. Speed regulation maintains 1 pu speed and nominal frequency (50 Hz output voltage)

**VII. CONCLUSIONS**

In this paper, we performance of synchronous machines dynamics using a novel equivalent circuit model. The goal has been to develop a simulation method that gives a sufficiently detailed picture of the electric and magnetic conditions to enable accurate loss calculations, but which still keeps simulation time reasonably short.

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## AUTHOR'S PROFILE



Rajnish Kumar Belong to Allahabad his Bachelor of Technology degree from U.P.Tech University, Lucknow in 2010. He is pursuing his M.Tech in Electrical and Electronics Engg.(Power System) from SHIATS,Allahabad,UP-India.  
Email: [rajnishkumarnishad@gmail.com](mailto:rajnishkumarnishad@gmail.com)



Er.Pratibha Tiwari Belong to Allahabad, She obtained her M.Tech in Electrical Engineering from Motilal Nehru National Institute of Technology, Allahabad, U.P.India. Presently she is working Asst. Professor in Electrical and Electronics Engg. Dept. SSET SHIATS (Formally Allahabad Agriculture Institute, Allahabad-India).  
Email: [p-tiwari28@rediffmail.com](mailto:p-tiwari28@rediffmail.com)