Algorithm for PAPR Reduction using Trimming and Differential Topping in OFDM Systems

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Abstract— **In this paper, we propose a simple technique for the reduction of high Peak to Average Power Ratio (PAPR), based on Clipping and Differential Scaling, in Orthogonal Frequency Division Multiplexing (OFDM) systems. In this technique, the amplitude of complex OFDM signal is clipped and then scaled in such a way so that the PAPR is reduced without causing much degradation in bit error rate (BER). We have determined the threshold values for clipping and scaling using Monte Carlo Simulations. We have presented PAPR and BER of the system considered using simulations for QPSK constellation. We have also compared the performance of the proposed PAPR reduction technique with the performance of the existing techniques.**

Keywords— **Differential Scaling, Orthogonal Frequency Division Multiplexing, Bit Error Rate**

Introduction

OFDM supports high data-rate transmission [1], [2]. It is currently being implemented in some of the newest and most advanced communication standards like WLAN IEEE 802.11, WiMAX IEEE 802.16 and 4G. Unlike single carrier schemes, OFDM uses multiple orthogonal sub-carriers. Superimposition of these sub-carriers to form the OFDM signal results in high PAPR which drives the transmit power amplifier (TPA) into saturation. In the saturation region non-linearities are introduced into the signal. As a result the signal is distorted and efficiency of TPA reduces. Therefore, PAPR reduction is inevitable for reliable transmission.

A number of techniques have been proposed to reduce the PAPR for example clipping [3], companding [4], [6] selective mapping(SLM) [7],[8], partial transmit sequences (PTS) [9], [10], tone reservation [11] etc. However, there are some limitations with these techniques. Significant PAPR reduction can be achieved with Clipping and companding. But both the

into its corresponding time domain representation.

techniques introduce significant distortion, which results in increased BER. On the contrary, SLM and PTS schemes do not degrade the BER performance much. However, these schemes require transmission of side information (SI) which reduces the data rate. They also require excessive amount of IFFT calculations and therefore the complexity associated is very high.

In this paper, we propose a simple PAPR reduction technique based on Clipping and Differential Scaling. First the complex amplitudes are clipped and confined in a certain amplitude range. Thereafter, the amplitudes of the signal are scaled in a way to reduce the PAPR. We determine the threshold values for clipping and scaling using Monte Carlo simulations and using these values, we present PAPR and BER for the proposed technique. We have also compared our results with the results available in the literature for existing techniques.

The rest of the paper is organized as follows. Section-II describes the system model and in Section-III, we present details of the proposed clipping and scaling PAPR reduction technique. In Section-IV, we present simulation results of the proposed technique. The paper is concluded in Section-V.

II. SYSTEM MODEL

As shown in Fig. 1, the incoming data bits are mapped onto the constellation plane for the corresponding M-PSK or M-QAM scheme and complex symbols are generated. These symbols are to be transmitted independently on to the sub-carriers. To achieve this, they are fed parallel to the input of the N-point IFFT. They represent the frequency domain data set. Inverse Fourier transform converts this frequency domain data set

Specifically, IFFT is useful for OFDM because it generates samples of waveforms with orthogonal frequency components. PAPR reduction is applied at the output of the IFFT block and the OFDM symbols are then transmitted over the channel with energy per bit as E_b. The channel considered here is an Additive White Gaussian Noise (AWGN) channel with mean zero and variance N_o. At the receiver the inverse PAPR reduction technique is applied and FFT block is used to get the frequency domain data set from the time domain values. The signal in frequency domain represents the data symbols which were mapped to M-PSK or M-QAM. After parallel to serial conversion, these symbols are used to estimate the original data values.

Let N denote the number of subcarriers used for the parallel information transmission and let S_k $(0 \le k \le N-1)$ denote the k^{th} mapped symbol in a block of N information symbols. The outputs s_n of the N- point IFFT of S_k are OFDM signal samples over one symbol interval, or mathematically

$$s_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k e^{\frac{j2\pi k n}{N}}, \quad 0 \le n \le N-1 \qquad (1)$$

The input information symbols are assumed to be statistically independent and identically distributed. So, when N is large e.g. $N \ge 64$, the real and imaginary parts of s_n denoted by $Re\{s_n\}$ and $I_m\{s_n\}$ are independent and identically distributed Gaussian random variables with zero mean and a common variance $\sigma^2 = E[|S_k|]^2/2$ $(0 \le k \le N-1)$, according to the central limit theorem [12]. The amplitude of the OFDM signal s_n is given by

$$|s_n| = \sqrt{R_e\{s_n\}^2 + I_m\{s_n\}^2} \qquad (2)$$

The amplitude $|s_n|$ has Rayleigh distribution. The PAPR of OFDM signals in one symbol period is then defined as

$$PAPR = 10 \log_{10} \frac{|s_{max}|^2}{E[|s_n|]} \, dB \qquad (3)$$

where $s_{max} = \max(s_n)$ for $0 \le n \le N-1$. The peak power occurs when N modulated symbols are added with the same phase. By using the clipping and differential scaling technique, the OFDM symbols s_n are modified as per the PAPR reduction function $h(.)$. The modified signal t_n $(0 \le n \le N-1)$ after PAPR reduction is given by

$$t_n = h(s_n) \qquad (4)$$

where $h(.)$ is the PAPR reduction function that changes only the amplitudes of the input signals. Considering an Additive White Gaussian Noise (AWGN) channel, the received sig-nals r_n after the analog-to-digital (A/D) conversion can be expressed as

$$r_n = t_n + w_n \qquad (5)$$

$$= h(s_n) + w_n \qquad (6)$$

where $w_n \sim CN(0, N_o)$ are the samples of AWGN signal. After the inverse PAPR reduction operation at the receiver, we obtain

$$\hat{s}_n = h^{-1}(r_n) \qquad (7)$$

where $h^{-1}(x)$ is the inverse PAPR reduction function, or the inverse of h(x) and $\hat{s}_n$ is an estimation of the OFDM symbol, as it was prior to PAPR reduction. Each estimated OFDM symbol is then fed one-by-one to the FFT at the receiver to generate another symbol which would be used to estimate the data symbols by de-mapping. Thus, the data bits can be estimated.

III.CLIPPING AND DIFFERENTIAL SCALING

We propose in this section a new technique called Clipping and Differential Scaling. The probability distribution of amplitudes of the OFDM signal follows Rayleigh distribution [12] and thus the probability of high peaks is very less. An upper threshold above which the signal amplitudes do not contribute much to the signal is determined as follows. Using simulations, we have determined BER for the modified signals alongwith PAPR. We select the clipping threshold at which the BER is degraded from 1.5×10^{-3} to 3.5×10^{-3} at SNR of 10dB and the amplitudes above this clipping threshold are clipped. Instead of clipping the signal further to reduce the PAPR, we consider a reversible process - Differential Scaling which would reduce the PAPR but not deteriorate the BER. Since different ranges of amplitudes of the signal are scaled in a different manner, it is called Differential Scaling. We have considered three types of scaling as described below.

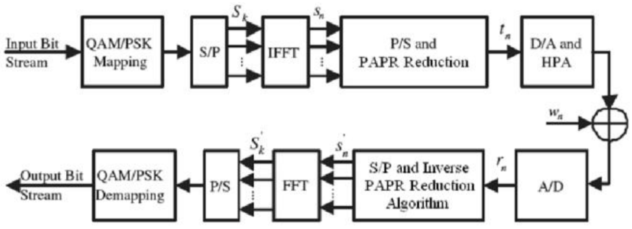

Fig. 1. OFDM system model with PAPR reduction block

Scale Up: In this method, we scale up the lower amplitudes of the signal by a factor of β. This leads to increase the average value without affecting the peak values. Therefore, the resulting PAPR reduces. The PAPR reduction function can be defined as

$$h(x) = \begin{cases} \alpha x_p, & if\ x > \alpha x_p \\ \beta x, & if\ x < A \\ x, & if\ A \le x \le \alpha x_p \end{cases} \qquad (8)$$

where x_p is the amplitude peak value occurring in an OFDM symbol block, α is the factor deciding the clipping threshold in terms of percentage of the peak value and β is the scaling factor for the range $[0, A)$ whose value is greater than one. The values of the parameters used are mentioned at the end of this section.

Scale Down: In this method, we scale down the higher amplitudes of the signal by a factor of γ. This leads to decrease the peak value. Although the average value would also fall down, the resulting PAPR reduces. Because the reduction in peak power is greater than the reduction in the average power. The PAPR reduction function can be defined as

$$\begin{aligned} h(x) &= \alpha x_p, && \text{if } x > \alpha x_p \\ &= \gamma x, && \text{if } B \le x \le \alpha x_p \\ &= x, && \text{if } x < B \qquad (9) \end{aligned}$$

where x_p is the amplitude peak value occurring in an OFDM symbol block, α is the factor deciding the clipping threshold in terms of percentage of the peak value and γ is the scaling factor for the range $[\beta, \alpha x_p]$ whose value is less than one. The values of the parameters used are mentioned at the end of this section.

TABLE I
SIMULATION PARAMETERS

Clipping Threshold (α)	0.47
Scale down factor (γ)	0.8
Lower limit for Scale down (B)	1.2
Scale up factor (β)	2
Upper limit for Scale up (A)	0.5

Scale Up and Down: In this method, we combine both the above-mentioned approaches i.e. up-scaling and down-scaling. This method exploits the advantages of both the methods. Hence, a PAPR can be reduced considerably. The PAPR reduction function can be defined as

$$\begin{aligned} h(x) &= \alpha x_p, && \text{if } x > \alpha x_p \\ &= \gamma x, && \text{if } B \le x \le \alpha x_p \\ &= \beta x, && \text{if } x < A \\ &= x, && \text{if } A \le x \le B \qquad (10) \end{aligned}$$

where x_p is the amplitude peak value occurring in an OFDM symbol block, α is the factor deciding the clipping threshold in terms of percentage of the peak value. β is the scaling factor for the range $[0, A)$ and γ is the scaling factor for the range $[B, \alpha x_p]$.

In order to make all these scaling techniques realizable, a marker needs to be used. The marker is basically a small set of signal values that needs to be transmitted along with the information signal. Its job is to keep track of values which have been scaled at the transmitter. The same values would be reversibly scaled at the receiver.The marker may be accomodated like the pilot carriers or sent on another frequency orthogonal to the carriers.

Using extensive simulations, the variation in PAPR with the simulation parameters A and β for the scale-up technique was observed at an SNR of 10 dB. From the 3-D plot obtained, we deduced the optimum values of A and β for which the PAPR is minimum. The optimum value of A and β is 0.5 and 2 respectively. Moreover, the BER obtained for the optimum values is 4×10^{-3} whereas the BER for the performance bound at 10 dB SNR is 2×10^{-3}. Thus, there is only a marginal compromise in the BER although we have reduced the PAPR significantly.

In the same manner for scale-down technique, the optimum values of γ and B for which the PAPR is minimum can be obtained. All these values are documented in Table-I

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IV. SIMULATION RESULTS

In this section, we present simulation results for the proposed techniques. The simulation is based on the system model in Fig. 1. PAPR reduction is achieved by clipping and there-after scaling the signal based on the algorithm. The marker signal is also generated depending on whether the signal value has been scaled or not. We have considered N = 64 and QPSK constellation. The average SNR per bit is shown as E_b/N_o in dB. We have shown the BER performance nd PAPR performance by means of complementary cumulative distribution function (CCDF) on values of the signal PAPR. The CCDF describes the probability that a real-valued random variable X with a given probability distribution will be found at a value greater than x i.e. $P\ [X > x]$. The CCDFs in Fig. 2 and Fig. 4 indicate the probabilities of signal PAPR occurring above certain thresholds.

Fig. 2 shows the CCDF with Scale up, Scale down and Scale up-down techniques. We have also shown the performance without PAPR reduction technique. It can be seen that all the three techniques significantly reduce the PAPR and the best PAPR reduction is achieved by Scale up-down technique as expected.

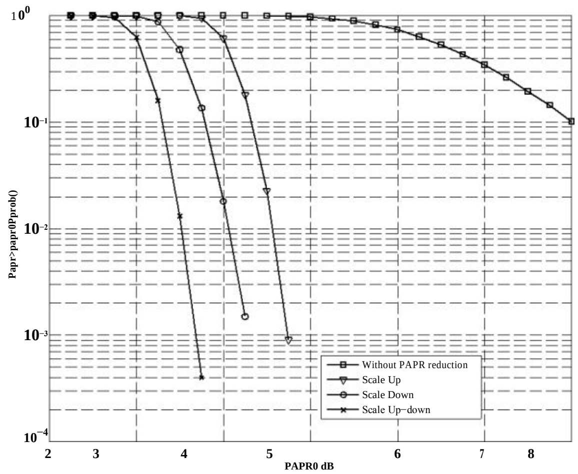

Fig. 2. Comparison of PAPR performance(CCDF) of Clipping and Differential Scaling with the CCDF of original OFDM signal

Fig. 3 shows the BER with Scale up, Scale down and Scale up-down techniques.

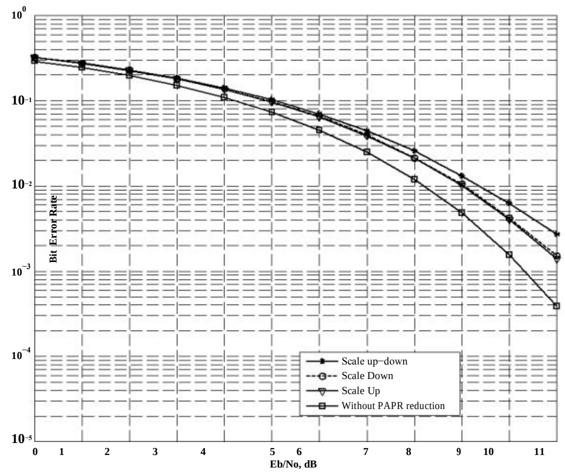

Fig. 3. Comparison of BER performance of Clipping and Differential Scaling with original OFDM signal

We have also shown the performance without PAPR reduction technique. The BER performances of all the three techniques are very close to the performance of the OFDM signal without PAPR reduction. Therefore, the proposed technique does not cause much degradation in BER. Fig. 4 and Fig. 5 show the PAPR and BER performance respectively for the prevailing Clipping [3], Companding [4] and Clipped Companding [5] techniques. In Clipping tech-nique, we consider the clipping threshold as 40 % of the peak value, in Companding we consider *M u*= 50 and in Clippied Companding we consider clipping threshold as 67 % of the peak value and value of *M u* = 50 for companding. In the same figures, we have also shown the performance of the proposed scale up-down technique for comparison.

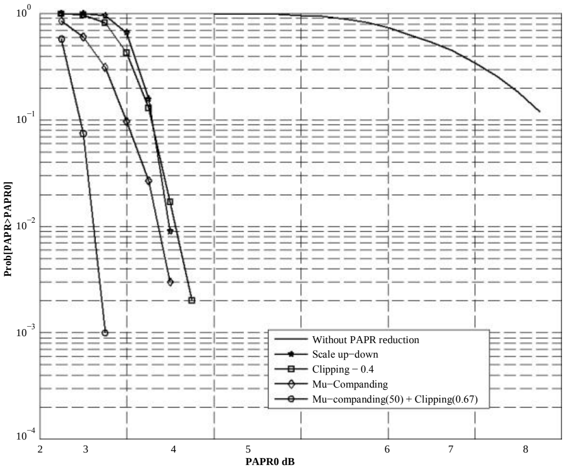

Fig. 4. Comparison of PAPR performance(CCDF) of different PAPR reduction techniques

From Fig. 4, it can be observed that the Clipped Companding and Companding techniques are able to achieve 2.5 dB and 3 dB values of PAPR respectively. The proposed Clipping and Differential Scaling also achieves PAPR which is comparable to these techniques. However, it can be seen in Fig. 5 that BER of the proposed technique is better than that of both the Companding technique and the Clipped Companding technique. The BER in both Companding and Clipped Companding is almost 20 times higher than that of the proposed technique at 10 dB SNR.

Moreover, from Fig. 4, the PAPR performance of Clipping is also comparable with the proposed technique but the BER performance of Clipping in Fig. 5 is not as good as the BER performance of the proposed technique. It can be seen that the BER of Clipping technique is 7 times higher than that of the proposed technique at 10 db SNR. Thus, the proposed technique is better as compared to the other techniques in terms of the PAPR reduction achieved at the corresponding BER performance.

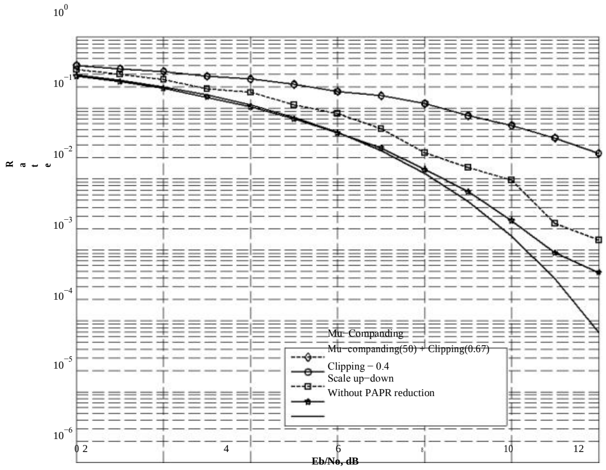

Fig. 5. Comparison ofBER performance of different PAPR reduction techniques

V. CONCLUSION

In this paper, we have used a simple approach based on Clipping and Differential Scaling to reduce the PAPR of OFDM signals. We have used Clipping along with three different scaling methods, namely up scaling, down scaling and up-down scaling. Using simulations, we obtained the values of threshold for clipping and parameters for scaling with a view to reduce PAPR without degradation in BER. We have presented the PAPR and BER performance for all the techniques considered. The proposed up-down scaling technique is able to achieve PAPR reduction of the order of 8.5 dB from 12 dB PAPR initially. The proposed technique is able to achieve a PAPR of 3.5 dB while maintaining the BER within a margin of 3 times the BER value at the performance bound at an SNR of 10 dB. We have also compared PAPR and BER of the up-down scaling with the existing Clipping and Clipped Companding techniques.

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