

# ***DESIGN AND VERIFICATION OF UNIQUE KEY BASED SUDOKU FOR STEGANOGRAPHY IN IMAGES***

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**Abstract:** This work intends to give an overview of image steganography, its uses & techniques. Paper work is an implementation of Image Steganography of same plaintext, the implementation of a system that design of unique Sudoku generation for Sudoku based data hiding and high security is the main concern of this paper. The system is based on a hybrid algorithm that applies the techniques of Sudoku and steganography to offer different security features to images transmitted between entities in internet. Based on the proposed algorithm, the authenticity and integrity of the transmitted images can be verified either in the spatial domain or in the encrypted domain or in both domains. The work is implemented on MATLAB design and simulation tool. Paper work uses modified Sudoku based data protection and SNR is very high in present paper work.

**Keywords:** Peak Signal to Noise Ratio (PSNR), Mean Square Error (MSE), Bit Per Pixel (BPP)

## **I-INTRODUCTION**

The paper work is the Implementation for steganography tools for hiding more data or information which may be text or another image files. Transform techniques will be used for identifying appropriate data. Available methods in area for steganography are as used as following:-

- Initially LSB substitution in image was popular<sup>[15],[16]</sup>
- Different schemes for LBS substitution developed<sup>[14]</sup>
- Key based LSB substitution developed<sup>[13]</sup>
- Multiple layer for steganography developed like steganography with steganography<sup>[12]</sup>
- DCT based image analysis for finding out area where data may be hiding appropriately<sup>[3]</sup>
- DWT based image analysis for finding out area where data may be hiding appropriately & found better than DCT based image analysis<sup>[7],[6]</sup>
- Sudoku based data hiding method in cover image<sup>[1],[2]</sup>

## **II-METHODOLOGY**

Proposed design resolve the problems of available work with significant modification done in interpolation method and Sudoku generation method, one 8 digit public and one 8 digit privet key are using for the proposed work and association of both 8 digit public and privet key develop one 8 bit digit Sudoku Key. Proposed work is a new design which uses Sudoku developed with 8 digits Sudoku Key provided by user, a unique Sudoku solution develop for each different key, this concept of unique Sudoku and combination of public and privet key makes proposed method robust against external intrusion. Proposed work is also using a interpolation and new pixel generation with help of four surrounding pixels instead of two pixels as was use by Chin-Chen Chang et al [1], four pixels interpolation develops more accurate new pixel then two pixels interpolation. The remaining processes of hiding are similar as use by Chin-Chen Chang et al [1].

### **2.1 DATA HIDING METHOD ADOPTED:**

Figure 1 below explains the flow diagram of the proposed work, it can be explains as in following steps

Step1: Input an 8 digit decimal public key, the key can be any value of 8 digit and must be pass when data hiding required.

Step2: perform logical XOR between 8 digit public key and 8 digit privet key which was already in the steganography module. The output key is known as Sudoku Key.

Step 3: develop a unique t1 to t9 digits out of 8 digits Sudoku key, the t1 to t9 can be anything between 1 to 9 but all t1 to t9 will be different from each other.

Step 4: Assign the values of t1 to t9 into a fixed bottom up approach Sudoku problem. Total 23 unknown values of Sudoku problem will be assigning using t1 to t9 digits.

Step 5: Solve the bottom up Sudoku problem with using Sudoku Rules, proposed method use the method of solving Sudoku by Ahmed Abdulkarim Almuhrj et al [27] from University of Manchester.

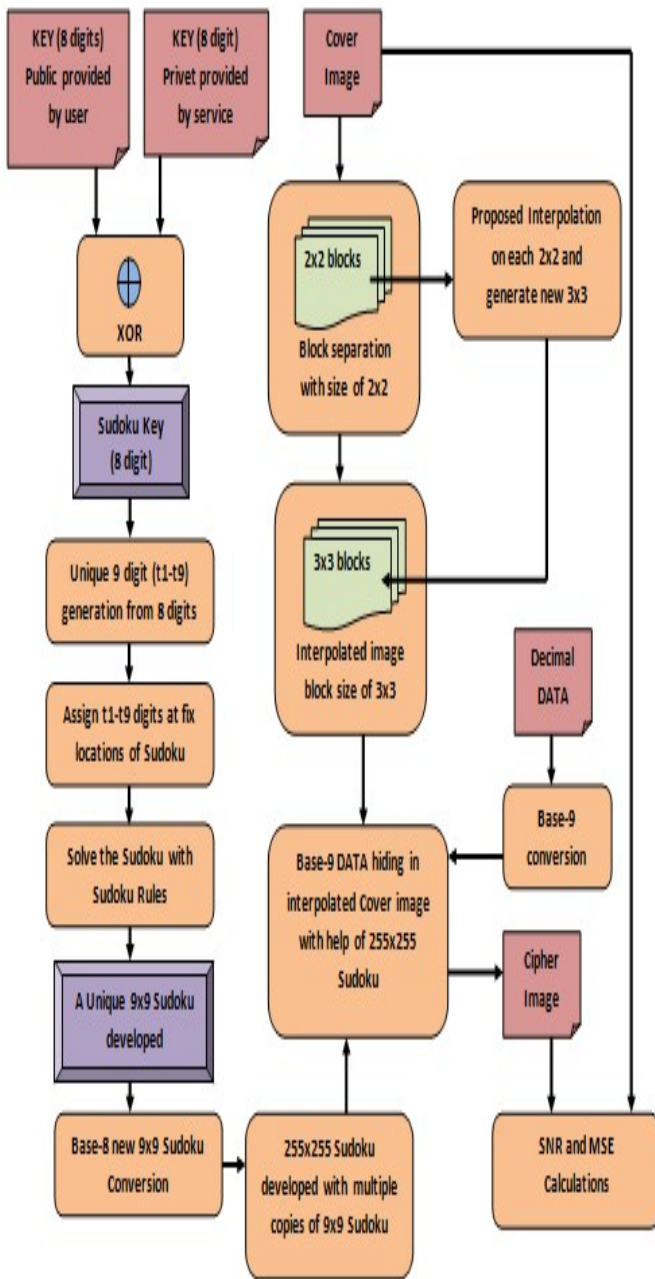


Figure 1 Data hiding Flow diagram

Step 6: convert the base-9 Sudoku into base-8 Sudoku because we can hide digits 0-8 only.

Step 7: develop a 255x255 Sudoku matrix with help of base-8 developed Sudoku, the size of 255x255 is fixed because any pixel of image cannot be greater than 255.

Step 8: input the cover image and isolate its 2x2 blocks

Step 9: perform interpolation on each 2x2 block and develop new 3x3 interpolated block, total 4 pixels was available in 2x2 block and total 9 pixels develop in 3x3 block, means 5 new pixels developed. These 5 new pixels can be modified according to the data which is to be hidden in the cover image.

The roll of interpolation is that original pixels (4 pixel of 2x2 block) must not be change and new 5 interpolated pixels can be modified with small amount, hence overall image will modified less as required in steganography.

Step 10: input the data which can be character or any number first convert it into its ASCII values then develop a single string of decimal digits.

Step 11: convert the number into base-9

Step 12: start hiding the data in cover image with help of 255x255 Sudoku matrix. Actually the original digit is not hiding in the cover image, the digits of data simply modify the interpolated pixels of cover image so small modification done. Each 2x2 block of interpolated image can hide 3 digits

## 2.2 PROPOSED ALGORITHM FOR DATA HIDING :

Input a 8 digit public key its 8 digits are K1, K2....K8

$$KEY=K1K2K3K4K5K6K7K8$$

Shift row for making Key complex as also done in AES

$$NK=K2K3K4K5K6K7K8K1$$

Logical XOR NK with privet key 'PK' for public Key encryption

$$MK=NK \text{ xor } PK$$

Now develop sub-keys from MK

$$MK1=99999999-MK$$

$$MK2=88888888-MK$$

$$MK3=77777777-MK$$

$$MK4=66666666-MK$$

$$MK5=55555555-MK$$

$$MK6=44444444-MK$$

$$MK7=33333333-MK$$

$$MK8=22222222-MK$$

On the other hand Let Sud is a 9x9 matrix where the position of t1,t2....t9 are fixed

$$Sud = \begin{matrix} t1 & UN & UN & UN & UN & 2 & UN & t3 & UN \\ UN & t4 & UN & UN & t5 & UN & UN & U & 6 \\ UN & UN & t7 & t8 & UN & UN & t9 & UN & UN \\ UN & UN & t1 & t2 & UN & UN & t3 & UN & UN \\ t6 & UN & UN & UN & UN & UN & t8 & UN & UN & UN \\ t9 & UN & UN & UN & UN & UN & UN & UN & t1 & UN \\ UN & t2 & UN & UN & UN & UN & UN & UN & t3 \\ UN & UN & t4 & UN & UN & UN & t5 & UN & UN \end{matrix}$$

From the Sub-Keys following method will generate t1, t2.....t9, the values of t1, t2 ....t9 can never be same.

Initially i=1 and j=1 and p=1

'i' can be range maximum from 1 to 23

'j' can be range maximum from 1 to 9

'p' can be range maximum from 1 to 8

For i

= 1:23

For j

= 1:8

if (MK(j)<sub>d(p+1)</sub> ≠ (t<sub>1:i-1</sub>, t<sub>1:i-2</sub>, t<sub>1:i-3</sub> ..... t<sub>1</sub>))

t<sub>i</sub> = MK(j)<sub>d(p)</sub>

elseif (MK(j)<sub>d(p+1)</sub> ≠ (t<sub>1:i-1</sub>, t<sub>1:i-2</sub>, t<sub>1:i-3</sub> ..... t<sub>1</sub>))

t<sub>i</sub> = MK(j)<sub>d(p+1)</sub>

elseif (MK(j)<sub>d(p+2)</sub> ≠ (t<sub>1:i-1</sub>, t<sub>1:i-2</sub>, t<sub>1:i-3</sub> ..... t<sub>1</sub>))

t<sub>i</sub> = MK(j)<sub>d(p+2)</sub>

⋮

⋮

else

t<sub>i</sub> = MK(j)<sub>d(p+7)</sub>

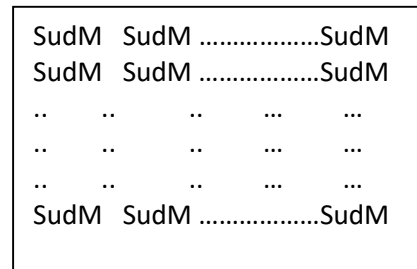
end

end

t1	U1	U2	U3	U4	t2	U5	t3	U6
U7	t4	U8	U9	t5	U10	U11	U12	t6
U13	U14	t7	t8	U15	U16	t9	U17	U18
U19	U20	t10	t11	U21	U22	t12	U23	U24
Sud1 = U25	t13	U26	U27	t14	U28	U29	U30	t15
t16	U31	U32	U33	U34	t17	U35	U36	U37
t18	U38	U39	U40	U41	U42	U43	t19	U44
U45	t20	U46	U47	U48	U49	U50	U51	t21
U52	U53	t22	U54	U55	U56	t23	U57	U58

'SudM' will contains digits from 0 to 8 only can be develop as: SudM=Sud1 -1

Next make copies of SudM such that a matrix of 255x255 generated



Ones the value of t1 to t9 computed, it will be according to public key provided by user and privet key by the service provider, a unique Sudoku need to be design and according to computed values of t1 to t9.

The values of UN in Sudoku matrix 'sud' will be computed using Sudoku solver according the Sudoku rules

- Any row cannot have repeated numbers 1 to 9
- Any column cannot have repeated numbers 1 to 9
- Any 3x3 square (cornering and central )matrix numbers cannot have repeated numbers 1 to 9

A MATLAB define function names SudokuSolver.m can solve this Sudoku and it can compute all the unknown using above rules, the complete Sudoku is represented below where t1-t9 initially computed with the help of the KEY and U1-U59 computed with the help of SodokuSolver and Sudoku rules.

This Sudoku is completlay define by the key as the key changes the complete Sudoku will also change, and this method will provide a good avalanche which is small change in input KEY cause major changes in output Sudoku, hence the system can be consider a chaotic system.



Figure 2: 255x255 Sudoku matrix

Input the cover image and convert it into matrix form which shows the pixels of the image. Let one part of the cover image as below

$$img = \begin{bmatrix} r1_1 & r1_2 & r1_3 \\ r2_1 & r2_2 & r2_3 \\ r3_1 & r3_2 & r3_3 \end{bmatrix}$$

Perform interpolation using the formula below

$$rx_{ni} = \left[ rx_i + \left( \frac{rx_i + rx_{i+1}}{2} \right) \right] / 2 \quad \text{where } x \text{ is constant}$$

$$cx_{ni} = \left[ ri_x + \left( \frac{ri_x+r(i+1)_x}{2} \right) \right] /_2 \quad \text{where } x \text{ is constant}$$

$$rcx_{ni} = \left\{ \left[ rx_{ni} + \left( \frac{rx_{ni}+rx_{n(i+1)}}{2} \right) \right] /_2 + \left[ cx_{ni} + \left( \frac{cx_{ni}+cx_{n(i+1)}}{2} \right) \right] /_2 \right\} /_2 \quad \text{where } x \text{ is constant}$$

process keep on until all the digits of the new data (ND) are not replace,

$r1_1$	$Y_{n1}$	$r1_2$	$Y_{n2}$	$r1_3$	$Y_{n3}$
$Y_{n4}$	$rc1_{n1}$	$Y_{n5}$	$rc1_{n2}$	$Y_{n6}$	$rc1_{n3}$
$r2_1$	$Y_{n7}$	$r2_2$	$Y_{n8}$	$r2_3$	$Y_{n9}$
$Y_{n10}$	$rc2_{n1}$	$Y_{n11}$	$rc2_{n2}$	$Y_{n12}$	$rc2_{n3}$
$r3_1$	$Y_{n13}$	$r3_2$	$Y_{n14}$	$r3_3$	$Y_{n15}$
$Y_{n16}$	$rc3_{n1}$	$Y_{n17}$	$rc3_{n2}$	$Y_{n18}$	$rc3_{n3}$

$r1_1$	$r1_{n1}$	$r1_2$	$r1_{n2}$	$r1_3$	$r1_{n3}$
$c1_{n1}$	$rc1_{n1}$	$c1_{n2}$	$rc1_{n2}$	$c1_{n3}$	$rc1_{n3}$
$r2_1$	$r2_{n1}$	$r2_2$	$r2_{n2}$	$r2_3$	$r2_{n3}$
$c2_{n1}$	$rc2_{n1}$	$c2_{n2}$	$rc2_{n2}$	$c2_{n3}$	$rc2_{n3}$
$r3_1$	$r3_{n1}$	$r3_2$	$r3_{n2}$	$r3_3$	$r3_{n3}$
$c3_{n1}$	$rc3_{n1}$	$c3_{n2}$	$rc3_{n2}$	$c3_{n3}$	$rc3_{n3}$

Input data ‘D’ and convert it into ASCII format and convert the data into base-9 formats ND=(D)<sub>9</sub>

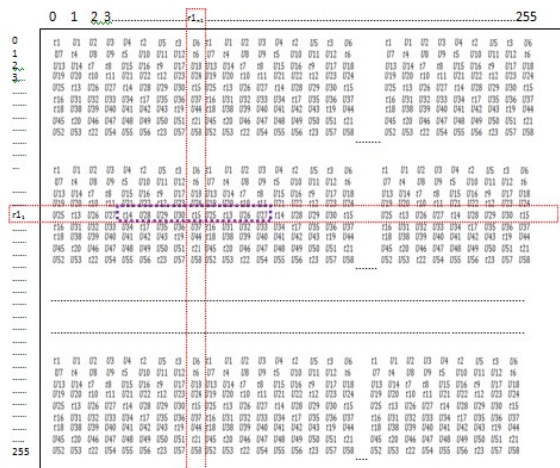


Figure 3 data in cover image modified according this 255x255 Sudoku matrix

Now from the interpolated cover image ‘img1’ select  $(r1_1, r1_{n1})$  position from Sudoku obtain at step 8 and then consider 4 pixel ahead and 4 pixels from back from the position  $(r1_1, r1_{n1})$ . The nine digits of Sudoku [t14 U28 U29 U30 t15 U25 t13 U26 U27] search for the first digit of new data ‘ND(1)’ and detect its position  $(r1_1, Y_{n1})$ . Replace the searched new position  $(r1_1, Y_{n1})$  with pixel of  $(r1_1, r1_{n1})$  of interpolated image, this

The whole idea is that we are not making any change in the original information pixels of the original cover image, the pixel values of interpolated pixel gets modified which was generated using interpolation formula, and making slight change in this pixel does not affect quality on the image significantly.

### 2.3PROPOSED METHOD FOR DATA EXTRACTION

Step1: Input an 8 digit decimal public key, the key can be any value of 8 digit and must be pass when data extraction required and it must be same as the time of data hiding.

Step2: perform logical XOR between 8 digit public key and 8 digit privet key which was already in the steganography module. The output key is known as Sudoku Key.

Step 3: develop a unique t1 to t9 digits out of 8 digits Sudoku key, the t1 to t9 can be anything between 1 to 9 but all t1 to t9 will be different from each other.

Step 4: Assign the values of t1 to t9 into a fixed bottom up approach Sudoku problem. Total 23 unknown values of Sudoku problem will be assigning using t1 to t9 digits.

Step 5: Solve the bottom up Sudoku problem with using Sudoku Rules, proposed method use the method of solving Sudoku by Ahmed Abdulkarim Almuhrif et al [27] from University of Manchester.

Step 6: convert the base-9 Sudoku into base-8 Sudoku because we can hide digits 0-8 only.

Step 7: develop a 255x255 Sudoku matrix with help of base-8 developed Sudoku, the size of 255x255 is fixed because any pixel of image cannot be greater than 255.

Step 8: Input the cipher image and isolate its 2x2 blocks



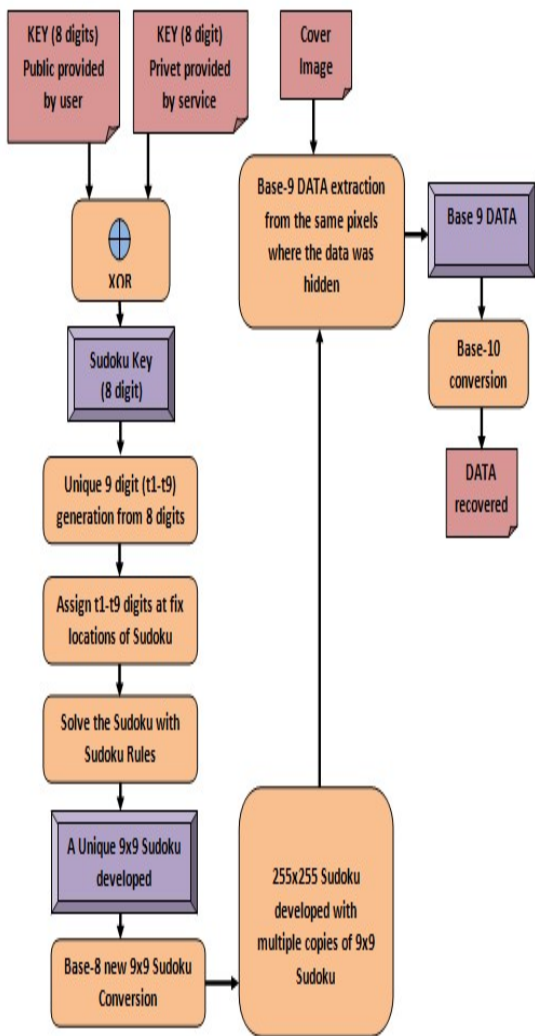


Figure 4 Data Extraction flow diagram

Step 9: With the help of 255x255 Sudoku matrix generate 3 decimal digits from each 2x2 block. The digit must be put as a sting of data.

Step 10: convert the number into base-10

Step 11: develop ASCII from the digit string and then convert ASCII into Characters.

### 2.4 ALGORITHM FOR DATA EXTRACTION

The process of 255x255 Sudoku will same as was in data hiding

Input the cover image and convert into matrix form which shows the pixels of the image. Let one part of the cover image as below:-

img =

P11	P12	.....	P1(N - 1)	P1N
P21	P22	.....	P2(N - 1)	P2N
P31	P32	.....	P3(N - 1)	P3N
P41	P42	.....	P4(N - 1)	P4N
.....	.....	.....	.....	.....
.....	.....	.....	.....	.....
P(N - 1)1	P(N - 1)2	.....	P(N - 1)	P(N - 1)N
PN1	PN2	.....	PN(N - 1)	PNN

Isolate 2x2 block let say

Blk= $\begin{matrix} P11 & P12 \\ P21 & P22 \end{matrix}$

Now from the cipher image 'img1' select (P11, P12), (P11, P21) and (P11, P22) position and search corresponding value in 255x255 Sudoku, the values in Sudoku will be data in base-9 form.



Figure 5: date digits will be find in the 255x255 matrix

Collect values from Sudoku according to positions (P11, P12), (P11, P21) and (P11, P22) and develop a string of base-9 data values  
 Convert base-9 data string into decimal data string  
 Convert decimal data string into ASCII and then convert ASCII into alphanumeric characters, these characters will be the final extracted DATA.

### III-RESULTS

MSE, SNR and BPP are the result parameters, this parameters will help with observation of results and comparison of proposed work other.

$$MSE = \frac{\{\sum_{i=1}^{rw} \sum_{j=1}^{cl} [P_{cipher}(i,j) - P_{cover}(i,j)]\}^2}{rw * cl}$$

$$PSNR = 10 * \log_{10} * \frac{256^2}{MSE}$$

$$BPP = \frac{8 * Bd}{Bi}$$

$P_{cipher}$  is the pixel of cipher image  
 $P_{cover}$  is pixel of cover image  
 Rw is number of row in cipher image  
 Cl is the number of column in cipher image  
 Bi number of bits in the cipher image  
 Bd number of bits in data

**BPP:** Bit Per Pixel is the number of bits that can be hidden inside a pixel, The capacity of data which can be hidden inside the proposed work can be explained with an test image let say image of 'Lena' with 512x512 pixels  
 512x512=2, 62, 144 pixels  
 Total 256x256 number of 2x2(4 pixel block) can be develop with 512x512 image  
 256x256x2x2=2,62,144 pixels  
 And after interpolation 2x2 block convert into 3x3 block means total  
 256x256x3x3=5,89,824 pixels  
 And as it's a color image with 3 frames total pixels can be  
 256x256x3x3x3= 17,69,472 pixels  
 As proposed method can hide three digits (12 bit) in each 2x2 block of cover image  
 17,69,472/4 = 4,42,386 total 2x2 blocks available  
 And each block can hide 12 bits hence 4,42,386 x 12 = 53,08,632 bits can be hidden in interpolated image size of 256x256x3x3x3 pixels and each pixel of 8 bit hence 256x256x3x3x3x8= 1,41,55,776 bits in interpolated image  
 BPP= 53,08,632 x 8 / 141,55,776 = 3.000122  
 Hence the maximum BPP observe for the proposed method is 3.000122

Number of bits	PSNR	MSE x 10 <sup>-11</sup>
2000	54.53	5.41
4000	53.47	6.91
8000	53.01	7.68
16000	52.15	9.37

32000	50.95	12.37
64000	49.84	15.95
128000	48.83	20.13

Table 1 PSNR observation for various size of data

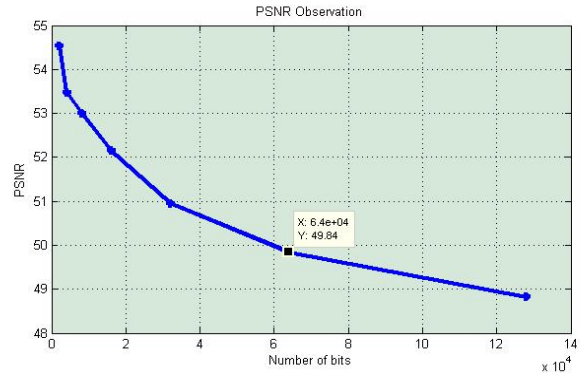


Figure 6 PSNR measure for different numbers of bits

### 3.1 COMPARATIVE RESULTS

PSNR Results observe for the 512x512 Lena Standard image			
2000 bits of data		1,20,000 bits of data	
Chang-Tsun Li et al [13]	Proposed Work	Thai-Son Nguyen et al [2]	Proposed Work
46.25	54.53	48.67	48.83

Table 2 PSNR Comparison with available work

Sr. NO.	AUTHOR	Results
1.	Fan Li et al [3]	PSNR observed is 52.99 when BPP taken as 0.063 for 512x512 Lena image

2.	Chin-Chen Chang et al [1]	2.23 bpp hidden in Lena Image obtain PSNR of 26.86
3.	Proposed work	3.000122 BPP observe for Lena image of 512x512 pixels

Table 3 BPP Comparison with available work

From the comparative results it can be observe that proposed work PSNR is better than available works and also the proposed work can hide more bits per pixel then available work.

#### IV-CONCLUSION

The original objective for paper work was to develop an optimized technique for hiding data inside cover image also to reduce amount for data in the channel while stenograph data transmission which is been achieved. A new 8 digit decimal number based unique Sudoku developed for enhancing the robustness of the work and also a modified interpolation is been developed for maintain good quality of image after hiding the data. problem with steganography is that it need lots of data (image) for sending few small amount for data, so proposed work is a good solution for this problem it can be confidently say that because we have achieved very good PSNR proposed design can be use for secure communication in cloud services also. The data bits are embedded in random into the cover-image pixels instead of it sequentially can be done to improve the security of the system in future.

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