Performance Analysis of OFDM and LDPC Coded OFDM System over AWGN and Rayleigh channel

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Abstract— **To fulfil the quench of high data rate and reliable data transmission, spectrally efficient modulation technique and efficient forward-error correction technique is required. Orthogonal frequency division multiplexing (OFDM) has high tolerance to multipath signals and it is spectrally efficient. Low-density parity-check (LDPC) coding is the efficient forward error correction technique that reduces the PAPR problem of OFDM system and used for highly reliable data transmission. In addition, cyclic-prefix (CP) between consecutive OFDM symbols can reduce effect of inter-symbol interference (ISI) more. In this paper, LDPC coding is combined with OFDM system so that performance of system can be improved in term of bit error rate (BER) versus signal to noise ratio (SNR). For simulation, quasi-cyclic (QC) LDPC code belonging to family of structured LDPC code is used. Iterative probabilistic decoding algorithm based on likelihood difference is used for decoding at receiver end. The proposed system also uses zero forcing equalizer along with CP to make system more immune to ISI. The simulation result shows that proposed system with LDPC code and CP improves BER performance than without LDPC coding. The system performance is simulated by using the software MATLAB 7.12.0.635. The performance was observed considering BPSK modulation and code rate=0.5.**

Keywords— **OFDM, Quasi-Cyclic LDPC Codes, Iterative Probabilistic Decoding Algorithm, Cyclic-prefix, PAPR.**

I. INTRODUCTION

The objective of any communication system is to transmit data at maximum possible data rate without error. A noisy communication channel provides interference to the signal. To cope with this problem channel coding technique is combined with the suitable modulation scheme. Thus channel coding and modulation have the same objective of producing the appropriate signal waveforms to face with the noisy channel. Low density parity check (LDPC) codes are the forward error correction code that provides best performance in noisy channels like additive white gaussian noise (AWGN) and Rayleigh (fading) channel.

As communication systems increase their information transfer speed, the time for each transmission necessarily becomes shorter. Since the delay time caused by multipath remains constant, inter-symbol interference (ISI) becomes a limitation in high-data rate communication. Orthogonal frequency division multiplexing (OFDM) avoids this problem by sending many low speed transmissions simultaneously. LDPC channel coding provides excellent error performance under noisy channel conditions with suitable modulation technique. LDPC coding reduces peak to average power ratio (PAPR) problem of OFDM system and improves error performance of the system in terms of bit error rate (BER) versus signal to noise (SNR) as well [1]. In this paper, we compare the BER performance of OFDM system and proposed LDPC Coded OFDM system over AWGN and Rayleigh fading channel. The effects of increase in FFT size on both systems are also observed.

II. BACKGROUND

A. OFDM System

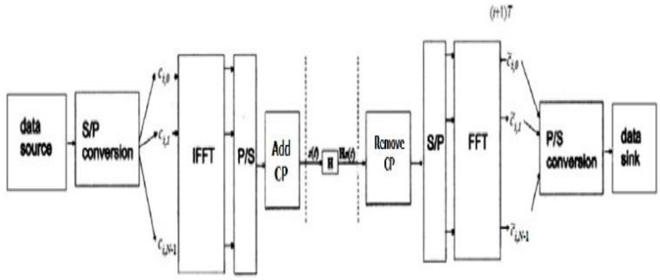

Fig.1 Block Diagram of OFDM System

OFDM stands for Orthogonal Frequency Division Multiplexing. OFDM is a multicarrier modulation technique widely used in both wireless and wired applications. OFDM is a special form of multicarrier transmission technique in which a single high rate data stream is divided into multiple low rate data streams [2]. These data streams are then modulated using sub-carrier which are orthogonal to each other. Since the sub-carriers are orthogonal to each other they can be packed closely together without causing interference between adjacent sub-carriers as shown in the Fig.2

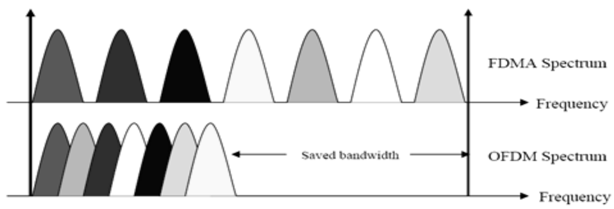

Fig.2 Bandwidth comparison between FDMA and OFDM

In this OFDM system, the incoming data stream is broken into various sub-channels. These sub-channels are assigned with different sub-carriers. Dividing of the incoming data stream is done by serial to parallel block. Once the bit stream have been assigned with different sub-carriers, each sub-carries is modulated as if it was an individual channel and combined back together to be transmitted as a whole.

The inverse fast fourier transform (IFFT) operation is performed on binary phase shift keying (BPSK) modulated bit streams on each sub-carrier. In this block, the modulation scheme is also chosen that is completely independent of specific channel. Parallel to serial conversion stage is the process of summing all the sub-carriers and combining them into a one complex OFDM symbol. A cyclic prefix is a repetition of the end section of a symbol that is appended to the front of the symbol, so as to completely eliminate ISI. The work of CP insertion block is to add the CP. Once the cyclic prefix has been added to OFDM symbols, they must be transmitted as one signal.

The signal coming out from CP insertion block is passed through the AWGN channel for this simulation. After passing through the channel Gaussian noise is added to represent the noises generated by thermal effects in the receiver. In the receiver, the reverse operation of the transmitter is performed by using blocks like CP removal block, serial to parallel converter, FFT block, and parallel to serial converter. The equalization is done to recover the symbols.

Advantages and disadvantages of OFDM [2]:

Advantages:

- OFDM makes efficient use of spectrum by allowing overlap of sub-carriers.
- By dividing the channel into narrow band flat fading sub-channel OFDM is more resistant to frequency selective fading than single carrier system.
- It eliminates ISI through use of cyclic prefix.
- Using adequate channel coding and interleaving one can recover symbols lost due to frequency selectivity of channel.
- OFDM is computationally efficient by using FFT techniques to implement the modulation and demodulation functions

Disadvantages:

- The OFDM signal has a noise like amplitude with very large dynamic range, also known as the peak-to-average-power ratio (PAPR). Therefore it requires RF power amplifier with a high peak to average power ratio.
- It is more sensitive to carrier frequency offset and drift than single carrier systems.

B. PAPR Reduction Techniques

B.1 Cyclic Prefix [1]:

The cyclic prefix, which is transmitted during the guard interval, consists of the end of the OFDM symbol copied into the guard interval, and the guard interval is transmitted followed by the OFDM symbol as shown in Fig.3. CP is a crucial feature of OFDM to combat the effect of multipath. The necessary condition for removal of ISI or ICI is that multipath delay should be less than CP as shown in Fig.4. Inter symbol Interference (ISI) and inter channel interference (ICI) are avoided by introducing a guard interval at the front, which, specifically, is chosen to be a replica of the back of OFDM time domain waveform. In CP each symbol is cyclically extended and as cyclic prefix carries no new information some loss in efficiency.

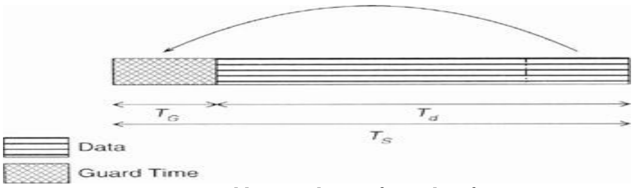

Fig .3 Adding Cyclic–prefix to data frame

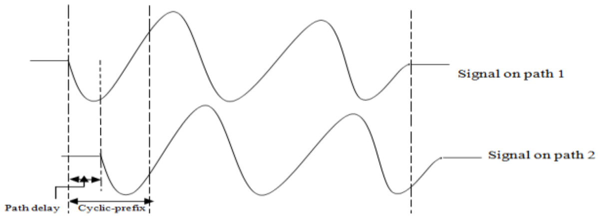

Fig.4 Effect of multipath on symbol with cyclic prefix

The idea behind CP is to convert the linear convolution (between signal and channel impulse response) to a circular convolution. In this way, the FFT of circularly convolved signals is equivalent to a multiplication in the frequency domain. The length of the prefix must be longer than the impulse response of the channel. Naturally this method introduces a lower overall bit rate, but the reduction of ISI more than outweighs the loss in data rate. In addition, OFDM facilitates the equalization at the receiver.

B.2 Low Density Parity Checking Codes

LDPC codes are linear block codes specified by a sparse parity-check matrix with the number of one's per columns (column weight) and number of one's per row (row weight) both of which are very small compared to the block-length (D.J.C. Mackay 1999).

There are two obvious characteristics for LDPC codes [3]:

- Parity-check: LDPC codes are represented by a parity-check matrix H, where H is a binary matrix that, must satisfy $cH^T = 0$, where c is a codeword.
- Low-density: H is a sparse matrix (i.e. the number of '1's is much lower than the number of '0's). It is the sparseness of H that guarantees the low computing complexity.

B.2.1 LDPC Representation

The example of a (7, 4) Hamming code is taken to show the way to represent the LDPC code into the factor graph.

$$H=\begin{bmatrix}1&0&0&1&0&1&1\\0&1&0&1&1&1&0\\0&0&1&0&1&1&1\end{bmatrix} \quad (1)$$

The syndrome equations are therefore:

$s_1=x_1 + x_4 + x_6+ x_7$
$s_2=x_2 + x_4 + x_5+ x_6$
$s_3= x_3 + x_5 + x_6+ x_7$

The factor graph of this code is shown in Fig.5. The nodes corresponding to the codeword bits are called the noise symbols and that corresponding to the parity check nodes are named check symbols [8]. The row of the parity check matrix indicate the check nodes or check symbols and the column of it indicate the bit node or codeword bits or noise symbols. In Fig.5, circular node and square node are used to indicate bit node and check node of parity-check matrix respectively.

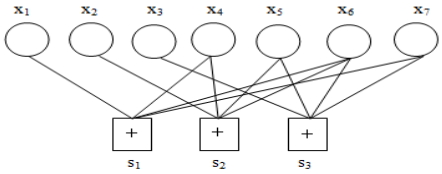

Fig .5 Factor graph of the Hamming code given by the parity check matrix in Equation (1)

B.2.2 Construction of LDPC code

The most obvious method for the construction of LDPC codes is via constructing a parity-check matrix with these characteristics in [3]. A larger number of construction designs have been researched and introduced in the literature to implement efficient encoding and decoding and to obtain near-capacity performance. Several methods for constructing good LDPC codes can be summarized into two main classes: random and structural constructions. Normally, for long code lengths, random constructions [4], [5] of irregular LDPC codes have been shown to closely approach the theoretical capacity limits for the AWGN channel. Generally, these codes outperform algebraically constructed LDPC codes. But because of their long code length and the irregularity of the parity-check matrix, their implementation becomes quite complex.

On the other hand, for short or medium-length LDPC codes, the situation is different. Irregular constructions are generally not better than regular ones, and graph-based or structured constructions can outperform random ones [6].

In this paper, quasi-cyclic (QC) LDPC codes are used for simulation that lies in the family of structured LDPC codes. An algebraic method of construction of structured LDPC codes is used to construct QC-LDPC as described in [7]. The rows or columns in sub-matrices of QC-LDPC have similar and cyclic connections. A QC-LDPC code can be simply represented by shift values of all of its sub-matrices. In this system, QC-LDPC code was generated which is formed by shifted identity sub-matrix and zero sub-matrixes. A shifted identity sub-matrix is obtained by shifting each row of an identity sub-matrix to right or left by some amount. The shifted identity sub-matrix and zero sub-matrixes are arranged in such a way to get larger size parity check matrix of LDPC code having girth at least 4. Few sub-matrices arrangement for QC-LDPC code is illustrated as shown in Fig.6. I_{xy} is a $p\times p$ shifted identity sub-matrix, O is a $p\times p$ zero sub-matrix where p is a positive integer in Fig.6. A higher the girth of the system, sparseness of the code increases and this reduces the decoding complexity.

I_{11}	I_{12}	I_{13}	I_{14}
I_{21}	I_{22}	I_{23}	I_{24}
I_{31}	I_{32}	I_{33}	I_{34}

(a)

I_{11}	O	O	O	I_{12}	I_{13}	O	I_{14}
O	I_{21}	I_{22}	I_{23}	O	O	I_{24}	O
I_{31}	I_{32}	O	I_{33}	O	O	O	I_{34}
O	I_{41}	I_{42}	O	I_{43}	I_{44}	O	O
O	O	I_{51}	O	O	I_{52}	I_{53}	I_{54}
I_{61}	O	O	I_{62}	I_{63}	O	I_{64}	O

Fig.6. QC code sub-matrices arrangement (a) with all non-zero sub-matrices (b) with zero sub-matrices.

The reason behind using QC-LDPC is that it provides a compact representation of the matrix and easy construction [7]. Due to quasi-cyclic structure it has low encoding complexity and low memory requirement, while preserving a high error correcting performance [3].

B.2.3 Encoding Message Blocks

The message bits are conventionally labelled by $u=[u_1,\ldots,u_k]$, where the vector 'u' holds the k message bits. The codeword 'c' corresponding to the binary message 'u' can be found by using matrix equation

$$c= uG \quad (2)$$

and for binary code with k message bits and length n codewords the generator matrix G, is a k×n binary matrix. The ration k/n is called rate of the code. A code with k message bits contains 2^k codewords. These codewords are a subset of the total possible 2^n binary vectors of length n.

A generator matrix for a code with parity check matrix H can be found by performing Gauss-Jordan elimination on H to obtain it in the form in equation (3).

$$H= [A, I_{n-k}] \quad (3)$$

Where A is an (n-k) ×k binary matrix and I_{n-k} is the identity matrix of the order n-k. The generator matrix is then given by

$$G= [I_k, A^T] \quad (4)$$

The row space of G is orthogonal to H. Thus if G is the generator matrix for a code with parity check matrix H then

$$GH^T=0 \quad (5)$$

B.2.4 Decoding Algorithm for LDPC Codes

The iterative decoding algorithm based on likelihood difference is derived from message-passing algorithm, as illustrated with example in [8], will be used for LDPC decoding in this simulation. The difference of likelihood is used instead of the likelihood in regard to the fact that all the symbols are binary.

Few terms used in algorithm are defined as follows:

- $N(i)=\{j : h_{ij}=1, 1\le j \le n\}$ is the set of codeword bits taking part in parity check i, n is the codeword length
- $M(j)=\{i : h_{ij}=1, 1\le i \le J\}$ is the set of parity checks that check the codeword bit j, J is the number of parity checks, or the number of rows of H.
- *N (i)\j*: The set of codeword bits taking part in the parity check i, excluding the codeword bit j.
- *M (j)\i*: The set of parity checks that check the codeword bit j, excluding parity check i.

In this algorithm, four parameters are defined for each nonzero element h_{ij} in the parity check matrix H: $\Psi_{ij}^{a=0}, \Psi_{ij}^{a=1}, \Omega_{ij}^{a=0}$ and $\Omega_{ij}^{a=1}$.

- Ψ_{ij}^{a} is the probability that code bit j gets the value a, given the information from all the parity checks excluding check i.
- Ω_{ij}^{a} is the probability that parity check i is satisfied if code bit x_j=a and the probabilities that other noise symbols get their values are given by $\{\Psi_{ij'}^{a} : j' \epsilon N(i)\backslash j, a = 0,1\}$

At the beginning, a posterior probabilities of the noise symbols can be initialized at p (r|+1) and p (r|-1).

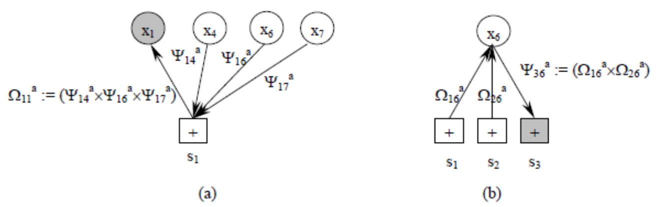

Fig .7 (a) Calculating the message from a check symbol to a noise symbol. (b) Calculating the message from a noise symbol to a check symbol.

In Fig.7, we can see that, when s_1 has received the Ψ messages from x_4, x_6 and x_7, it can calculate the Ω message to send to x_1. Similarly, the codeword node x_6 use the Ω messages from s_1 and s_2 to compute the Ψ message for s_3.

1. Initialisation:

The probabilities of channel outputs given the transmitted symbols are provided by Equation (6):

$$p(r_j|-1)=\frac{1}{1+e^{\left(\frac{-2r_j}{\sigma^2}\right)}} \text{ and } p(r_j|+1)=\frac{e^{\left(\frac{-2r_j}{\sigma^2}\right)}}{1+e^{\left(\frac{-2r_j}{\sigma^2}\right)}} = 1-p(r_j|-1) \quad (6)$$

At the beginning, and Ψ_{ij}^{0} and Ψ_{ij}^{1} are initialised at p $(r_j|x_j=-1)$ and p $(r_j|x_j=1)$, respectively. In the matrices $\{\Psi_{ij}^{0}\}$and$\{\Psi_{ij}^{1}\}$, the messages a noise symbol sends to all the parity checks it is connected to are the same and equal to $p(r_j|x_j=-1)$ and $p(r_j|x_j=1)$, respectively.

2. Iterative Decoding:

(a) Horizontal step:

Define the difference $\delta\Psi_{ij}=\Psi_{ij}^{0}-\Psi_{ij}^{1}$. For every pair (i, j), with a = 0 and 1, we update the Ω messages from the check symbol s_i to the noise symbol x_j:

$$\delta\Omega_{ij}=\prod_{j'\epsilon N(i)\backslash j}\delta\Psi_{ij'} \quad (7)$$

$$\Omega_{ij}^{a}=\frac{1}{2}\left[1+(-1)^{a}\delta\Omega_{ij}\right] \quad (8)$$

(b) Vertical step:

For every pair (i, j), with a = 0 and 1, we update the Ψ messages from the noise symbol x_j to the check symbol s_i:

$$\Psi_{ij}^{a}=\alpha_{ij}\, p\,(r_j|x_j=2a-1)\prod_{i'\epsilon M(j)\backslash i}\Omega_{i'j}^{a} \quad (9)$$

Where α_{ij} is a normalising constant chosen to give $\Psi_{ij}^{0}+\Psi_{ij}^{1}=1$. For each j and a=0, 1, update the "pseudo *a posterior* probabilities" [9] Ψ_{j}^{0} and Ψ_{j}^{1} using the equation:

$$\Psi_{j}^{a}=\alpha_j\, p\,(r_j|x_j=2a-1)\prod_{i\epsilon M(j)}\Omega_{ij}^{a} \quad (10)$$

Where α_j is a normalising constant chosen to give $\Psi_{j}^{0}+\Psi_{j}^{1}=1$

Evaluation:

- A bit-by-bit decoded value is $\hat{x}_j$ chosen using the rule: If $\Psi_{j}^{1}>0.5$, $\hat{x}_j$=1, if $\Psi_{j}^{1}\le 0.5$, $\hat{x}_j$=0

-If $\hat{x}H^{T}$=0 then $\hat{x}$ is a valid codeword and the algorithm stops successfully.

-Else

- If the maximum number of iterations has been reached, a failure is recorded and the algorithm stops.
- Else: Go back to the beginning of Iterative Decoding.

III. SIMULATION

A. System Model

In this paper we study and compare the performance of the OFDM system and LDPC Coded OFDM system where both system uses CP to combat with effect of multipath and BPSK modulation technique is used for simulation. The simulation is carried out in MATLAB 7.12.0.635 (R 2011a). Matlab model was built to simulate the transmitter, channel and receiver. The OFDM system model used for simulation is designed as shown in Fig.1. The proposed system model of LDPC Coded OFDM system for improved performance is shown in Fig.8. A BER vs. SNR plot of OFDM and LDPC system are compared over AWGN and Rayleigh fading channel. Similarly, we also observe the performance of each system in different channel separately. Then we observe the effect of FFT/IFFT size on these systems.

A.1 System Model for LDPC Coded OFDM system

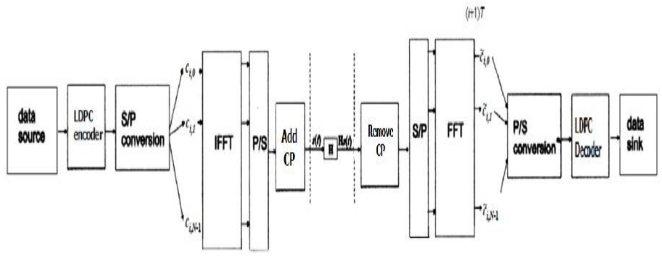

Fig.8 Block Diagram of LDPC-OFDM system

The proposed model of LDPC coded OFDM system for simulation is as shown in Fig 8. In this system, QC-LDPC code was generated which is formed by shifted identity sub-matrix and zero sub-matrixes. The shifted identity sub-matrix and zero sub-matrixes are arranged in such a way to get larger size parity check matrix of LDPC code having girth at least 4. The higher the girth of the system, sparseness of the code increases and this reduces the decoding complexity. The code rate is chosen to be 1/2 by doubling the column size than the row size of parity check matrix of LDPC code. Generator matrix 'G' is determined from parity check matrix 'H' using equation (3), (4) and (5).

In this system, incoming data is initially LDPC coded by using LDPC encoder before applying the OFDM technique to the data stream. After encoding message block with QC-LDPC code, data stream is broken into various sub-channels as in OFDM. These sub-channels are assigned with different sub-carriers. Dividing of the LDPC encoded data stream is done by serial to parallel block. Once the encoded bit stream have been assigned with different sub-carriers, each sub-carries is modulated as if it was an individual channel and combined back together to be transmitted as a whole.

The BPSK modulation is performed on channel encoded data in each sub-carrier and then IFFT operation is performed on these modulated signals. In this block, the modulation scheme is also chosen that is completely independent of specific channel. Parallel to serial conversion stage is the process of summing all the sub-carriers and combining them into a one complex OFDM symbol. After addition of CP in this symbol it is transmitted as one signal over AWGN channel or Rayleigh fading channel for this simulation.

Assumption is made for simulation that receiver knows the channel state information by providing channel coefficient value h_{ij}. The Zero-Forcing Equalizer applies the inverse of the channel frequency response to the received signal, to restore the signal after the channel. The name Zero Forcing corresponds to bringing down the ISI to zero in a noise free case. This will be useful when ISI is significant compared to noise. This equalizer is used before the CP is removed from received data. This equalizer is generally used for spatial decoding in MIMO wireless system. However to make system more immune to the ISI, the proposed system uses it.

In the receiver, the reverse operation of the transmitter is performed by using blocks like CP removal block, serial to parallel converter, FFT block, parallel to serial converter and LDPC decoder. In the receiver end, for this system LDPC decoder iterative probabilistic decoding algorithm based on likelihood difference is used [8].

B. Simulation Results

To study OFDM and LDPC Coded OFDM system we plot the BER performance of both system over AWGN and Rayleigh fading channel up to SNR point 20 dB. We also observed effect of increase in FFT/IFFT size on OFDM and LDPC Coded OFDM system. The system used for simulation uses QC-LDPC code with code rate 0.5 and BPSK modulation. The simulation results are listed below:

- From Fig.9(a),In AWGN channel BER of OFDM system is approximately 10^{-3} whereas that of LDPC Coded OFDM system is 10^{-5} at SNR= 8 dB. Similarly, from Fig.9(b) in Rayleigh fading channel BER of OFDM system is approximately 10^{-2} whereas that of LDPC Coded OFDM system is 10^{-5} at SNR=10 dB.
- Fig.10 shows that BER performance of both OFDM and LDPC Coded OFDM system is far better in AWGN channel than Rayleigh fading channel .
- The effect of variation of FFT/IFFT size on OFDM and LDPC Coded OFDM system is observed separately keeping other system parameter constant. From Fig.11 we observed that increase in FFT/IFFT size have only slight decrease in BER performance in both systems.

From the simulation results, it verifies that use of LDPC code in OFDM system improves the BER performance of the system. These systems perform better in AWGN channel than in Rayleigh fading channel environment . Theoretically, the increase in FFT/IFFT size increases the OFDM symbol length making it more vulnerable to phase noise and resulting increase in BER of the system. However, the simulation results shows that FFT/IFFT size has no significant effect on BER .It reduces slightly on increasing FFT size.

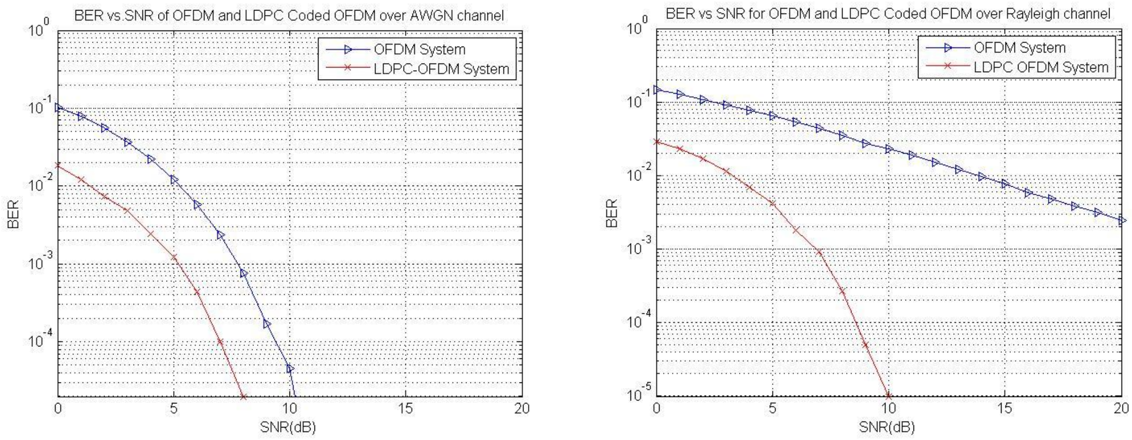

Fig.9.BER Performance of OFDM and LDPC Coded OFDM System on various channel: a) AWGN channel b) Rayleigh fading channel

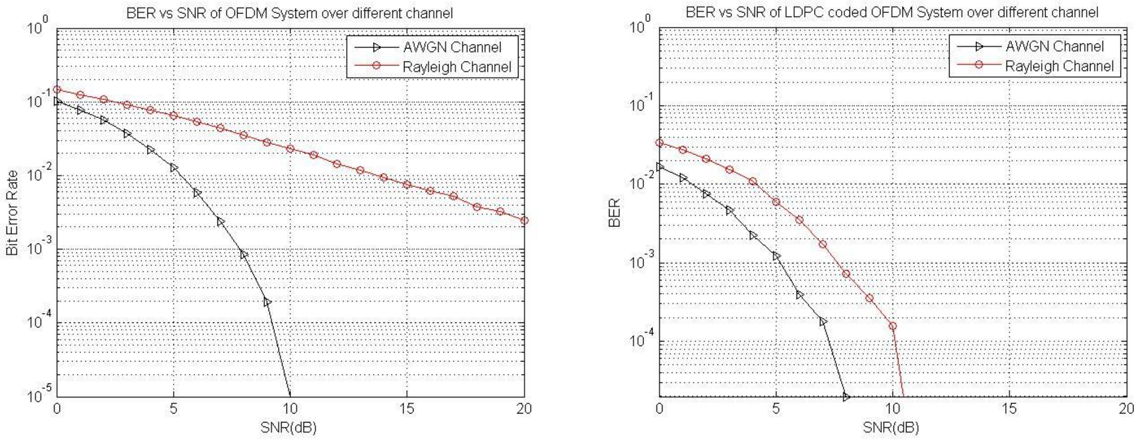

Fig.10. Effect of channel environment on BER performance of different system: a) OFDM System b) LDPC Coded OFDM System

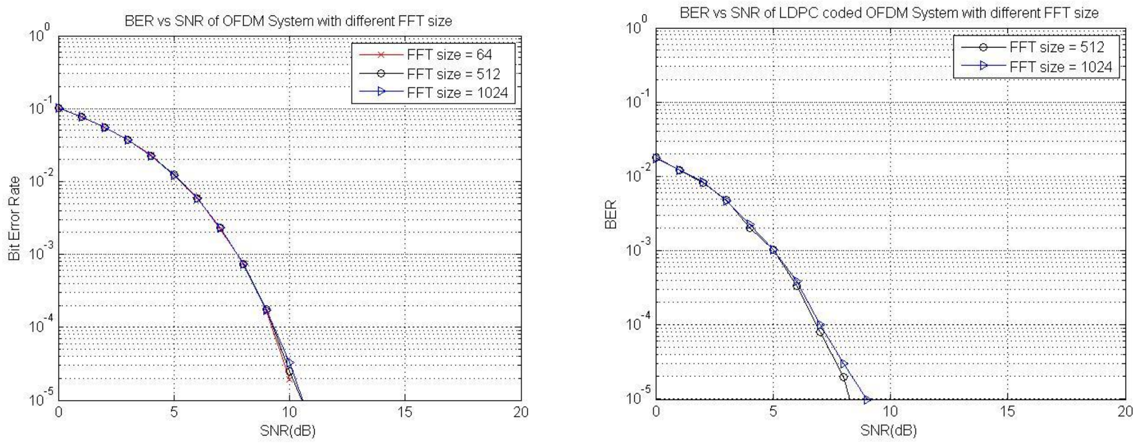

Fig.11. Effect of FFT/IFFT size on BER performance of different system: a) OFDM System b) LDPC Coded OFDM System

IV. CONCLUSIONS

In this paper, we compared the performance of OFDM system and LDPC coded OFDM system over AWGN AND Rayleigh fading channel. The BER vs. SNR curve is plotted for comparision and analysis of these systems up to 20 dB using BPSK modulation. The proposed LDPC coded OFDM system uses QC LDPC code belonging to the family of structured LDPC code with code rate of 0.5. This system uses both LDPC code and CP method to improve system performance as well as to reduce PAPR problem of OFDM system. Iterative decoding algorithm based on likelihood difference is used at receiver end for LDPC decoding. From this simulation we concluded that using LDPC code in OFDM system improves the system performance in comparision with system without coding in terms of BER. From simulation results we observed that BER performance is improved in LDPC Coded OFDM system in comparision to OFDM system over both AWGN and Rayleigh fading channel environment. We also concluded that FFT/IFFT size has no significant effect on BER performance in both OFDM and LDPC Coded OFDM system. BER reduces only slightly on increasing FFT/IFFT size.

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