Solution to Economic Load Dispatch including transmission losses using Pattern Search Method

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Abstract—**This paper presents pattern search method to solve the economic load dispatch problem including transmission losses. Economic load dispatch (ELD) is a process of finding optimal generation scheduling of committed generators in a power system to meet the demand of system, at minimum cost. Pattern search (PS) is a Direct search (DS) method that do not require the gradient of the problem for optimization. Hence, Pattern search can be used for problems that are not continuous or differentiable. Effectiveness of the proposed PS method has been demonstrated by solving economic load dispatch problem including transmission losses on 3, 6 and 20 generating units respectively. Performance of the proposed method is compared with the genetic algorithm (GA) and previously published results and is found to be superior in terms of solution quality and speed.**

Keyword—**economic load dispatch, power system, direct search method, pattern search method, line losses**

1. INTRODUCTION

The continuously increasing power demand has made the modern power system operation highly complex and unpredictable. Hence, the focus has shifted toward increasing reliability, performance, customer satisfaction and environmental perspective. Conventional methods for solving ELD problem, like Lamda iteration, Base point participation factor, Gradient methods etc. are dependent on the convexity assumption of generator cost curves and hence approximate these curves using quadratic or piecewise quadratic monotonically increasing cost functions [1]. Practically, power generators neither have equal fuel cost nor have equal distance from the load, so we have to select a more economic generator considering transmission line losses for the fulfillment of power demand for a specific time. Economic load dispatch has highly non linear objective function with rigid equality and inequality constraints [2, 3].

In recent year, many heuristic methods such as evolutionary programming [4,5], tabu search [6], genetic algorithm[10, 19] and particle swarm optimization (PSO) [7]have been applied for ELD problem, which do not depend on convexity assumptions, but require large computational time. These heuristic methods do not always guarantee global best solutions, but are often found to achieve a fast and near global optimal solution. Methods based on artificial intelligence techniques were also presented in many references [8, 9].

In this paper, pattern search (PS) method has been applied for solving economic load dispatch problem including line losses. Pattern Search method is a kind of Direct search (DS) methods. DS methods search a set of points that has smaller objective value than the present value [11]. The Direct search methods include Pattern Search, Simplex method (SM), Powell optimization etc. DS methods don't require the gradient of the objective function to be optimized and are called the derivative free or black-box optimization methods [12, 13].

2. PROBLEM FORMULATION:

Economic load dispatch problem is required to be solved to find the optimal generation sharing among committed generators for minimization of power generation cost. The cost formulation of ELD problem includes fuel cost and maintenance cost but here for simplicity only fuel cost considered. The cost of power generation, particularly in fossil fuel plants, is very high and economic load dispatch helps in saving a significant amount of revenue. The mathematical expressions for ELD and transmission loss are as follows [14, 19].

$$C_t = \sum_{i=1}^{N} F_i(P_i) \tag{1}$$

$$\sum_{i=1}^{N} P_i = P_d + P_l \tag{2}$$

$$F_i(P_i) = a_i P_i^2 + b_i P_i + c_i \tag{3}$$

And generating capacity constraint as

$$P_i^{min} < P_{gi} < P_i^{max} \tag{4}$$

The transmission losses can be described as

$$P_l = \sum_{i=1}^{N} \sum_{j=1}^{N} P_i B_{ij} P_j + \sum_{i=1}^{N} B_{0i} P_i + B_{00}$$
(5)

Where
C_t = total cost of generation
P_i = output generation of unit i
N = number of generators in the system
$F_i(P_i)$ = fuel cost function of the i_{th} unit
P_d = total demand of power system
P_l = total transmission loss of power system
a_i, b_i, c_i = constant of the fuel function
$B_{ij}, B_{0i} B_{00}$ = transmission loss coefficient

3. PATTERN SEARCH METHOD

The Pattern Search method is a kind of direct search method that can be used for several problems that lies outside the scope of general optimization method. PS is a method for solving optimization problems that does not require any information about the gradient of the objective function [17, 20]. Unlike the traditional optimization methods that use information about the gradient or higher order derivatives to search for an optimal point, a direct search algorithm explores a set of points around the current point, looking for one where the value of the objective function is lower than the value at the current point. Direct search methods can be used to solve problems for which the objective function is not differentiable, or is not even continuous [18]. The easy implementation and accuracy of the method increase is usefulness. Unlike other algorithms Neural Networks, PSO, GA [4, 9, 10], Pattern Search method shows the good response for optimization problem. The various types of direct search methods for unconstraint optimization are discussed in [12].

3.1 MESH AND PATTERN

The pattern search algorithm starts with a set of initial points, called a mesh around the given point. This starting point can be given by the user or it may be the taken from last step. The mesh is made by adding the current point to a scalar multiple of a set of vector called a pattern. When a point in the mesh is found to improve the objective function at the current point, the new point become current point for the next iteration

Initially, the pattern search begins with the initial point X0 that is defined by the user. At the first iteration, taking scalar equal to 1 called the mesh size, the pattern vectors are: [1 0], [0 1], [-1 0], [0 -1]. These may be called as direction vectors [13]. The PS algorithm adds the each direction vector to the initial point to compute the next direction vector or it may be called the pattern vector:

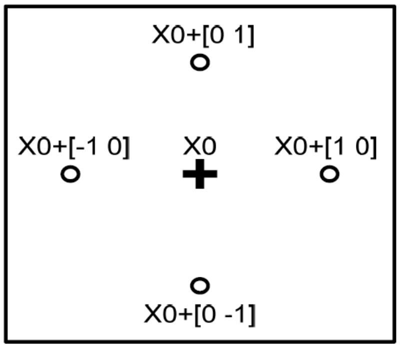


Figure 1: pattern search mesh points

X0 + [1 0], X0 + [0 1], X0 + [-1 0] and X0 + [0 -1]

The formulation of the mesh points and pattern vector are shown in Figure 1. The algorithm computes the objective function at the mesh points in the order shown above. The algorithm polls the mesh points by computing their objective function values until it finds one whose value is smaller than the objective function value at present point X0. If such a point is found, then the poll is successful and the algorithm sets this point equal to X1 that will be starting point for the next iteration [16]. When the poll is successful, the algorithm moves (steps) to second iteration and multiplies the current mesh size by 2 (this is called expansion factor and has a default value of 2). The mesh at second iteration contains the points:

2*[1 0] + X1, 2*[0 1] X1, 2*[-1 0] + X1 and 2*[0 -1] + X1.

The algorithm polls the mesh points until it finds one whose value is smaller the objective function value of last one (X1). The point is called X2 and the poll is said to be successful. As the poll is successful, the algorithm multiplies the current mesh size by 2 to get a mesh size of 4 at the third iteration (the expansion factor is taken as 2). If in iteration 3 (mesh size= 4), none of the mesh points has a smaller objective function value X2, then the poll is said to be unsuccessful. The algorithm does not change the current point at the next iteration. That is, X3 = X2 at the next iteration and the algorithm multiplies the current mesh size by a factor 0.5, called contraction factor. Thus, the mesh size at the next iteration is smaller. The algorithm then polls with a smaller mesh size [15]. The Pattern search optimization algorithm will repeat the steps until it finds the optimal solution for the minimization of the objective function. The PS algorithm stops when any of the following conditions occurs [20]:

- Time reaches its maximum value.
- The mesh size reaches its maximum tolerance.
- The number of iterations performed by the algorithm reaches its maximum iteration.
- Objective function evaluations performed by the algorithm reaches the value of Maximum function evaluations.
- The distance between the point found at one successful poll and the point found at the next successful poll is less than the given tolerance.

The Flowchart of Pattern Search method implementation has been shown in Figure 2. It is to be noted that all the parameters in the pattern search optimization algorithm can be pre-defined subject to the nature of the problem being solved [17].

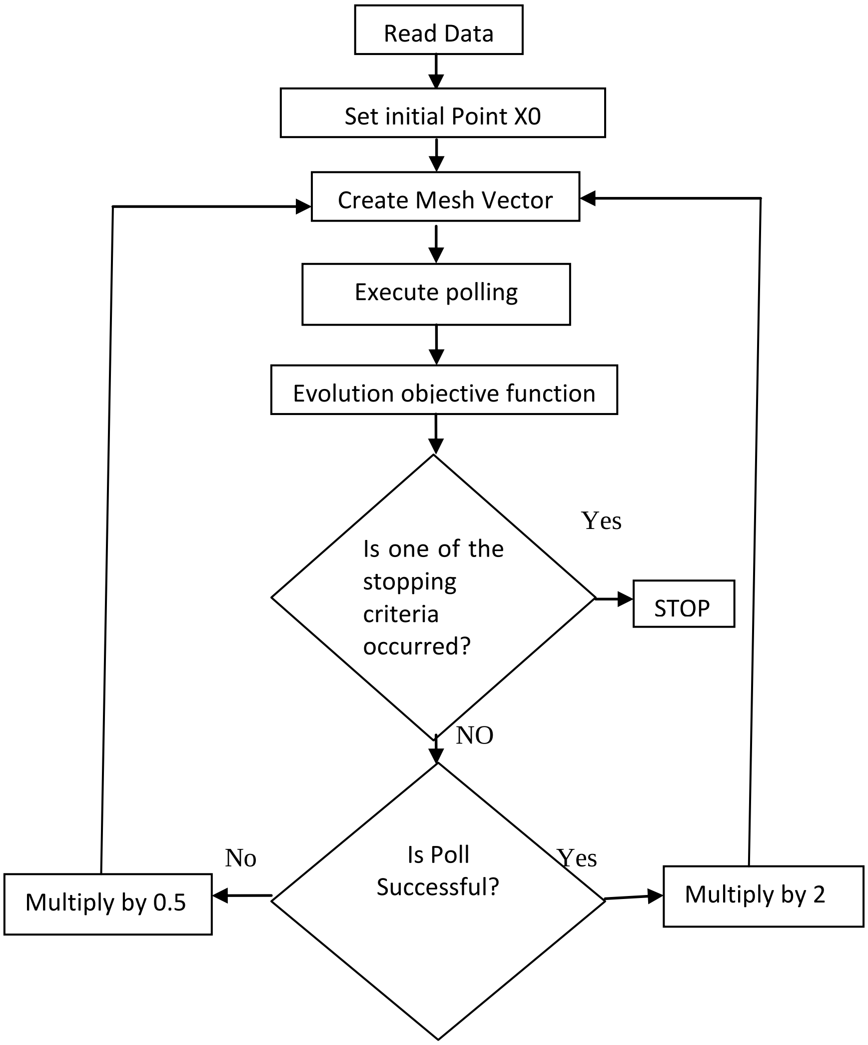


Figure 2: Flow chart of Pattern Search algorithm

4. TEST RESULT AND ANALYSIS

The program for PS method has been developed using MatLab software and executed on Celeron M processor having 1.7 GHZ 512GB DDR2 RAM. Initially, several runs have been carried out with different values of the key parameters of PS such as the initial point; mesh size, expansion and contraction factors. In this study, the mesh size and the mesh expansion and contraction factor are selected as 1, 2 and 0.5, respectively.

In order to assess the effectiveness and robustness of the proposed Pattern Search method, three test cases of economic load dispatch with transmission loss have been considered. The non-linear minimization problem formulation of all test cases has been solved using the predefined function pattern search incorporated in the GA & DS toolbox of Matlab R2009b [13, 14].

4.1 CASE I – THREE GENERATING UNIT

The test case consists of three generating units with quadratic cost function combined with the transmission losses. The unit data (upper and lower bounds) along with the cost coefficients for the fuel cost (*a*, *b*, *c*) for the three generators with loss coefficients are taken from [21]. The solutions obtained using PS method has been compared with the results of other evolutionary method Genetic Algorithm (GA) applied to the same test system in [21].

Table 1: Generator loading and fuel cost with Transmission loss for 150(MW)

Unit output power	Iterative method	GA	PSO	PS
P_{g1}	73.5275	73.8324	73.9723	73.8343
P_{g2}	69.5074	69.9595	69.6656	69.9607
P_{g3}	75.7826	75.0201	75.1645	75.0223
Power loss(MW)	8.8165	8.827	8.7546	8.8173
Generation cost($/h)	3163.9	3163.63	3162.938	3163.69

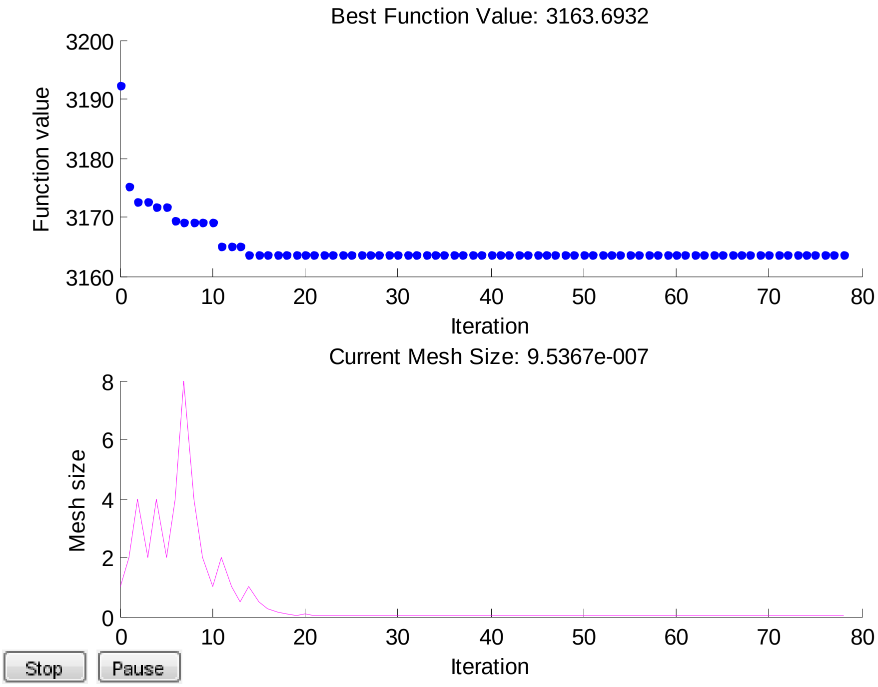


Figure 3: Best function valve and current mesh size for Case I

The convergence characteristic of PS method is shown in Figure 3, where only 19 iterations were needed to find the optimal solution. PS method stops after 60 more iteration and returns the optimal value. It is apparent that the mesh size decreases until the algorithm terminates, in this case at a mesh size of 9.5367e-007 which is more than the stopping criteria, thus indicating that this particular run did not terminate using the mesh size tolerance. Figure 3 shows that for the first 2 iterations the poll was successful since the mesh size keeps increasing as the algorithm had to expand the scope of the search. This is accomplished by multiplying the current mesh size by the expansion factor, in this study taken as 2. This scenario continued until iteration number 2 when the mesh size reached 4. At iteration number 3 the mesh size decreased by half due to multiplying the current mesh size by the contracting factor, indicating an unsuccessful poll in the previous iteration. This process continues until reaching one of the termination criteria.

4.2 CASE II - SIX GENERATING UNIT

This test system assumes 6 generating units with quadratic cost function combined with the transmission losses. The unit data (upper and lower bounds) and cost coefficients for the fuel cost for the 6 generators with transmission loss are given as.

Table 2: Generator loading with Transmission losses for 700 (MW)

Generator	GA	PS
P_{g1}	318.1615	323.6373
P_{g2}	82.3083	76.6857
P_{g3}	157.1374	158.4360
P_{g4}	50.4707	50.0000
P_{g5}	50.2831	51.9764
P_{g6}	52.3972	50.0000
Total power	710.7641	710.7354
Power loss(MW)	10.7641	10.7354
Generation cost($/h)	8356.7884	8352.6109

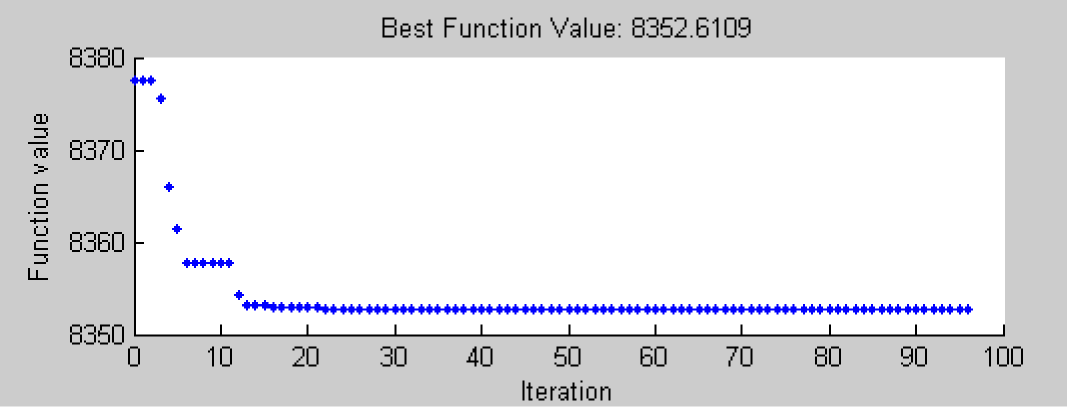


Figure 4: Convergence of PS for Case II

The convergence of the PS algorithm is shown in Figure 5. A total of 97 iterations have been performed. The function value varies till the 22 iterations after that the program run only to improve confidence in result.

The dynamics of the mesh size is depicted by Figure 6. As before, the initial polling is successful leading to mesh size increases, As for Case I, the termination criteria for the mesh size have not been reached. Now the current mesh size reaches to 9.5367 e-007.

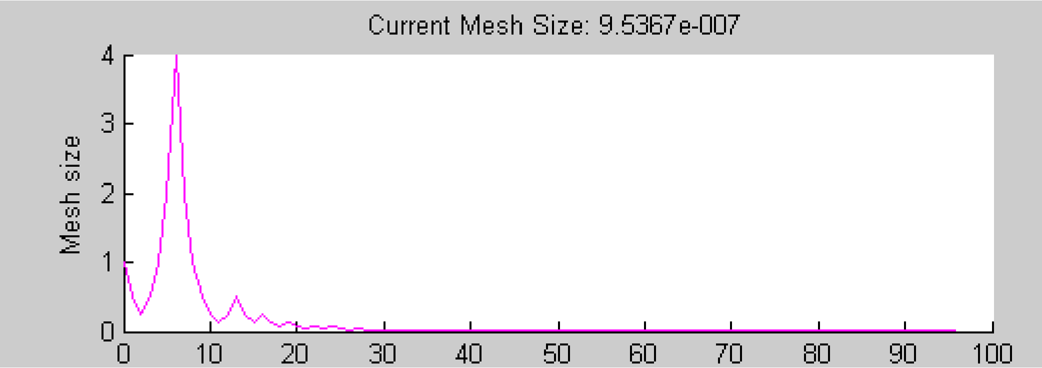


Figure 5: Convergence of PS Mesh Size Case II

4.3 CASE III -TWENTY GENERATING UNIT

This test system assumes 20 generating units with quadratic cost function combined with the transmission losses. The unit data (upper and lower bounds) and cost coefficients are taken from reference [20].

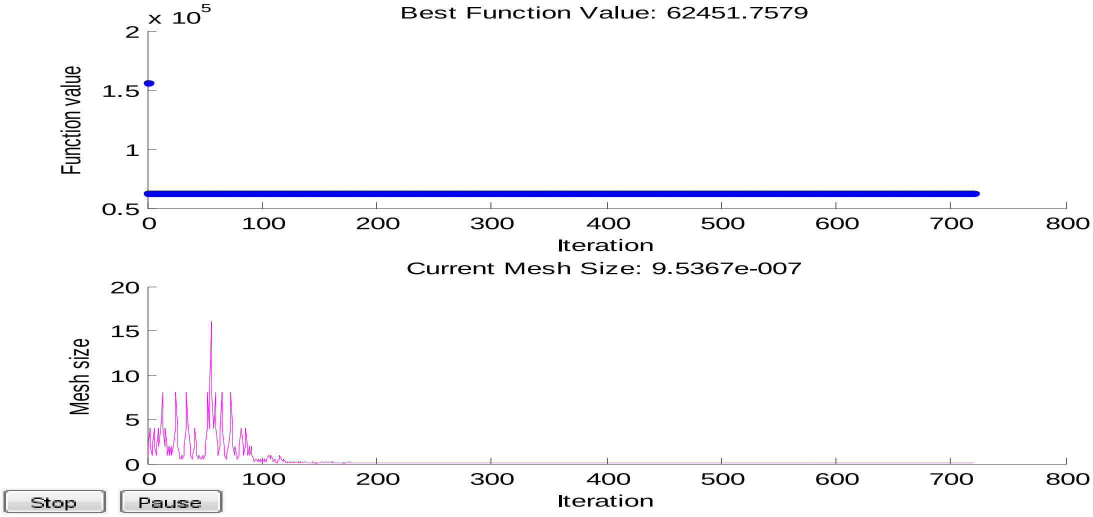


Figure 6: Convergence of PS method for Case III

Table 3: comparison of PS for 20 generating units System

Unit generated	Lambda	Hopfield	GAMS	PS
P_{g1}	512.7805	512.7804	513.111	508.5342
P_{g2}	169.1033	169.1035	167.385	200.0000
P_{g3}	126.8898	126.8897	126.986	126.2190
P_{g4}	102.8657	102.8656	102.841	102.4924
P_{g5}	113.6836	113.6836	113.790	112.3397
P_{g6}	73.5710	73.5709	73.589	69.6603
P_{g7}	115.2878	115.2876	114.775	109.8962
P_{g8}	116.3994	116.3994	116.541	114.2453
P_{g9}	100.4062	100.4063	100.612	99.9297

P_{g10}	106.0267	106.0267	106.639	92.9281
P_{g11}	150.2394	150.2395	150.468	148.5874
P_{g12}	292.7648	292.7647	292.698	297.4286
P_{g13}	119.1154	119.1155	119.190	120.6135
P_{g14}	30.8340	30.8342	31.075	29.4211
P_{g15}	115.8057	115.8056	115.841	115.4462
P_{g16}	36.2545	36.2545	36.263	36.1093
P_{g17}	66.8590	66.8590	66.966	65.3035
P_{g18}	87.9720	87.9720	87.926	89.7383
P_{g19}	100.8033	100.8033	100.913	100.0001
P_{g20}	54.3050	54.3050	54.400	53.2683
Power loss	91.9670	91.9669	92.009	92.272
Fuel cost	62456.6391	62456.6341	62458.093	62451.7579

I. CONCLUSIONS

This paper proposes Pattern Search optimization method for solving the power system economic dispatch problem including transmission losses. The proposed method has been compared with a Genetic Algorithm. The results analysis has demonstrated that PS outperforms the other methods in terms of a better optimal solution. On the other hand, the PS overcomes the problem of local minima that is usualy seen in GA problems this makes more reliable to PS. However, the much improved speed of computation allows for additional searches to be made to increase the confidence in the solution. The Pattern Search method outperforms other reported methods in terms of solution quality, computational efficiency, convergence and robustness.

APPENDIX

Table 4: Fuel cost data for three generator system [21]

Unit	a_i	b_i	c_i	P_{imin}	P_{imax}
1	213.1	11.660	0.00533	050.0	200.0
2	200.0	10.330	0.00889	037.5	150.0
3	240.0	10.833	0.00741	045.0	180.0

Loss Coefficients for three generating unit are

$$B_{11} = 0.01*\begin{bmatrix} 0.06760 & 0.00953 & -0.00507 \\ 0.00953 & 0.05210 & 0.00901 \\ -0.00507 & 0.00901 & 0.0294 \end{bmatrix}$$

B_{10}= [-0.0766 -0.00342 0.01890]
B_{00} = 4.0357

Table 5: Fuel cost data for six generator system

	a	*b*	*c*	*lb*	*ub*
g_1	0.007	7	240	100	500
g_2	0.0095	10	200	50	200
g_3	0.009	8.5	220	80	300
g_4	0.009	11	200	50	150
g_5	0.008	10.5	220	50	200
g_6	0.0075	12	120	50	120

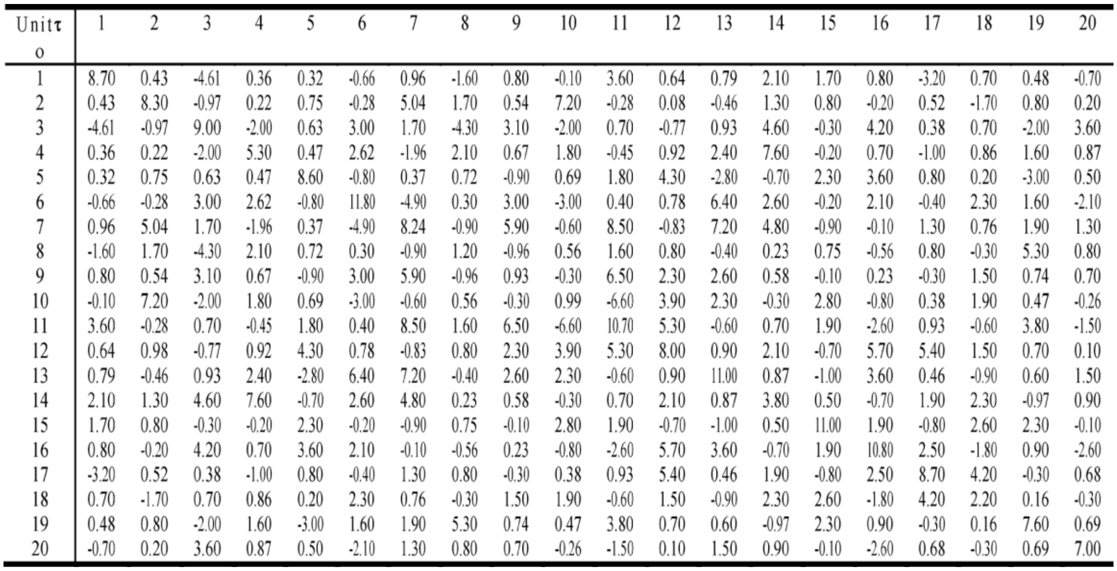
Loss Coefficients for three generating units are as follow:

$$B = \begin{bmatrix} 0.14 & 0.10 & 0.15 & 0.19 & 0.26 & 0.22 \\ 0.17 & 0.60 & 0.13 & 0.16 & 0.15 & 0.20 \\ 0.15 & 0.13 & 0.65 & 0.17 & 0.24 & 0.19 \\ 0.19 & 0.16 & 0.17 & 0.71 & 0.30 & 0.25 \\ 0.26 & 0.15 & 0.24 & 0.30 & 0.69 & 0.32 \\ 0.22 & 0.20 & 0.19 & 0.25 & 0.32 & 0.85 \end{bmatrix}$$

Table 6: Generator parameters for 20-unit ELD problem [20]

Unit	Generation limit(MW)		Generation Cost Parameters		
	Max.	Min.	a_i	b_i	c_i
1.	600	150	1000	18.19	000068
2.	200	50	970	19.26	0.00071
3.	200	50	600	19.80	0.00650
4.	200	50	700	19.10	0.00500
5.	160	50	420	18.10	0.00738
6.	100	20	360	19.26	0.00612
7.	125	25	490	17.14	0.00790
8.	150	50	660	18.92	0.00813
9.	200	50	765	18.27	0.0052
10.	150	30	770	18.92	0.00573
11.	300	100	800	16.69	0.00480
12.	500	150	970	16.76	0.00310
13.	160	40	900	17.36	0.00850
14.	130	20	700	18.70	0.00511
15.	185	25	450	18.70	0.00398
16.	80	20	370	14.26	0.07120
17.	85	30	480	19.14	0.00890
18.	120	30	680	18.92	0.00713
19.	120	30	850	19.79	0.00773
20.	100	30	850	19.79	0.00773

Table 7: loss coefficients for 20 unit ELD problem



Unitτ o	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	8.70	0.43	-4.61	0.36	0.32	-0.66	0.96	-1.60	0.80	-0.10	3.60	0.64	0.79	2.10	1.70	0.80	-3.20	0.70	0.48	-0.70
2	0.43	8.30	-0.97	0.22	0.75	-0.28	5.04	1.70	0.54	7.20	-0.28	0.08	-0.46	1.30	0.80	-0.20	0.52	-1.70	0.80	0.20
3	-4.61	-0.97	9.00	-2.00	0.63	3.00	1.70	-4.30	3.10	-2.00	0.70	-0.77	0.93	4.60	-0.30	4.20	0.38	0.70	-2.00	3.60
4	0.36	0.22	-2.00	5.30	0.47	2.62	-1.96	2.10	0.67	1.80	-0.45	0.92	2.40	7.60	-0.20	0.70	-1.00	0.86	1.60	0.87
5	0.32	0.75	0.63	0.47	8.60	-0.80	0.37	0.72	-0.90	0.69	1.80	4.30	-2.80	-0.70	2.30	3.60	0.80	0.20	-3.00	0.50
6	-0.66	-0.28	3.00	2.62	-0.80	11.80	-4.90	0.30	3.00	-3.00	0.40	0.78	6.40	2.60	-0.20	2.10	-0.40	2.30	1.60	-2.10
7	0.96	5.04	1.70	-1.96	0.37	-4.90	8.24	-0.90	5.90	-0.60	8.50	-0.83	7.20	4.80	-0.90	-0.10	1.30	0.76	1.90	1.30
8	-1.60	1.70	-4.30	2.10	0.72	0.30	-0.90	1.20	-0.96	0.56	1.60	0.80	-0.40	0.23	0.75	-0.56	0.80	-0.30	5.30	0.80
9	0.80	0.54	3.10	0.67	-0.90	3.00	5.90	-0.96	0.93	-0.30	6.50	2.30	2.60	0.58	-0.10	0.23	-0.30	1.50	0.74	0.70
10	-0.10	7.20	-2.00	1.80	0.69	-3.00	-0.60	0.56	-0.30	0.99	-6.60	3.90	2.30	-0.30	2.80	-0.80	0.38	1.90	0.47	-0.26
11	3.60	-0.28	0.70	-0.45	1.80	0.40	8.50	1.60	6.50	-6.60	10.70	5.30	-0.60	0.70	1.90	-2.60	0.93	-0.60	3.80	-1.50
12	0.64	0.98	-0.77	0.92	4.30	0.78	-0.83	0.80	2.30	3.90	5.30	8.00	0.90	2.10	-0.70	5.70	5.40	1.50	0.70	0.10
13	0.79	-0.46	0.93	2.40	-2.80	6.40	7.20	-0.40	2.60	2.30	-0.60	0.90	11.00	0.87	-1.00	3.60	0.46	-0.90	0.60	1.50
14	2.10	1.30	4.60	7.60	-0.70	2.60	4.80	0.23	0.58	-0.30	0.70	2.10	0.87	3.80	0.50	-0.70	1.90	2.30	-0.97	0.90
15	1.70	0.80	-0.30	-0.20	2.30	-0.20	-0.90	0.75	-0.10	2.80	1.90	-0.70	-1.00	0.50	11.00	1.90	-0.80	2.60	2.30	-0.10
16	0.80	-0.20	4.20	0.70	3.60	2.10	-0.10	-0.56	0.23	-0.80	-2.60	5.70	3.60	-0.70	1.90	10.80	2.50	-1.80	0.90	-2.60
17	-3.20	0.52	0.38	-1.00	0.80	-0.40	1.30	0.80	-0.30	0.38	0.93	5.40	0.46	1.90	-0.80	2.50	8.70	4.20	-0.30	0.68
18	0.70	-1.70	0.70	0.86	0.20	2.30	0.76	-0.30	1.50	1.90	-0.60	1.50	-0.90	2.30	2.60	-1.80	4.20	2.20	0.16	-0.30
19	0.48	0.80	-2.00	1.60	-3.00	1.60	1.90	5.30	0.74	0.47	3.80	0.70	0.60	-0.97	2.30	0.90	-0.30	0.16	7.60	0.69
20	-0.70	0.20	3.60	0.87	0.50	-2.10	1.30	0.80	0.70	-0.26	-1.50	0.10	1.50	0.90	-0.10	-2.60	0.68	-0.30	0.69	7.00

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