

# Resource Allocation in OFDM-Based Cognitive Network Using Sub-Optimal Scheme

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**Abstract**—This paper analyses the performance of secondary users (SUs) in a cognitive radio (CR) environment. First, an optimal algorithm is presented that allocates the unused frequency spectrum to unlicensed users (secondary users) dynamically. In doing so, various interference and power constraints are also taken into account. However, regarding the computational complexity of the optimal algorithm, in this paper, a “sub-optimal algorithm” is presented that demands less computing time and is less complex. For the sub-optimal algorithm, singular value decomposition technique is used to represent the channels. Moreover, Water filling algorithm for Rayleigh fading channel is proposed. Simulation results depict that the proposed algorithm can maximize the overall system’s capacity keeping the interference caused in PUs’ bands within a tolerable range.

**Keywords**— Cognitive Radio, MIMO-OFDM, Throughput, Water Filling, Resource Management, Transceiver

## I. INTRODUCTION

The word “Cognitive Radio” refers to a system that has an ability to detect unused (available) channels in its neighborhood and uses those channels for the transmission and/or reception of data. Cognition is a process of dynamic spectrum management. The trend so far is to allocate frequency spectrum bands statically ([1] and [2]) to particular services. Due to this static strategy, some of the frequency bands (for instance, Cellular networks) are overcrowded whereas most of the bands remain unused most of the time (example includes frequency bands used by VIPs and those used for emergency purpose). Moreover, fixed spectrum allocation prevents rarely used frequencies from being used, even when any unlicensed users would not cause noticeable interference to the assigned service. In [3] Liying et al. presented a joint relay selection and power allocation algorithm where the cognitive relay system is prevented from inducing severe interference to the primary system by limiting its maximum transmission power. In [4] the authors proposed an algorithm to select the best transmit way between the network nodes. In [5] the authors have proposed the time sharing condition for spectrum access. What cognitive radio does is it seeks the unused frequency bands in its vicinity and accomplishes transmission/ reception of data using that frequency band. The cognitive radio transceiver can also shift to another frequency band whenever necessary. In the process

of channel selection, data transmission or shifting to another band, the secondary user must ensure that the interference limit caused by it to the primary users is kept well below the pre-specified interference limit. Thus, cognition is a smart way of accessing the spectrum, dynamically.

A lots of research work have been done before on non-cognitive systems and Software Defined Radio (SDR) ([6]–[9]). Quite a few number of works have also been done on Cognitive Radio ([10]–[12]). This letter describes about orthogonal frequency division multiplexing (OFDM) based cognitive system. The resources, like power, subcarriers and relay selection, are optimized jointly in order to maximize the total system capacity. The resources are allocated in accordance with per node power constraint as well as the interference to the primary system constraint. Since the optimal algorithm is highly complex in terms of computation, a sub-optimal algorithm is proposed. The suboptimal algorithm allocates the different resources and in the meanwhile, also considers the channel qualities, interference to the primary system, and individual power budgets.

This letter has been organized as follows: section II gives the system model. In section III, the optimal algorithm is presented while the sub-optimal scheme is presented in the fourth section. Section V shows the simulation results and concludes the paper.

## II SYSTEM MODEL

As shown in figure 1, the primary users (PUs) and the secondary users (SUs) co-exist in the same geographical location. The data sent by the cognitive source is decoded by the nearby relay in the first time slot. After decoding, the relay acquires sufficient information about the data and destines it within the second time slot. Thus, the relays work in half duplex mode, which means that they receive in the first time slot and transmit in the second one. In most of the cases, there is no direct link between the source and the destination so that the source tries to communicate with destination through its favorable relay/s. . Assumption is made that we have perfect knowledge of all the channels. The CR system’s frequency is divided into  $N$  subcarriers, each subcarrier is assumed to have a bandwidth of  $\Delta f$ . Moreover, it is assumed that the CR

system can use the inactive(unused or available) PU bands provided that the total interference introduced to the PU band,  $I_{th}$ , does not exceed the maximum interference power that can be tolerated by PU.

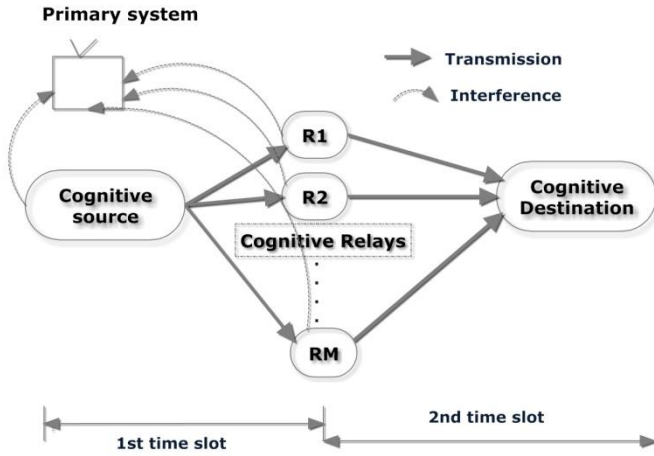


Fig 1: Cooperative relay cognitive radio network

The  $j^{th}$  subcarrier in the source should be paired with only one subcarrier  $k$  in the destination. In OFDM systems, the mutual interference introduced by the  $i^{th}$  subcarrier to PU,  $I_i(d_i, P_i)$ , can be expressed as [6]:

$$I_i(P_i, d_i) = \int_{d_i - B/2}^{d_i + B/2} G_i P_i T_s (\sin \pi f T_s / \pi f T_s)^2 df \quad (1)$$

$d_i$  is the spectral distance between the  $i^{th}$  subcarrier and the PU band.  $G_i$  is the square of the channel gain between the  $i^{th}$  subcarrier and the PU band.  $P_i$  is the total transmit power emitted by the  $i^{th}$  subcarrier and  $T_s$  is the symbol duration. Likewise, the interference power introduced by PU signal with power spectrum density  $y(e^{j\omega})$  into the band of the  $i^{th}$  subcarrier is given as:

$$J_i = \int_{d_i - \Delta f/2}^{d_i + \Delta f/2} Y_i Y(e^{j\omega}) d\omega \quad (2)$$

where,  $Y_i$  is the square of the channel gain between the  $i^{th}$  subcarrier and the PU signal. If  $j^{th}$  subcarrier in the source side coupled with the  $k^{th}$  subcarrier in the destination and assigned to the  $m^{th}$  relay, the data rate can be obtained by using the equation:

$$Rate_{(j,k,m)} = 0.5 \min \{ \log_2(1 + P_{SRm}^j H_{SRm}^j / N_o), \log_2(1 + P_{RmD}^k H_{RmD}^k / N_o) \} \quad (3)$$

where,  $P_{SRm}^j (P_{RmD}^k)$  is the power transmitted over the  $j^{th}(k^{th})$  subcarrier in the source to the relay  $R_m$  ( $R_m$  to destination).  $N_o$  is the noise variance. Let's assume that there are  $N$  subcarriers and  $M$  relays. Moreover,  $H_{SRm}^j (H_{RmD}^k)$  is the square of the  $j^{th}(k^{th})$  subcarrier fading gain from source to  $R_m$  ( $R_m$  to Destination) link.

### III. OPTIMAL SOLUTION

Before the  $j^{th}$  subcarrier is paired with  $k^{th}$  subcarrier in the destination, via  $m^{th}$  relay, the following are the constraints that have to be taken care of [13]:

- (C1: Source power constraint):  $\sum_{m=1}^M \sum_{j=1}^N P_{SRm}^j \leq P_S$
- (C2: Relays individual power constraints):  $\sum_{k=1}^N P_{RmD}^k, \forall m$
- (C3: Interference at the first time slot):  $\sum_{m=1}^M \sum_{j=1}^N P_{SRm}^j \Omega_j \leq I_{th}$
- (C4: Interference at second time slot):  $\sum_{m=1}^M \sum_{k=1}^N P_{RmD}^k \Omega_{k,m} \leq I_{th}$
- (C5: Subcarrier pairing constraint) :  $\sum_{k=1}^N t_{j,k} \leq 1, \forall j$   
and  $\sum_{j=1}^N t_{j,k} \leq 1, \forall k$
- (C6: Relay Assignment constraint):  $\sum_{m=1}^M \pi_{(j,k,m)} = 1, \forall j,k \quad (4)$

$I_{th}$  is the interference threshold prescribed by PU.  $P_S$  and  $P_{Rm}$  are the available power budget in the source and the  $m^{th}$  relay respectively.  $\Omega_j$  and  $\Omega_{k,m}$  are the  $j^{th}(k^{th})$  subcarrier interference factor to the PU band from the source and the  $m^{th}$  relay respectively and is given as:

$$\Omega_j = \int_{d_i - B/2}^{d_i + B/2} G_j T_s (\sin \pi f T_s / \pi f T_s)^2 df \quad (5)$$

The subcarrier pairing constraint ensures that subcarrier in the source is paired with only one subcarrier in the destination i.e.  $t_{j,k} = 1$  if the  $j^{th}$  subcarrier in the source is paired with the  $k^{th}$  in the destination, and zero otherwise. Moreover,  $\pi_{(j,k,m)}$  is the relay assignment indicator which equals to one when the pair  $(j,k)$  is assigned to the  $m^{th}$  relay and zero otherwise. Thus, the optimization problem can be formulated as:

$$\begin{aligned} & \text{Max} \\ & \sum_{m=1}^M \sum_{j=1}^N \sum_{k=1}^N \pi_{(j,k,m)} t_{j,k} Rate_{(j,k,m)} \\ & P_{SRm}^j \geq 0, P_{RmD}^k \geq 0 \\ & \text{s.t the constraints C1-C6.} \end{aligned} \quad (6)$$

From (3) we can easily find that maximum data rate can be obtained when:

$$\begin{aligned} P_{SRm}^j H_{SRm}^j &= P_{RmD}^k H_{RmD}^k \\ \text{And thus we have:} \\ P_{RmD}^k &= P_{SRm}^j H_{SRm}^j / H_{RmD}^k \end{aligned} \quad (7)$$

Using (3) and (7), new optimization problem can be re-written as:

$$\begin{aligned} & \text{Max} \\ & \sum_{m=1}^M \sum_{j=1}^N \sum_{k=1}^N 0.5 \pi_{(j,k,m)} t_{j,k} P_{SRm}^j \log_2(1 + P_{SRm}^j H_{SRm}^j / N_o) \end{aligned} \quad (8)$$

The constraints C2 and C4 are updated accordingly as:

$$\sum_{j=1}^N \sum_{k=1}^N \pi_{(j,k,m)} t_{j,k} \frac{p_{SRm}^j H_{SRm}^j}{H_{RmD}^k} \leq P_{Rm}; \forall m$$

$$\text{And } \sum_{m=1}^M \sum_{j=1}^N \sum_{k=1}^N \pi_{(j,k,m)} t_{j,k} \frac{p_{SRm}^j H_{SRm}^j}{H_{RmD}^k} \Omega_{k,m} \leq I_{th}$$

The problem associated with (8) can be written as:

$$\min_{\beta \geq 0; \gamma_m \geq 0; \lambda \geq 0; \mu \geq 0} g(\beta, \gamma_m, \lambda, \mu) \quad (9)$$

where,  $\beta$  and  $\gamma_m$  are dual variables associated to C1 and the C2 respectively whereas,  $\lambda$  and  $\mu$  are associated to C3 and C4 respectively. The function  $g(\beta, \gamma_m, \lambda, \mu)$  is defined as:

$$g(\beta, \gamma_m, \lambda, \mu) = \max_{P_{SRm}^j > 0, t_{j,k}, \pi_{(j,k,m)}} \zeta$$

s.t (C5), (C6)

Where,  $\zeta$  is Lagrangian operator [13] and is given as:

$$\begin{aligned} \zeta = & \sum_{m=1}^M \sum_{j=1}^N \sum_{k=1}^N 0.5 \pi_{(j,k,m)} t_{j,k} \log\left(1 + \frac{p_{SRm}^j H_{SRm}^j}{N_0}\right) + \\ & \beta (P_S - \sum_{m=1}^M \sum_{j=1}^N p_{SRm}^j) + \lambda (I_{th} - \sum_{m=1}^M \sum_{j=1}^N p_{SRm}^j \Omega_j) + \\ & \sum_{m=1}^M \gamma_m (P_{Rm} - \sum_{j=1}^N \sum_{k=1}^N \pi_{(j,k,m)} t_{j,k} \frac{p_{SRm}^j H_{SRm}^j}{H_{RmD}^k}) + \\ & \mu (I_{th} - \sum_{m=1}^M \sum_{j=1}^N \sum_{k=1}^N \pi_{(j,k,m)} t_{j,k} \frac{p_{SRm}^j H_{SRm}^j}{H_{RmD}^k} \Omega_{k,m}) \end{aligned} \quad (10)$$

Moreover, “g” can be rewritten as:

$$g(\beta, \gamma_m, \lambda, \mu) = \max [ \sum_{m=1}^M \sum_{j=1}^N \sum_{k=1}^N \pi_{(j,k,m)} t_{j,k} D(P_{SRm}^j, k) + \beta P_S + \sum_{m=1}^M \gamma_m P_{Rm} + I_{th}(\lambda + \mu) ] \quad (11)$$

$$\text{s.t } \pi_{(j,k,m)}, t_{j,k}, P_{SRm}^j \geq 0$$

C5 and C6 are retained. And,

$$\begin{aligned} D(P_{SRm}^j, k) = & 0.5 \log\left(1 + \frac{p_{SRm}^j H_{SRm}^j}{N_0}\right) - \beta p_{SRm}^j - \gamma_m \frac{p_{SRm}^j H_{SRm}^j}{H_{RmD}^k} \\ & - \lambda p_{SRm}^j \Omega_j - \mu \frac{p_{SRm}^j H_{SRm}^j}{H_{RmD}^k} \Omega_{k,m} \end{aligned} \quad (12)$$

To obtain the optimum solution, we can start by assuming any initial values for the variables  $\beta$ ,  $\gamma_m$ ,  $\lambda$ , and  $\mu$ . We also assume that  $(j,k)$  is a valid pair of subcarrier which is assigned to the  $m^{\text{th}}$  relay. The optimum power allocation can be found by solving the following for every  $(j,k,m)$  pair

$$\begin{aligned} \text{Max } & D(P_{SRm}^j, k) \\ \text{s.t } & P_{SRm}^j \geq 0 \end{aligned} \quad (13)$$

On solving (13) for optimal power we have:

$$P_{SRm}^{*j} = \left[ \frac{1}{\beta + \frac{\gamma_m H_{SRm}^j}{H_{RmD}^k} + \lambda \Omega_j + \mu \frac{H_{SRm}^j \Omega_{k,m}}{H_{RmD}^k}} - \frac{N_0}{H_{SRm}^j} \right]^+ \quad (14)$$

where,  $[x]^+ = \max(0, x)$ .

Thus, (11) becomes:

$$\begin{aligned} \text{max} [ & \sum_{m=1}^M \sum_{j=1}^N \sum_{k=1}^N \pi_{(j,k,m)} t_{j,k} D(P_{SRm}^{*j}, k) + \beta P_S \\ & + \sum_{m=1}^M \gamma_m P_{Rm} + I_{th}(\lambda + \mu) ] \\ \text{s.t } & \text{(C6)} \end{aligned}$$

Hence, the optimal relay assignment strategy is achieved by allocating the  $(j, k)$  pair to the relay which maximizes the function  $D(P_{SRm}^{*j}, k)$  in which case  $\pi_{(j,k,m)}^* = 1$  and zero otherwise. After finding the optimal solution, i.e.  $P_{SRm}^{*j}$ ,  $t_{j,k}^*$  and  $\pi_{(j,k,m)}^*$  of the function “g” at given points  $\beta$ ,  $\gamma_m$ ,  $\lambda$  and  $\mu$ , the variables at  $(i+1)^{\text{th}}$  iteration are updated as:

$$\begin{aligned} \beta(i+1) &= \beta(i) - \delta(i) (P_S - \sum_{m=1}^M \sum_{j=1}^N P_{SRm}^{*j}) \\ \gamma_m(i+1) &= \gamma_m(i) - \delta(i) (P_{Rm} - \sum_{j=1}^N \sum_{k=1}^N \pi_{(j,k,m)}^* t_{j,k}^* \frac{P_{SRm}^{*j} H_{SRm}^j}{H_{RmD}^k}); \\ & \forall m \\ \lambda(i+1) &= \lambda(i) - \delta(i) (I_{th} - \sum_{m=1}^M \sum_{j=1}^N P_{SRm}^{*j} \Omega_j) \\ \mu(i+1) &= \mu(i) - \\ & (\delta(i) (I_{th} - \sum_{m=1}^M \sum_{k=1}^N \sum_{j=1}^N \pi_{(j,k,m)}^* t_{j,k}^* \frac{P_{SRm}^{*j} H_{SRm}^j}{H_{RmD}^k} \Omega_{k,m})) \end{aligned} \quad (15)$$

$\delta(i)$  is the step size and can be updated according to non-summable diminishing step size policy [14]. The updated values of the variables  $\beta$ ,  $\gamma_m$ ,  $\lambda$  and  $\mu$  are again used to for optimal power allocation and subcarrier matching. This method is used iteratively until convergence.

#### IV. SUB-OPTIMAL ALGORITHM

The key innovation of this paper is sub-optimal algorithm using which the different system resources are allocated jointly with lower computational complexity than that of the optimal solution. The sub-optimal algorithm takes into account the interference introduced to the primary system, the different available channels and also the available power budgets with less effort than that required in the case of optimal algorithm. In fact, there are many sub-optimal schemes proposed before, ([15],[17]) however, their performances were less as compared to optimal algorithm. Therefore, in this paper we present a new sub-optimal scheme whose performance is as good as that of the optimal algorithm.

We commence the sub-optimal scheme by taking up a MIMO system transceiver, assuming that the CR transceiver has ‘T’ transmitting and ‘N’ receiving antennas respectively. The channel of secondary user (SU) for the  $i^{\text{th}}$  sub-carrier can be denoted by a matrix  $C_i$  having dimensions  $N \times M$ . The element  $C_{i,n,m}$  denotes the channel gain between the  $m^{\text{th}}$

transmit and the  $n^{\text{th}}$  receive antenna. Moreover, channel gain between the SU transmitter and the  $K^{\text{th}}$  PU receiver for the  $i^{\text{th}}$  sub-carrier is denoted by a  $K \times M$  matrix. Again, the channel gain for the channel between the SU's  $m^{\text{th}}$  transmit antenna and the  $k^{\text{th}}$  PU receiver antenna is denoted by  $g_{i,m}$ . Here, we use singular value decomposition (SVD) technique to decompose the sub-carrier channel into parallel independent sub-channels s.t:

$$C_i = S_i V_i D_i^H \tag{16}$$

where,  $S_i \in A^{R \times R}$  and  $D_i \in A^{R \times R}$  are unitary matrices and  $H$  is called Hermitian operator.

Here,

$$D_i^H = (D_i^T)^* = (D_i^*)^T$$

where,  $*$  denotes the complex conjugate of each element and  $T$  is the transpose operation.

Moreover,  $V_i \in A^{R \times T}$  is a rectangular matrix whose diagonal elements are non-negative real numbers. Also, the diagonal elements of  $A$  are singular values of the matrix  $C_i$ . The SVD technique decouples the channel matrix in spatial domain in a way similar to the DFT decoupling the channel in the frequency domain. The sub channels are characterized by the channel gains (denoted by the singular values of the MIMO channel matrix on each sub-carrier). We multiplex independent data onto these independent channels. The capacity of  $i^{\text{th}}$  subcarrier can be obtained by using (3). For the sub-optimal scheme, we present water-filling algorithm for allocating unused channels to each subcarrier.

(i) WATER-FILLING ALGORITHM FOR SINGLE USER

Before allocating channel to a subcarrier, channel gain to noise ratio(CNR) is calculated for each channel; and is given as:

$$CNR_n = |C_n|^2 / No_n, n=1, \dots, N \tag{17}$$

According to waterfilling algorithm, power on each sub-channel is given by:

$$E_n = [L_o - CNR^{-1}_n]^+ \text{ where, } [x]^+ = \max(0, x)$$

the “water level”  $L_o$  has to be chosen such that

$$E_{tot} = \sum_{n=1}^N E_n \tag{18}$$

Power is allocated to that subcarrier whose CNR is sufficiently high and which also satisfies (18)

(ii) MULTI-USER WATER-FILLING ALGORITHM

In the multiple channels’ case, the received symbol on sub channel  $n$  is given by:

$$Y_n[k] = \sum_{u=1}^U C_{u,n} x_u[k] + rn[k] \tag{19}$$

where,

$rn[k]$  is random noise signal and  $U$  denotes the number of users and the CNRs in this case are defined by

$$CNR_{u,n} = |C_{u,n}|^2 / No_n \tag{20}$$

We can generalize the water-filling algorithm by assuming an equivalent channel  $C^*_{u,n} = C_{u,n} / \sqrt{x_u}$ . Thus, we can obtain the equivalent transmit power

$$E^*_{u,n} = x_u \cdot E_{u,n}.$$

Hence, this approach makes it possible to combine the water filling diagrams of different users. For each user, the multiplier  $x_u$  is chosen such that the water level is unity. The power constraint of user  $u$  is denoted by  $E_{max}(u)$ :

$$E_{tot}(u) = \sum_{n=1}^N E_{u,n} \leq E_{max}(u) \tag{21}$$

This system is solved for  $x_u$  so that the energy allocation and the sub channel allocation can be derived easily.

V SIMULATION RESULTS AND CONCLUSION

The simulations are performed under the scenario given in Fig.1. An OFDM system of  $N = 64$  subcarriers is assumed with  $M = 5$  relays. The values of  $T_s$ ,  $\Delta f$ , and  $No$  are assumed to be  $4\mu$  seconds,  $0.3125$  MHz and  $10^{-6}$  respectively. The channel gains are outcomes of independent Rayleigh distributed random variables with mean equal to 1. All the results have been averaged over 1000 iterations. In the simulations, we present the result obtained by using both *Optimal* and *Sub-optimal* algorithms.

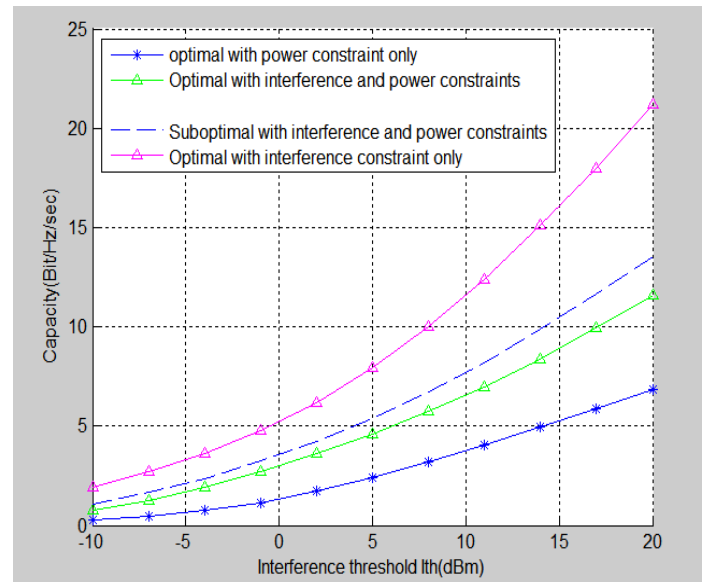


Fig 2: simulation results using optimal and sub-optimal algorithm

Graph of data rate or throughput (capacity) is plotted against given interference threshold. The curves with solid line represent simulation results for optimal algorithm, either with interference constraint only or power constraint only or both interference and power constraints. The one with dashed line represent simulation result obtained by using the suboptimal algorithm considering both interference and power constraints. As the simulation results suggest, the sub-optimal algorithm can produce almost the same capacity (rate) as that produced by using the optimal algorithm. The advantage in sub-optimal scheme is that it requires less computational complexity which saves both time and energy.

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