Analysis of fuzzy working vacations queueing models using DSW algorithm

K.Julia Rose Mary[#1], Sheela Jancy.S[#2]

#1 Department of mathematics ,Nirmala College for Women(Autonomous),Coimbatore-18,Tamil Nadu,India.
#2 Department of mathematics ,Nirmala College for Women(Autonomous),Coimbatore-18,Tamil Nadu,India.
roseyvictor@hotmail.com,sheela.moon18@gmail.com

Abstract: **We consider the queuing systems FM/FM/1/SWV and FM/FM/1/MWV with finite capacity. An approximate method namely DSW (Dong, Shah and Wong) algorithm is applied for our models, which is based on α-cut representation for fuzzy sets in a standard interval analysis. For FM/FM/1/SWV and FM/FM/1/MWV queuing models DSW algorithm is used to define the membership functions of its performance measures. Further the performance measures are analyzed for fuzzy working vacation queuing models, which provides more information. Moreover numerical example is also illustrated to check the validity of the proposed models.**

Key words: **Working vacation, α-cut, membership function and DSW algorithm.**

I. INTRODUCTION

During classical vacation the server may stop the primary services and perform other supplementary jobs or may not be fully available for a period of time. Class of semi vacation policy was introduced by Servi and Finn in 2002, in which the server works at lower service rate instead of completely stopping service. Servi and Finn first studied the M/M/1 queue with working vacation in 2002.By using the quasi-birth and death model Servi and Finn obtained the stationary queue length and waiting time. Later Liu et al (2007), Zahang and Xu (2008) and Tian et al (2008), Jamila parveen et al (2009) have analyzed the M/M/1 working vacation .But Wu and Tackagi(2006) extended Servi and Finn 's M/M/1 queue with working vacation model to M/G/1 queue with working vacations. Due to the applications in computer systems and analysis of communication, the discrete-time queues with various working vacation policies are considered by Tian et al(2008a), Li and Tian (2007b), Li et al(2007) ,Li and Tian(2008) and yi et al(2007).

In ordinary queuing theory, the inter-arrival time, service times are characterized as random variables, it is impossible to obtain the data's needed in queuing theory in many situations. In fact, in modeling queuing systems, we would describe the Inter-arrival times and services times by the terms very fast, fast, slow. Thus, fuzzy queuing systems are more useful and realistic than the ordinary queuing theory. In fuzzy analysis Zadeh (1965) provide the concept of fuzzy set from membership function. By using the Zadeh extension principle,Li and Lee (1989) investigated the analytical results for two typical fuzzy queues namely FM/FM/1/∞ and M/F/1/∞.

W.Ritha and Lilly Robert (2010) have analyzed the DSW Algorithm in fuzzy queues with priority discipline. Julia Rose Mary and Angel jenita (2014) have discussed the optimal operating policy of M^X /M/1 multiple working vacations in fuzzy environment and later R.Srinivasan (2014) has also analyzed the fuzzy queuing models using DSW algorithm. With the aid of the available literature, we are evaluating the performance measures for fuzzy working vacation queuing models.

II. PROBLEM FORMULATION

Consider a M/M/1 queuing system with working vacation in which there is one server. The inter arrival time A, service time (regular) S, working in vacation S_r and vacation parameter η are approximately known.

Let $\mu_A(a), \mu_s(s), \mu_{S_r}(u)$ and $\mu_\eta(v)$ are denoted as the membership functions of A,S,S_r and η respectively. We get the following fuzzy sets as:

$A = \{(a,\mu_A(a))/a \in X\}$; $S = \{(s,\mu_s(s))/S \in Y\}$;
$S_r = \{(u,\mu_{S_r}(u)\ /u \in U\}$;$\eta = \{(v,\mu_\eta(v))/v \in V\}$

where X, Y, U and V are crisp universal sets of A, S,$S_r$and η . The α-cuts or α-levels sets of A, S,S_r and η are

$A(\alpha) = \{a \in X/\mu_A(a) \geq \alpha\}$; $S(\alpha) = \{s \in Y/\mu_s(s) \geq \alpha\}$;

$S_r(\alpha) = \{u \in U/\mu_{S_r}(u) \geq \alpha\}$; $\eta(\alpha) = \{v \in V/\mu_\eta(v) \geq \alpha\}$
where $A(\alpha)$, $S(\alpha)$,$S_r(\alpha)$ and $\eta(\alpha)$ are the crisp sets using α-cuts.

Now, the inter arrival time A, regular service time S, working vacation service time S_r and vacation parameter η can be represented by different levels of confidence intervals. Then, the membership function p (A, S,S_r,η) is constructed as

$$\mu_{p(A,S,S_r,\eta)} = \begin{cases} L(z) & z_1 \leq z \leq z_2 \\ & \\ & \end{cases}$$

1 $z_2 \le z \le z_3$

R (z) $z_3 \le z \le z_4$

where $z_1 \le z_2 \le z_3 \le z_4$ and $L(z_1) = R(z_4)$=0.

Moreover, we note that an attractive feature of the α-cut is that, all α-cuts form a nested structure with respect to α.These nested structures are analyzed by Chen (2005). Further an approximate method of extension is propagating, fuzziness of continuous valued mapping which determines the membership functions for the output variables.

III. FOR SINGLE WORKING VACATION(SWV) AND MULTIPLE WORKING VACATION (MWV) QUEUING MODELS:

Jamila Parveen, Afthab Begum and Julia Rose Mary(2009) have analized M/M/1 queuing SWV and MWV. According to them,

A. *For single working vacation model :*

The expected number of customers in the system is given by

$$L_s = \frac{\rho}{1-\rho} + \left(1 - \frac{\mu_v}{\mu}\right)\left(\frac{r_v}{1-r_v}\right)\{\left(1 - \frac{r_v\mu_v}{\mu}\right) + \frac{\eta}{\lambda}(1-r_v)\}^{-1}$$

The expected number of customers in the queue as:

$$L_q = \frac{\rho}{1-\rho} + \left(1 - \frac{\mu_v}{\mu}\right)\left(\frac{r_v}{1-r_v}\right)\{\left(1 - \frac{r_v\mu_v}{\mu}\right) + \frac{\eta}{\lambda}(1-r_v)\}^{-1} - \rho$$

B. *For multiple working vacation :*

The expected number of customers in the system

$$L_s = \frac{\rho}{1-\rho} + \left(1 - \frac{\mu_v}{\mu}\right)\left(\frac{r_v}{1-r_v}\right)\left(1 - \frac{r_v\mu_v}{\mu}\right)^{-1}$$

The expected number of customers in the queue

$$L_q = \frac{\rho}{1-\rho} + \left(1 - \frac{\mu_v}{\mu}\right)\left(\frac{r_v}{1-r_v}\right)\left(1 - \frac{r_v\mu_v}{\mu}\right)^{-1} - \rho$$

where r_v is the characteristic root and $r_v \in$[0,1].

IV. DSW ALGORITHM:

DSW (Dong, Shah and Wang) Algorithm is one of the approximate method makes use of intervals at various α-cut levels in defining membership functions. The DSW algorithm greatly simplifies manipulation of the extension principle for continuous valued fuzzy variables, such as fuzzy numbers defined on the real line. Any continuous membership function can be represented by a continuous sweep of α-cut in term from $\alpha = 0$ *to* α =1.Suppose we have single input mapping given by y=f(x) that is to be extended for fuzzy sets B=f(A) and we want to decompose A into the series of α-cut intervals say I_α . It utilizes the entire α-cut intervals in a standard intervals analysis.

Now, the DSW algorithm consists of the following steps:

1. Select a α-cut value where $\alpha \in [0,1]$
2. Find the intervals in the input membership function that correspond to this α
3. Using standard binary intervals operations, compute the interval for the output membership function for the selected α-cut level.
4. Repeat steps 1-3 for different values of α to complete a α-cut representation of the solution.

Srinivasan (2014) has analyzed fuzzy queuing model with the aid of DSW Algorithm. Based on that we have evaluated the Performance Measures for FM/FM/1/SWV and FM/FM/1/MWV.

V. NUMERICAL EXAMPLE:

Consider a FM/FM/1/WV queue, where λ, μ ,μ_v ,η are fuzzy numbers and r_v ∈[0,1].

λ =[6,7,8,9] μ_v=[10,11,12,13]
μ=[15,16,17,18] η=[1,2,3,4] r_v=0.5

Now the interval of confidence at possibility level α for [6+α,9-α],for μ [$15+\alpha, 18-\alpha$],
for μ_v [$10+\ \alpha, 13-\alpha$] and for η [1+$\alpha, 4-\alpha$].

Hence for SWV model:

$$L_s = \frac{\rho}{1-\rho} + \left(1 - \frac{x_v}{x}\right)\left(\frac{r_v}{1-r_v}\right)\{\left(1 - \frac{r_v x_v}{x}\right) + \frac{z}{y}(1-r_v)\}^{-1}$$

and

$$L_q = \frac{\rho}{1-\rho} + \left(1 - \frac{x_v}{x}\right)\left(\frac{r_v}{1-r_v}\right)\{\left(1 - \frac{r_v x_v}{x}\right) + \frac{z}{y}(1-r_v)\}^{-1} - \rho$$

For MWV:

$$L_s = \frac{\rho}{1-\rho} + \left(1 - \frac{x_v}{x}\right)\left(\frac{r_v}{1-r_v}\right)\left(1 - \frac{r_v x_v}{x}\right)^{-1}$$

and

$$L_q = \frac{\rho}{1-\rho} + \left(1 - \frac{x_v}{x}\right)\left(\frac{r_v}{1-r_v}\right)\left(1 - \frac{r_v x_v}{x}\right)^{-1} - \rho$$

where $y =$ [6+ α, 9- α], $x =$[$15+\alpha, 18-\alpha$],
x_v= [$10+\ \alpha, 13-\alpha$] , $z =$ [1+ $\alpha, 4-\alpha$] and ρ=$\frac{y}{x}$

By substituting the above values, the effect of parameters on the Ls and Lq for SWV and MWV are tabulated and their graphical representations are also shown below.

	SWV		MWV	
	Ls	Lq	L s	Lq
$\alpha = 0$	[0.6262,2.2152]	[0.0262,1.8819]	[0.6845,2.2852]	[0.0845,1.9519]
$\alpha = 0.1$	[0.6574,2.122]	[0.068,1.7813]	[0.72,2.1959]	[0.1306,1.8552]
$\alpha = 0.2$	[0.6893,2.0345]	[0.1104,1.6862]	[0.7558,2.1121]	[0.1769,1.7638]
$\alpha = 0.3$	[0.722,1.952]	[0.1534,1.5961]	[0.7922,2.033]	[0.2236,1.6771]
$\alpha = 0.4$	[0.7559,1.8729]	[0.1975,1.5093]	[0.8296,1.9566]	[0.2712,1.593]
$\alpha = 0.5$	[0.7905,1.7977]	[0.2422,1.4263]	[0.8673,1.884]	[0.319,1.5126]
$\alpha = 0.6$	[0.8263,1.7261]	[0.2879,1.3468]	[0.906,1.8146]	[0.3676,1.4353]
$\alpha = 0.7$	[0.8629,1.6582]	[0.3343,1.271]	[0.9453,1.7486]	[0.4167,1.3614]
$\alpha = 0.8$	[0.901,1.5927]	[0.3821,1.1974]	[0.9859,1.6846]	[0.467,1.2893]
$\alpha = 0.9$	[0.9402,1.5304]	[0.4308,1.1269]	[1.0272,1.6236]	[0.5178,1.2201]
$\alpha = 1$	[0.9804,1.4706]	[0.4804,1.0589]	[1.0693,1.5648]	[0.5693,1.1531]

TABLE I: THE α- CUTS OF LS, LQ FOR SWV AND MWV

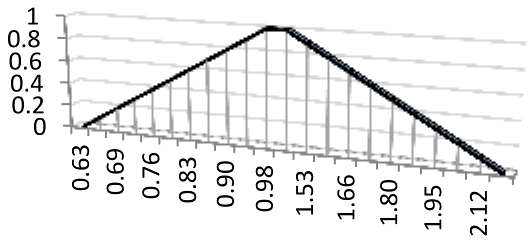

Fig 1: The membership function of the expected system length Ls for swv

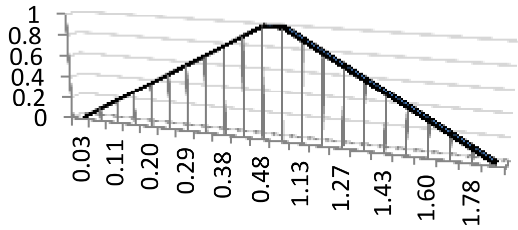

Fig 3: The membership function of the expected length Lq for swv

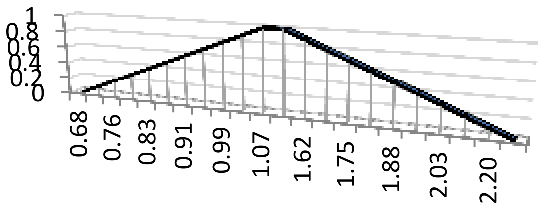

Fig 2:The membership function of the expected system length Ls for mwv

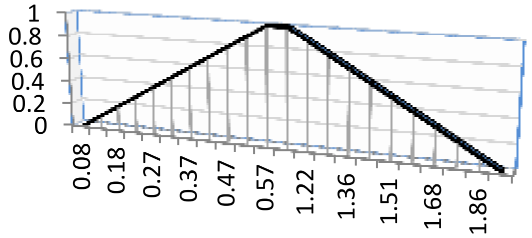

Fig 4: The membership function of the expected length Lq for mwv

Here we perform α-cuts of fuzzy performance measure for expected system length and expected queue length for SWV and MWV at eleven distinct α-levels 0,0.1,0.2,…,1.0.Crisp intervals for fuzzy performance measure for SWV and MWV at different possibilities α-level are presented in the table. From the table we observe that

A. For FM/FM/1/SWV model

Ls lie between 0.9804 and 1.4706 and it's value does not fall outside range 0.6262 and 2.2152.Also Lq lie between 0.4804 and 1.0589 and it's value does not fall outside range 0.0262 and 1.8819.

B. For FM/FM/1/MWV model

Ls lie between 1.0693 and 1.5648 and it's value does not fall outside range 0.6845 and 2.2852.Also Lq lie between 0.5693 and 1.1531 and it's value does not fall outside range 0.0845 and 1.9519.

Further Figures(1-4) depict a rough shape of Ls and Lq for SWV and MWV.The rough shape turns out rather fine and looks like continuous function. We also note that the α-cut represent the possibility of the performance measures will lie in the associated range of interval.The above technique is very useful for designing the queuing systems FM/FM/1/SWV and FM/FM/1/MWV.

Conclusion:

The queuing model has more applicability in the real environments than the crisp systems. In this paper fuzzy queuing systems with single working vacations and multiple working vacations are analyzed by using DSW algorithm which helps to find the membership function for the performance measures. It is interesting to note that the input values are fuzzy numbers then the performance measures such as the average system length and average queue length will also be fuzzy. Numerical example also shows the efficiency of the algorithm.

References:

Chen SP,Parametric nonlinear programming approach to fuzzy queues with bulk service, European Journal of Operational Research ,163:434-444,2005.

Jamila parveen.M,Aftab Begum .M.I and Julia Rose mary.K,"M/M/1 queue with working vacation" Proceeding of international conference on mathematical and computational models;Recent trends P.S.G college of technology,71-79,2009.

K.Julia Rose Mary and M.I Afthab Begum,Batch arrival queuing systems with Bi-level control policy,vacations,Breakdowns and heterogeneous service facilities,2011.

K.Julia Rose Mary and P.Angel jenitta, Optimal operating policy of FM^X/FM/1 multiple working vacation queuing system.International journal of Advanced and Innovative research (2278-2284)vol 3 Issue-1,2014.

Li,R.J., Lee,E.S,Analysis of fuzzy queues.Computer and Mathematical Application.17:1143-1147,1989.

Li.J. and Tian,n.,"The discrete time G1/Geo/1 queue with working vacations and vacation interruptions",Appl.Math. and Comput.,85,1-10,2007b.

Li,J. and Tian,,"Analysis of the discrete time Geo/Geo/1 queue with single working vacations",Quality Technology &Quantitative Management,5,77-89,2008.

Li,J.,Tian,N. and Liu,W,"Discrete time G1/Geo/1 queue with multiple working vacations",Queuing systems,56,53-63,2007.

Liu.W,Xu.X and Tian N,Some results on the M/M/1 queue with single working vacation,Oper. Res.Letters,vol.35,No;5,pp.595-600,2007.

W.Ritha and Lilly Robert, Fuzzy queues with priority discipline, applied mathematical sciences, Vol.4, no.12, 575-582, 2010.

L.D Servi and S.G Finn M/M/1 queue with working vacation,performance evaluation archive,vol 50,issue 1-4,page 41-52,2002.

R.Srinivasan, Fuzzy queuing model using DSW algorithm, International journal of advance researchin mathematics and applications volume: 1 issue: 1 08-jan-2014.

Tian,N.,Ma,Z.and Liu,M,"The discrete time Geo/Geo/1 queue with multiple working vacations",Appl.Math.Modeling,32,2941-2953, 2008a.

Tian.N,Wang.K,Zhao.X,M/M/1 queue with single working vacation,international journal of information and management science,vol.19,pp.621-634,2008.

Wu.D,and Takagi.H"An M/G/1 queue with multiple working vacations",Performance evaluation, vol.63,pp.654-681.2006

Yi,X.,Kim,J.,Choi,D.and chae,k.,"The Geo/G/1 queue with disaster and multiple working vacations",Stoch.Models,23,537-549 ,2007.

Zadeh,L.A:Fuzzy sets.Information and Control.8:338-353,1965.

Zang.Z and Xu,Analysis for the M/M/1 queue with multiple working vacations and N policy,Information and management science,vol.19,pp.495-506,2008.

