OPTIMAL OPERATING POLICY of FM^X/FM/1/MWV QUEUING SYSTEM

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Abstract

In this paper we investigate the cost and profit analysis of an M^X/M/1 multiple working vacation (MWV) queuing models in a fuzzy environment. A mathematical parametric Non Linear Programming (NLP) method is used to construct the membership function of the system characteristics in which arrival rate, service rate for busy and vacation period, vacation parameter, group size, service cost for busy and vacation period and holding cost are fuzzy numbers. The α cut and Zadeh's extension principle are used to transform a fuzzy queue into a family of conventional crisp queues. By means of membership function of the system characteristics a set of parametric non linear program is developed to calculate the lower and upper bound of the system characteristics function at α. Thus the membership functions of the system characteristics are constructed. Numerical example is also illustrated to check the validity of the proposed system.

Key words

Batch arrival, Working vacation policy, α cut, Membership function and Zadeh's extension principle.

I.INTRODUCTION

Servi and Finn[13] introduced a class of semi vacation policies, in which servers work at a lower rate rather than completely stopping primary service during vacation, such a vacation is called working vacation (WV).The M/M/1 queuing system with working vacation has been analyzed by Servi and Finn [13], Liu et al [9], Zahang and Xu[19] Tian et al [15] and Jamila Parveen et al [2].Wu and Tackagi [16] later extended Servi and Finn's M/M/1/WV to M/G/1/MWV model. They assumed that both service times during a regular service period and during working vacations are generally distributed. Moreover, Baba [1] analyzed GI/M/1 queue with multiple working vacations.

Among the various paradigmatic changes in Science and Mathematics, one such change concerns the concept of uncertainty. Based on uncertainty Zadeh [17] introduced a theory whose objects-fuzzy set is not a matter of affirmation or denial, but rather a matter of a degree. The available literature shows that the simplification of fuzzy queuing system opens the road for creating realistic queuing models, which are potentially more useful than the commonly used crisp queues. .

Li and Lee [8] investigated the analytical results for two typical fuzzy queues FM/FM/1/∞, M/F/1/∞ where F represents fuzzy time and FM represents fuzzified exponential distribution. Nagi and Lee [10] proposed a procedure using the α cut and two variables simulation to analyze fuzzy queues. Unfortunately their approach provides only crisp solutions. Using parametric programming Kao et al [7] constructed the membership functions of the system characteristics for fuzzy queues and applied them to four simple fuzzy queues namely F/M/1/∞, M/F/1/∞ and FM/FM/1/∞ successfully. Ritha and Lilly Robert[12] developed the profit analysis of FM/FE_K/1 queues system in which they convert the fuzzy queue into a family of conventional crisp queues and construct the membership function successfully by using the approaches of parametric NLP techniques. Later the same techniques were utilized by Jeeva and Rathanakumari[3] to analyze a batch arrival single server, Bernoulli feedback queue with fuzzy vacations and fuzzy parameters. Ramesh and Kumara [11] analyzed batch arrival queue with fuzzy parameters. Recently Jeeva and Rathanakumari [4] had also introduced the fuzzy cost computations of M/M/1 and M/G/1 queuing models also. With the help of these available literatures we determine the optimum operating policy of FM^X/FM/1/MWV queuing system.

II.COST ANALYSIS of M^X/M/1/MWV QUEUE

Consider the model M^X/M/1/MWV in which arrival stream forms a Poisson process and the actual number of customers in any arriving module are a random variable X. Let X(Z), E(X) and E(X^2) denote the probability generating function(PGF), first and second moment of random variable X respectively. The server provides service to the customers with exponential service rate μ in a regular busy period. Whenever the system becomes empty at service completion instant, the server starts a working vacation during which the service is done at lower rate. The vacation duration V follows an exponential distribution with parameter η. During working vacations, arriving customers are served with exponential service rate μ_v . When the vacation terminates and the server finds the system is empty, then he begins another working vacation (MWV). On the other hand, if the server finds the system is not empty at the vacation termination instant, then he switches to a regular period. Service rates for busy and vacation are both exponential but with different rates μ and μ_v. When the server changes his service mode from μ_v to μ the size of the service batch being served remains unchanged.

The above concept of M^X/M/1/MWV was analyzed by Julia Rose Mary and Afthab Begum [5], and they derived queue length (L_N) also. But for any queuing system cost and profit analysis constitute a very important aspect of its investigation. Hence for this model, we compute the total expected cost by using the concept of cost computation given by Kanti swarap [6] and Taha [14]. The expected profit of the system is the difference between the total expected revenue and total expected cost. Let R be the earned revenue for providing service to each customer then total expected revenue (TER) is given by RL_N , where L_N is the length of the system. Further, the Total expected cost (TEC) is given by the addition of service cost and waiting cost.

Thus, TEC = μC_s + $\mu_v C_{s1}$+ X$L_N C_h$, where c_s is the service cost for busy period, C_{s1} is the service cost for vacation period and c_h is the holding cost. Then the total expected profit of the system is given by,

$$\text{TEP = TER – TEC} = \left[\frac{\lambda[E(X)+E(X^2)]}{2(\mu-\lambda E(X))} + \frac{\lambda E(X)-\mu}{\eta} + \frac{z_1[\mu-\lambda E(X)]}{\mu(1-z_1\ )+\lambda z_1\ [X(z_1)-1]}\right](\text{R-X}c_h)- \mu c_s - \mu_v c_{s1} \quad (1)$$

III. COST ANALYSIS of FM^X/FM/1 with FUZZY MULTIPLE WORKING VACATION PERIOD

We extend the above queuing system in fuzzy environment. Suppose the arrival rate λ, service rate μ for busy period, service rate μ_v for vacation period, exponential distribution of vacation parameter η, expected group size X, service cost for busy c_s and vacation period c_{s1} and holding cost c_h are approximately known and can be represented as fuzzy set $\bar{\lambda}$, $\bar{\mu}$,$\overline{\mu_v}$,$\bar{\eta}$, $\overline{X}, \overline{c_s}, \overline{c_{s1}}, \overline{c_h}$. Using α cut for the arrival rate, service rate for busy and vacation period ,vacation parameter, service cost for busy and vacation period , holding cost are represented by different levels of confidence.

Let this interval of confidence be represented by $[x_{1\alpha}\ , x_{2\alpha}]$. Since the probability distribution for the α cuts can be represented by uniform distributions. We have

$$\text{P}(x_\alpha\ ) = \frac{1}{x_{2\alpha}-x_{1\alpha}} \quad [x_{1\alpha}\ \le x_\alpha \le x_{2\alpha}]$$

Then the mean and the second order moment of the distribution are obtained as $1/2[x_{2\alpha} + x_{1\alpha}\ ]$ and $\frac{{x_{2\alpha}}^3-{x_{1\alpha}}^3}{3(x_{2\alpha}-x_{1\alpha})}$. Further its variance is given by $1/12[x_{2\alpha} - x_{1\alpha}]^2$.

Let$\varphi_{\bar{\lambda},}(p), \varphi_{\bar{\mu}}(q), \varphi_{\overline{\mu_v}}(r), \varphi_{\bar{\eta}}(y), \varphi_{\overline{X}}(x), \varphi_{\overline{C_s}}(s)$ $\varphi_{\overline{C_{s1}}}(t), \varphi_{\overline{C_h}}(u)$ denote the membership function of $\bar{\lambda}$, $\bar{\mu}$,$\overline{\mu_v}$,$\bar{\eta}$, $\overline{X}, \overline{c_s}, \overline{c_{s1}}, \overline{c_h}$ respectively. Then we have the following fuzzy sets as,

$$\bar{\lambda} = \{ (\text{p} , \varphi_{\bar{\lambda},}(p)) / \text{p} \in P\} \quad (2)$$

$$\bar{\mu} = \{ (\text{q} , \varphi_{\bar{\mu}}(q)) / \text{q} \in Q\} \quad (3)$$

$$\overline{\mu_v} = \{ (\text{r} , \varphi_{\overline{\mu_v}}(r)) / \text{r} \in R\} \quad (4)$$

$$\bar{\eta} = \{ (\text{y} , \varphi_{\bar{\eta}}(y) \ / \text{y} \in \text{Y}\} \quad (5)$$

$$\overline{X} = \{ (\text{x} , \varphi_{\overline{X}}(x) / \text{x} \in X\} \quad (6)$$

$$\overline{C_s} = \{ (\text{s} , \varphi_{\overline{C_s}}(s) / \text{s} \in \text{S} \} \quad (7)$$

$$\overline{C_{s1}} = \{ (\text{t} , \varphi_{\overline{C_{s1}}}(t) / \text{t} \in \text{T}\} \quad (8)$$

$$\overline{C_h} = \{ (\text{u} , \varphi_{\overline{C_h}}(\text{u}) / \text{u} \in \text{U} \} \quad (9)$$

where P,Q,R,Y,X,S,T,U are the crisp universal sets of the batch arrival rate, service rate for busy and vacation period, vacation parameter, group size, service cost for busy and vacation period and holding cost respectively.

Let f (p,q,r,y,x,s,t,u) denote the system characteristics of interest. Since $\bar{\lambda}$,$\bar{\mu}$,$\overline{\mu_v}$,$\bar{\eta}$, $\overline{X}, \overline{C_s}, \overline{C_{s1}}, \overline{C_h}$ are fuzzy numbers, f($\bar{\lambda}$,$\bar{\mu}$,$\overline{\mu_v}$,$\bar{\eta}$, $\overline{X}, \overline{C_s}, \overline{C_{s1}}, \overline{C_h}$) is also a fuzzy number. By Zadeh's extension principle the membership function of the system characteristics $f(\bar{\lambda},\bar{\mu},\overline{\mu_v},\bar{\eta},\overline{X},\overline{C_s},\overline{C_{s1}},\overline{C_h})$ is defined as

$$\varphi_{f(\bar{\lambda},\bar{\mu},\overline{\mu_v},\bar{\eta},\overline{X},\overline{C_s},\overline{C_{s1}},\overline{C_h})}(z) = \sup_{\substack{p\in P,q\in Q,r\in R,y\in Y,\\ x\in X,s\in S,t\in T,u\in U}} \min \{ \varphi_{\bar{\lambda}}(p), \varphi_{\bar{\mu}}(q), \varphi_{\overline{\mu_v}}(r), \varphi_{\bar{\eta}}(y), \varphi_{\overline{X}}(x), \varphi_{\overline{C_s}}(s), \varphi_{\overline{C_{s1}}}(t), \varphi_{\overline{C_h}}(u) \ / \ z = f(p,q,r,y,x,s,t,u)\} \quad (10)$$

Also assume that if the system characteristics of interests are TER (total expected revenue), TEC (total expected cost), and TEP (total expected profit). We have to find the membership function of TER, TEC and TEP. First we consider TEP as

$$f(p,q,r,y,x,s,t,u) = \left[\frac{p[(E(x)+E(x^2)]}{2(q-pE(x))} + \frac{pE(x)-r}{y} + \frac{z_1(q-pE(x))}{q(1-z_1)+pz_1(x(z_1)-1)}\right](R - xu) - qs - rt$$

Then the membership function of the total expected profit is

$$\varphi_{\overline{TEP}}(z) = \sup_{\substack{p\in P,q\in Q,r\in R,y\in Y,\\ x\in X,s\in S,t\in T,u\in U}} \min \{\varphi_{\bar{\lambda}}(p), \varphi_{\bar{\mu}}(q), \varphi_{\overline{\mu_v}}(r), \varphi_{\bar{\eta}}(y), \varphi_{\overline{X}}(x), \varphi_{\overline{C_s}}(s), \varphi_{\overline{C_{s1}}}(t)\varphi_{\overline{C_h}}(u) \ / \ z = \left[\frac{p[(E(x)+E(x^2)]}{2(q-pE(x))} + \frac{pE(x)-r}{y} + \frac{z_1(q-pE(x))}{q(1-z_1)+pz_1(x(z_1)-1)}\right](R - xu) - qs - rt\} \quad (11)$$

Likewise, the membership function of the TER & TEC can also be obtained. The membership function in the above equation is not in the usual forms thus making it very difficult to imagine its shapes. For this we approach the problem using the mathematical programming techniques. Parametric NLPs are developed to find α cut of $f(\bar{\lambda},\bar{\mu},\overline{\mu_v},\bar{\eta},\overline{X},\overline{C_s},\overline{C_{s1}},\overline{C_h})$ based on the extension principle.

IV PARAMETRIC NON LINEAR PROGRAMMING

To construct the membership function φ_{TEP} (z), it is required to determine the α cuts of TEP. For that, the α cuts of $\bar{\lambda},\bar{\mu},\overline{\mu_v},\bar{\eta},\overline{X},\overline{C_s},\overline{C_{s1}},\overline{C_h}$ are represented by crisp intervals as follows

$$\lambda(\alpha) = [p_\alpha^L \ p_\alpha^U] = [\min\{ p\in P / \varphi_{\bar{\lambda}}(p) \ge \alpha \}, \max\{ p\in P / \varphi_{\bar{\lambda}}(p) \ge \alpha \}] \quad (12a)$$

$$\mu(\alpha) = [q_\alpha^L \ q_\alpha^U] = [\min\{ q\in Q / \varphi_{\bar{\mu}}(q) \ge \alpha \}, \max\{ q\in Q / \varphi_{\bar{\mu}}(q) \ge \alpha \}] \quad (12b)$$

$$\mu_v(\alpha) = [r_\alpha^L \ r_\alpha^U] = [\min\{ r \in R / \varphi_{\overline{\mu_v}}(r) \ge \alpha \}, \max\{ r \in R / \varphi_{\overline{\mu_v}}(r) \ge \alpha \}] \quad (12c)$$

$$\eta(\alpha) = [y_\alpha^L \ y_\alpha^U] = [\min\{ y\in Y / \varphi_{\bar{\eta}}(y) \ge \alpha \}, \max\{ y\in Y / \varphi_{\bar{\eta}}(y) \ge \alpha \}] \quad (12d)$$

$$X(\alpha) = [x_\alpha^L \ x_\alpha^U] = [\min\{x\in X / \varphi_{\overline{X}}(x) \ge \alpha \}, \max\{ x\in X / \varphi_{\overline{X}}(x) \ge \alpha \}] \quad (12e)$$

$$C_s(\alpha) = [s_\alpha^L \ s_\alpha^U] = [\min\{s\in S / \varphi_{\overline{C_s}}(s) \ge \alpha \}, \max\{s\in S / \varphi_{\overline{C_s}}(s) \ge \alpha \}] \quad (12f)$$

$$C_{s1}(\alpha) = [t_\alpha^L \ t_\alpha^U] = [\min\{t\in T / \varphi_{\overline{C_{s1}}}(t) \ge \alpha \}, \max\{ t\in T / \varphi_{\overline{C_{s1}}}(t) \ge \alpha \}] \quad (12g)$$

$$C_h(\alpha) = [u_\alpha^L \ u_\alpha^U] = [\min\{u\in U / \varphi_{\overline{C_h}}(u) \ge \alpha \}, \max\{ u\in U / \varphi_{\overline{C_h}}(u) \ge \alpha \}] \quad (12h)$$

Further, the bounds of these intervals can be described as functions of α and can be obtained as

$p_\alpha^L = \min \varphi_{\bar{\lambda}}^{-1}(\alpha)$ $\quad p_\alpha^U = \max \varphi_{\bar{\lambda}}^{-1}(\alpha)$

$q_\alpha^L = \min \varphi_{\bar{\mu}}^{-1}(\alpha)$ $\quad q_\alpha^U = \max \varphi_{\bar{\mu}}^{-1}(\alpha)$

$r_\alpha^L = \min \varphi_{\overline{\mu_v}}^{-1}(\alpha)$ $\quad r_\alpha^U = \max \varphi_{\overline{\mu_v}}^{-1}(\alpha)$

$y_\alpha^L = \min \varphi_{\bar{\eta}}^{-1}(\alpha)$ $\quad y_\alpha^U = \max \varphi_{\bar{\eta}}^{-1}(\alpha)$

$x_\alpha^L = \min \varphi_{\overline{X}}^{-1}(\alpha)$ $\quad x_\alpha^U = \max \varphi_{\overline{X}}^{-1}(\alpha)$

$s_\alpha^L = \min \varphi_{\overline{C_s}}^{-1}(\alpha)$ $\quad s_\alpha^U = \max \varphi_{\overline{C_s}}^{-1}(\alpha)$

$t_\alpha^L = \min \varphi_{\overline{C_{s1}}}^{-1}(\alpha)$ $\quad t_\alpha^U = \max \varphi_{\overline{C_{s1}}}^{-1}(\alpha)$

$u_\alpha^L = \min \varphi_{\overline{C_h}}^{-1}(\alpha)$ $\quad u_\alpha^U = \max \varphi_{\overline{C_h}}^{-1}(\alpha)$

Therefore by making use of the α-cuts for TEP we construct the membership functions of (11) which is parameterized by α. To derive the membership function of TEP it is suffice to find the left and right shape function of φ_{TEP}(z). This can be achieved by following the zadhe's extension principle for φ_{TEP}(z) which is the minimum of

$\varphi_{\bar{\lambda},}(p), \varphi_{\bar{\mu}}(q), \varphi_{\overline{\mu_v}}(r), \varphi_{\bar{\eta}}(y), \varphi_{\bar{X}}(x), \varphi_{\overline{C_s}}(s), \varphi_{\overline{C_{s1}}}(t), \varphi_{\overline{C_h}}(u)$. Now to derive $\varphi_{TEP}(z) = \alpha$, then at least one the following cases to be hold which satisfies $\varphi_{TEP}(z) = \alpha$. Thus,

Case(i)

$$\varphi_{\bar{\lambda},}(p) = \alpha, \varphi_{\bar{\mu}}(q) \geq \alpha, \varphi_{\overline{\mu_v}}(r) \geq \alpha, \varphi_{\bar{\eta}}(y) \geq \alpha, \varphi_{\bar{X}}(x) \geq \alpha, \varphi_{\overline{C_s}}(s) \geq \alpha, \varphi_{\overline{C_{s1}}}(t) \geq \alpha, \varphi_{\overline{C_h}}(u) \geq \alpha$$

Case(ii)

$$\varphi_{\bar{\lambda},}(p) \geq \alpha, \quad \varphi_{\bar{\mu}}(q) = \alpha, \quad \varphi_{\overline{\mu_v}}(r) \geq \alpha, \varphi_{\bar{\eta}}(y) \geq \alpha, \varphi_{\bar{X}}(x) \geq \alpha, \varphi_{\overline{C_s}}(s) \geq \alpha, \varphi_{\overline{C_{s1}}}(t) \geq \alpha, \varphi_{\overline{C_h}}(u) \geq \alpha$$

Case (iii)

$$\varphi_{\bar{\lambda},}(p) \geq \alpha, \varphi_{\bar{\mu}}(q) \geq \alpha, \varphi_{\overline{\mu_v}}(r) = \alpha, \varphi_{\bar{\eta}}(y) \geq \alpha, \varphi_{\bar{X}}(x) \geq \alpha, \varphi_{\overline{C_s}}(s) \geq \alpha, \varphi_{\overline{C_{s1}}}(t) \geq \alpha, \varphi_{\overline{C_h}}(u) \geq \alpha$$

Case(iv)

$$\varphi_{\bar{\lambda},}(p) \geq \alpha, \varphi_{\bar{\mu}}(q) \geq \alpha, \varphi_{\overline{\mu_v}}(r) \geq \alpha, \varphi_{\bar{\eta}}(y) = \alpha, \varphi_{\bar{X}}(x) \geq \alpha, \varphi_{\overline{C_s}}(s) \geq \alpha, \varphi_{\overline{C_{s1}}}(t) \geq \alpha, \varphi_{\overline{C_h}}(u) \geq \alpha$$

Case(v)

$$\varphi_{\bar{\lambda},}(p) \geq \alpha, \varphi_{\bar{\mu}}(q) \geq \alpha, \varphi_{\overline{\mu_v}}(r) \geq \alpha, \varphi_{\bar{\eta}}(y) \geq \alpha, \varphi_{\bar{X}}(x) = \alpha, \varphi_{\overline{C_s}}(s) \geq \alpha, \varphi_{\overline{C_{s1}}}(t) \geq \alpha, \varphi_{\overline{C_h}}(u) \geq \alpha$$

Case (vi)

$$\varphi_{\bar{\lambda},}(p) \geq \alpha, \varphi_{\bar{\mu}}(q) \geq \alpha, \varphi_{\overline{\mu_v}}(r) \geq \alpha, \varphi_{\bar{\eta}}(y) \geq \alpha, \varphi_{\bar{X}}(x) \geq \alpha, \varphi_{\overline{C_s}}(s) = \alpha, \varphi_{\overline{C_{s1}}}(t) \geq \alpha, \varphi_{\overline{C_h}}(u) \geq \alpha$$

Case (vii)

$$\varphi_{\bar{\lambda},}(p) \geq \alpha, \varphi_{\bar{\mu}}(q) \geq \alpha, \varphi_{\overline{\mu_v}}(r) \geq \alpha, \varphi_{\bar{\eta}}(y) \geq \alpha, \varphi_{\bar{X}}(x) \geq \alpha, \varphi_{\overline{C_s}}(s) \geq \alpha, \varphi_{\overline{C_{s1}}}(t) = \alpha, \varphi_{\overline{C_h}}(u) \geq \alpha$$

Case (viii)

$$\varphi_{\bar{\lambda},}(p) \geq \alpha, \varphi_{\bar{\mu}}(q) \geq \alpha, \varphi_{\overline{\mu_v}}(r) \geq \alpha, \varphi_{\bar{\eta}}(y) \geq \alpha, \varphi_{\bar{X}}(x) \geq \alpha, \varphi_{\overline{C_s}}(s) \geq \alpha, \varphi_{\overline{C_{s1}}}(t) \geq \alpha, \varphi_{\overline{C_h}}(u) = \alpha$$

This can be accomplished by using parametric NLP techniques. The NLP techniques to find the lower and upper bounds of α cut of $\varphi_{\overline{TEP}}(z)$ for case (i) is

$$[TEP]_\alpha^{L1} = \min\{[\frac{p[(E(x)+E(x^2)]}{2(q-pE(x))} + \frac{pE(x)-r}{y} + \frac{z_1(q-pE(x))}{q(1-z_1)+pz_1(x(z_1)-1)}] (R-xu)-qs-rt\} \quad (13a)$$

$$[TEP]_\alpha^{U1} = \max\{[\frac{p[(E(x)+E(x^2)]}{2(q-pE(x))} + \frac{pE(x)-r}{y} + \frac{z_1(q-pE(x))}{q(1-z_1)+pz_1(x(z_1)-1)}] (R-xu)-qs-rt\} \quad (13b)$$

For case (ii) as

$$[TEP]_\alpha^{L2} = \min\{[\frac{p[(E(x)+E(x^2)]}{2(q-pE(x))} + \frac{pE(x)-r}{y} + \frac{z_1(q-pE(x))}{q(1-z_1)+pz_1(x(z_1)-1)}] (R-xu)-qs-rt\} \quad (13c)$$

$$[TEP]_\alpha^{U2} = \max\{[\frac{p[(E(x)+E(x^2)]}{2(q-pE(x))} + \frac{pE(x)-r}{y} + \frac{z_1(q-pE(x))}{q(1-z_1)+pz_1(x(z_1)-1)}] (R-xu)-qs-rt\} \quad (13d)$$

For case (iii) as

$$[TEP]_\alpha^{L3} = \min\{[\frac{p[(E(x)+E(x^2)]}{2(q-pE(x))} + \frac{pE(x)-r}{y} + \frac{z_1(q-pE(x))}{q(1-z_1)+pz_1(x(z_1)-1)}] (R-xu)-qs-rt\} \quad (13e)$$

$$[TEP]_\alpha^{U3} = \max\{[\frac{p[(E(x)+E(x^2)]}{2(q-pE(x))} + \frac{pE(x)-r}{y} + \frac{z_1(q-pE(x))}{q(1-z_1)+pz_1(x(z_1)-1)}] (R-xu)-qs-rt\} \quad (13f)$$

For case (iv) as

$$[TEP]_\alpha^{L4} = \min\{[\frac{p[(E(x)+E(x^2)]}{2(q-pE(x))} + \frac{pE(x)-r}{y} + \frac{z_1(q-pE(x))}{q(1-z_1)+pz_1(x(z_1)-1)}] (R-xu)-qs-rt\} \quad (13g)$$

$$[TEP]_\alpha^{U4} = \max\{[\frac{p[(E(x)+E(x^2)]}{2(q-pE(x))} + \frac{pE(x)-r}{y} + \frac{z_1(q-pE(x))}{q(1-z_1)+pz_1(x(z_1)-1)}] (R-xu)-qs-rt\} \quad (13h)$$

For case (v) as

$$[TEP]_\alpha^{L5} = \min\{[\frac{p[(E(x)+E(x^2)]}{2(q-pE(x))} + \frac{pE(x)-r}{y} + \frac{z_1(q-pE(x))}{q(1-z_1)+pz_1(x(z_1)-1)}] (R-xu)-qs-rt\} \quad (13i)$$

$$[TEP]_\alpha^{U5} = \max\{[\frac{p[(E(x)+E(x^2)]}{2(q-pE(x))} + \frac{pE(x)-r}{y} + \frac{z_1(q-pE(x))}{q(1-z_1)+pz_1(x(z_1)-1)}] (R-xu)-qs-rt\} \quad (13j)$$

For case (vi) as

$$[TEP]_\alpha^{L6} = \min\{[\frac{p[(E(x)+E(x^2)]}{2(q-pE(x))} + \frac{pE(x)-r}{y} + \frac{z_1(q-pE(x))}{q(1-z_1)+pz_1(x(z_1)-1)}] (R-xu)-qs-rt\} \quad (13k)$$

$$[TEP]_{\alpha}^{U6} = \max\{[\frac{p[(E(x)+E(x^2)]}{2(q-pE(x))} + \frac{pE(x)-r}{y} + \frac{z_1(q-pE(x))}{q(1-z_1)+pz_1(x(z_1)-1)}] (R - xu) - qs - rt\} \quad (13l)$$

For case (vii) as

$$[TEP]_{\alpha}^{L7} = \min\{[\frac{p[(E(x)+E(x^2)]}{2(q-pE(x))} + \frac{pE(x)-r}{y} + \frac{z_1(q-pE(x))}{q(1-z_1)+pz_1(x(z_1)-1)}] (R - xu) - qs - rt\} \quad (13m)$$

$$[TEP]_{\alpha}^{U7} = \max\{[\frac{p[(E(x)+E(x^2)]}{2(q-pE(x))} + \frac{pE(x)-r}{y} + \frac{z_1(q-pE(x))}{q(1-z_1)+pz_1(x(z_1)-1)}] (R - xu) - qs - rt\} \quad (13n)$$

And for case (viii) as

$$[TEP]_{\alpha}^{L8} = \min\{[\frac{p[(E(x)+E(x^2)]}{2(q-pE(x))} + \frac{pE(x)-r}{y} \frac{z_1(q-pE(x))}{q(1-z_1)+pz_1(x(z_1)-1)}] (R - xu) - qs - rt\} \quad (13o)$$

$$[TEP]_{\alpha}^{U8} = \max\{[\frac{p[(E(x)+E(x^2)]}{2(q-pE(x))} + \frac{pE(x)-r}{y} + \frac{z_1(q-pE(x))}{q(1-z_1)+pz_1(x(z_1)-1)}] (R - xu) - qs - rt\} \quad (13p)$$

As λ(α),μ(α), μ_v(α),η(α),X(α),$C_s(\alpha)$, C_{s1}(α),C_h(α) given in equations (12 a-h) p ∈ λ(α), q ∈ μ(α), r ∈ $\mu_v(\alpha)$, y ∈ η(α), x ∈ X(α), s ∈ $C_s(\alpha)$, t ∈ C_{s1}(α), u ∈ C_h(α) can be replaced by p ∈ $[p_{\alpha}^L\ p_{\alpha}^U]$, q ∈ $[q_{\alpha}^L\ q_{\alpha}^U]$, r ∈ $[r_{\alpha}^L\ r_{\alpha}^U]$, y ∈ $[y_{\alpha}^L\ y_{\alpha}^U]$, x ∈ $[x_{\alpha}^L\ x_{\alpha}^U]$, s ∈ $[s_{\alpha}^L\ s_{\alpha}^U]$, t ∈ $[t_{\alpha}^L\ t_{\alpha}^U]$, and u ∈ $[u_{\alpha}^L\ u_{\alpha}^U]$ which are given by the α cuts and in turn they form a nested structure with respect to α which are expressed in (13a-p). Hence for given 0< α_2< α_1< 1, we have

$$[p_{\alpha 1}^L\ p_{\alpha 1}^U] \subseteq [p_{\alpha 2}^L\ p_{\alpha 2}^U]$$
$$[q_{\alpha 1}^L\ q_{\alpha 1}^U] \subseteq [q_{\alpha 2}^L\ q_{\alpha 2}^U]$$
$$[r_{\alpha 1}^L\ r_{\alpha 1}^U] \subseteq [r_{\alpha 2}^L\ r_{\alpha 2}^U]$$
$$[y_{\alpha 1}^L\ y_{\alpha 1}^U] \subseteq [y_{\alpha 2}^L\ y_{\alpha 2}^U]$$
$$[x_{\alpha 1}^L\ x_{\alpha 1}^U] \subseteq [x_{\alpha 2}^L\ x_{\alpha 2}^U]$$
$$[s_{\alpha 1}^L\ s_{\alpha 1}^U] \subseteq [s_{\alpha 2}^L\ s_{\alpha 2}^U]$$
$$[t_{\alpha 1}^L\ t_{\alpha 1}^U] \subseteq [t_{\alpha 2}^L\ t_{\alpha 2}^U]$$
$$[u_{\alpha 1}^L u_{\alpha 1}^U] \subseteq [u_{\alpha 2}^L\ u_{\alpha 2}^U]$$

Thus equations (13a), (13c), (13e), (13g), (13i), (13k), (13m), (13o) have the unique smallest element and equations (13b), (13d), (13f), (13h),(13j), (13l), (13n), (13p) have the unique largest element. Now, to find the membership function of $\varphi_{\overline{\text{TEP}}}$(z) which is equivalent to find the lower bound of $[TEP]_{\alpha}^L$ and upper bound of $[TEP]_{\alpha}^U$ is written as,

$$[TEP]_{\alpha}^{L} = \min\{[\frac{p[(E(x)+E(x^2)]}{2(q-pE(x))} + \frac{pE(x)-r}{y} + \frac{z_1(q-pE(x))}{q(1-z_1)+pz_1(x(z_1)-1)}] (R- xu) - qs - rt\} \quad (14a)$$

such that, $p_{\alpha}^L \le$ p $\le p_{\alpha}^U$, $q_{\alpha}^L \le$ q $\le q_{\alpha}^U$, $r_{\alpha}^L \le r \le r_{\alpha}^U$,

$y_{\alpha}^L \le y \le y_{\alpha}^U$, $x_{\alpha}^L \le x \le x_{\alpha}^U$, $s_{\alpha}^L \le s \le s_{\alpha}^U$, $t_{\alpha}^L \le t \le t_{\alpha}^U$,

$u_{\alpha}^L \le u \le u_{\alpha}^U$ and

$$[TEP]_{\alpha}^{U} = \min\{[\frac{p[(E(x)+E(x^2)]}{2(q-pE(x))} + \frac{pE(x)-r}{y} + \frac{z_1(q-pE(x))}{q(1-z_1)+pz_1(x(z_1)-1)}] (R- xu) - qs - rt\} \quad (14b)$$

Such that, $p_{\alpha}^L \le$ p $\le p_{\alpha}^U$, $q_{\alpha}^L \le$ q $\le q_{\alpha}^U$, $r_{\alpha}^L \le r \le r_{\alpha}^U$,

$y_{\alpha}^L \le y \le y_{\alpha}^U$, $x_{\alpha}^L \le x \le x_{\alpha}^U$, $s_{\alpha}^L \le s \le s_{\alpha}^U$, $t_{\alpha}^L \le t \le t_{\alpha}^U$,

$u_{\alpha}^L \le u \le u_{\alpha}^U$

(i.e) At least any one of p, q, r, y, x, s, t, u must hit the boundaries of their α cut that satisfy φ_{TEP}(z) = α

By applying the results of Zimmerman[18] and convexity property, we obtain $[TEP]_{\alpha 1}^L \ge [TEP]_{\alpha 2}^L$ and $[TEP]_{\alpha 1}^U \le [TEP]_{\alpha 2}^U$, where 0< α_2< α_1< 1.

In both $[TEP]_{\alpha}^L$ and $[TEP]_{\alpha}^U$ are invertible with respect to α then the left shape function L(z) =[$[TEP]_{\alpha}^L]^{-1}$ and right shape function R(z) =[$[TEP]_{\alpha}^U]^{-1}$ can be derived, such that

$$\varphi_{\overline{\text{TEP}}}(z) = \begin{cases} L(z) & [TEP]_{\alpha=0}^L \le z \le [TEP]_{\alpha=1}^L \\ 1 & [TEP]_{\alpha=1}^L \le z \le [TEP]_{\alpha=1}^U \\ R(z) & [TEP]_{\alpha=1}^U \le z \le [TEP]_{\alpha=0}^U \end{cases}$$

In many cases, the value of $\{(TEP)_{\alpha}^L\ (TEP)_{\alpha}^U$ / α∈ [0 1]} cannot be solved analytically, consequently a closed form membership function of TEP cannot be obtained. The numerical solutions for $(TEP)_{\alpha}^L$ *and* $(TEP)_{\alpha}^U$ at different levels of α can be collected that approximate the shape of L(z) and R(z) (i.e.,) the set of intervals$\{(TEP)_{\alpha}^L\ (TEP)_{\alpha}^U$ / α∈ [0 1] } will estimate the shapes.

V NUMERICAL EXAMPLE

Consider a FM^X/FM/1/MWV queuing system. The corresponding cost parameters such as the arrival rate λ, service rate μ for busy period, service rate μ_v for vacation period, exponential distribution of vacation parameterη, expected group size X, service cost for busy C_s and working vacation period C_{s1} and holding cost C_h are fuzzy numbers.

Let $\bar{\lambda}$ = [4,5,6,7], $\bar{\mu}$ =[2,3,4,5], $\overline{\mu_v}$ =[.005, .05,.5,.7], $\bar{\eta}$ =[.0011, .0012, .0013, .0014], $\bar{X}$ = [5, 6, 7, 8] $\overline{C_s}$ = [50, 100, 150, 200], $\overline{C_{s1}}$ = [70, 120, 170, 220], $\overline{C_h}$ =[5, 6, 7, 8] R = 300 and z_1= .8

Then the total expected Revenue, Cost and Profit per unit time per customer are given by,

$\overline{TER}$ = RL_N , $\overline{TEC}$ = qs + rt + XuL_N,

$\overline{TEP} = \overline{TER} - \overline{TEC}$ = L_N(R-Xu)-qs-rt

$where\ L_N = [\frac{p[(E(x)+E(x^2)]}{2(q-pE(x))} + \frac{pE(x)-r}{y} + \frac{z_1(q-pE(x))}{q(1-z_1)+pz_1(x(z_1)-1)}]$

and p,q,r,y,x,s,t,u are the fuzzy variable corresponding to $\bar{\lambda}$, $\bar{\mu}$,$\overline{\mu_v}$,$\bar{\eta}$, $\bar{X}$, $\overline{c_s}$, $\overline{c_{s1}}$, $\overline{c_h}$ respectively. Thus,

$[p_\alpha^L\ p_\alpha^U] = [4+\alpha\ \ 7-\alpha], [q_\alpha^L\ \ q_\alpha^U] = [2+\alpha\ \ 5-\alpha]$

$[r_\alpha^L\ r_\alpha^U] = [0.005 + 0.045\alpha\ \ 0.7 - 0.2\alpha]$,

$[y_\alpha^L\ y_\alpha^U\ ] = [0.0011 + 0.0001\alpha\ \ 0\ .0014 - 0.0001\alpha]$,

$[x_\alpha^L\ x_\alpha^U\ ] = [5+\alpha\ 8-\alpha]$, $[s_\alpha^L\ s_\alpha^U] = [50+50\alpha\ \ 200-50\alpha]$

$[\ t_\alpha^L\ \ t_\alpha^U] = [70+50\alpha\ \ 220-50\alpha]$ $[\ u_\alpha^L\ \ u_\alpha^U]$= [5+α, 8-α]

By substituting the above values, the effect of parameters on the total expected system of Revenue (TER), Cost (TEC), and profit (TEP) are tabulated and their graphical representations are also shown below

Table 1**:** The α cuts for the performance measure of TER

α	TER L	TER U
0	7087732	9598405.5
.1	7198337.6	9531497.7
.2	7306967.5	9463619.8
.3	7413674.1	9394750.8
.4	7518508.1	9324868.7
.5	7621518.4	9253951
.6	7722752.1	9181974.6
.7	7822254.9	9108915.5
.8	7920070.8	9034749.2
.9	8016242.2	8959450.2
1	8110810.5	8882992.6

Fig 1: The membership function for fuzzy TER

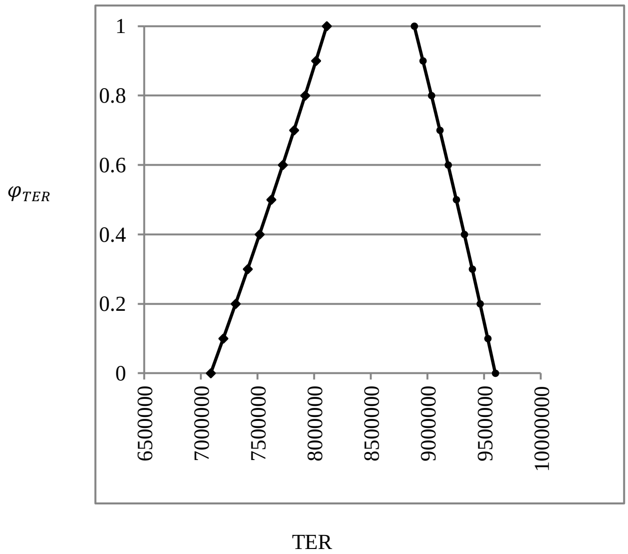


Table 2 **:** The α cuts for the performance measure of TEC

α	TEC L	TEC U
0	590744.7	2048813.8
.1	624212.1	1983970.9
.2	658734.5	1920272.7
.3	694318.1	1857716.6
.4	730969.1	1796300.1
.5	768693.2	1736020.3
.6	807496.2	1676874.6
.7	847383.5	1618860.1
.8	888360.4	1561973.9
.9	930432	1506212.8
1	973603.3	1451573.8

Fig 2: The membership function for fuzzy TEC

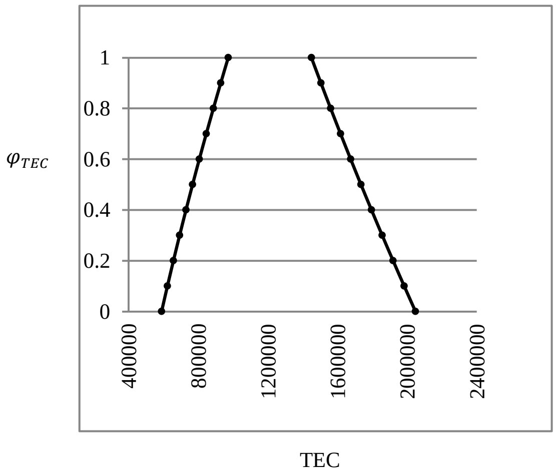
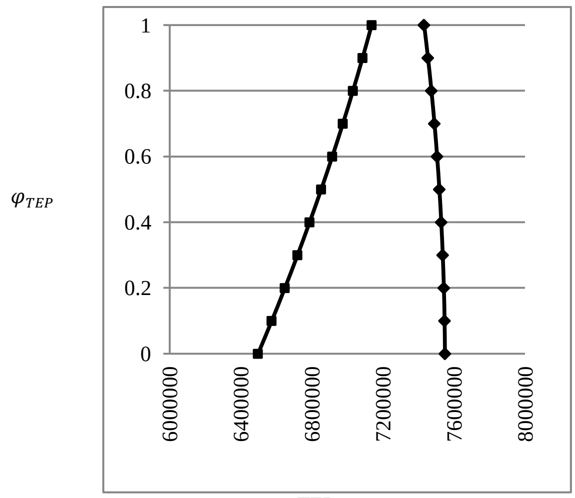


Table 3 **:** The α cuts for the performance measure of TEP

α	**TEP L**	**TEP U**
0	6496987.3	7549591.7
.1	6574125.5	7547526.7
.2	6648233	7543347.1
.3	6719356	7537034.2
.4	6787539.1	7528568.7
.5	6852825.2	7517930.7
.6	6915255.9	7505100
.7	6974871.4	7490055.4
.8	7031710.3	7472775.3
.9	7085810.2	7453237.4
1	7137207.2	7431418.8

Fig 3: The membership function for fuzzy TEP



Here we perform α cuts for fuzzy TER, TEC and TEP at eleven distinct α levels of 0,0.1,0.2……….1.0. Crisp intervals of fuzzy TER in the system for different possibilities of α level, are presented in table1. Similarly other performance measures such as TEC & TEP are presented in tables 2 & 3 respectively. Fig 1 depicts the rough shape of TER constructed from α value. The rough shape turns out rather fine and looks like a continuous function. Other performance measures are represented by fig 2 & fig 3 respectively.

The α cut depicts that these three performance measures will lie only in the specified associated cost range. Further, we find that the above information is very useful for designing the fuzzy queuing system.

VI CONCLUSION

The fuzzy queuing model has more applicability in the real environments than the crisp systems. This paper applies the concept of α cut and Zadhe's extension principle to FM^X/FM/1/MWV and thereby deriving the membership function of the total expected profit, for the model. By noting the total expected profit we find that it is more meaningful to express TEP as a membership function rather than by a crisp value. (i.e.,) it is a fuzzy performance measure. The benefit and significance of such a fuzzy performance measure include maintaining the fuzziness of input information completely. Thus it can be concluded that the fuzzy cost based queuing systems are much more useful than the commonly used crisp queues.

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