Solving Multi Objective Fully Fuzzy Linear Programming Problem Using Ranking Function

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Abstract— **In this paper we solve a multi objective fully fuzzy linear programming problems using trapezoidal fuzzy numbers are proposed. Then the fully fuzzy linear programming problem is solved by using Maleki and yager ranking function.**

Keywords— **Fuzzy linear programming, ranking function, trapezoidal fuzzy numbers.**

I. INTRODUCTION

The classical linear programming problem is used to find an optimal solution for a single objective function whereas many problems in real world involve multiple objectives. This type of problems can be formulated as multi objective linear programming **(**MOLPP) problems. Bellman and Zadeh[1] developed the concept of decision making using fuzzy environment. The first formulation of fuzzy linear programming was presented by Zimmerman [10]. Maleki[5] introduced fuzzy variable in linear programming problems and proposed a new method for solving linear programming problem using ranking function. Pandian and jayalakshmi [6] proposed a new method for solving integer linear programming problems with fuzzy variables. A multi objective approach for fuzzy linear programming problems was proposed by Tanaka,et. al,[8]. Pandian [7] used sum of objectives for solving multi objective programming problems. Dheyab[2] solved the decision maker in the form of non symmetrical trapezoidal fuzzy numbers and solved it by using ranking function.Vasanth **,**et**.**al **[**9] applied linear programming with fuzzy parameters for decision making in industrial production planning.Singh[4] proposed a new method for solving fully fuzzy linear programming problems using ranking function. Hashem[3] introduced the decision maker in the form of non symmetrical trapezoidal fuzzy numbers and solved it by using ranking function.In this paper,the fully fuzzy linear programming problems is solved by using Maleki linear ranking function when (α=β) and using Yager linear ranking function when (α=β) .

II. PRELIMINARIES

A. *Definition [1]*

If X is a collection of objects denoted generically by x, then fuzzy set $\tilde{A}$ in X is defined to be a set of ordered pairs.
$\tilde{A}$ = {(x, $\mu_{\tilde{A}}$ (x)/ x $\in X$)}, where $\mu_{\tilde{A}}$ (x) is called the membership function for the fuzzy set.The membership function maps each of X value between [0,1].Assume X is the real line R.

B. Definition [1]

The support of a fuzzy set $\tilde{A}$ is the set of point x in X with $\mu_{\tilde{A}}(x) > 0$, the case of a fuzzy set $\tilde{A}$ is the set of point x in X with $\mu_{\tilde{A}}(x)=1$.

C. Definition [1]

A fuzzy set $\tilde{A}$ is called normal . if its core is nonempty. There is at least one point $x \in X$ with $\mu_{\tilde{A}}(x)=1$.

D. Definition [1]

A fuzzy set $\tilde{A}$ on R is convex . if for any x, y $\in$ X and [0,1]; then $\mu_{\bar{A}}(\lambda_x + (1-\lambda)y) \geq \min\{\mu_{\bar{A}}(x), \mu_{\bar{A}}(y)\}$.

E. Definition [1]

A fuzzy number $\tilde{A}$ is a fuzzy set on the real line that satisfies the conditions of normality and convexity.

III.TRAPEZOIDAL FUZZY NUMBERS [1]

The fuzzy number is defined by its corresponding membership function. The trapezoidal membership functions for fuzzy number $\tilde{a}$ as follows.

$$\mu_{\tilde{a}}(x) = \begin{cases} 1-\left(\dfrac{a^l - x}{\alpha}\right) & when\ a^l - \alpha \leq x \leq a^l \\ 1 & when\ a^l \leq x \leq a^u \\ 1-\left(\dfrac{x - a^u}{\beta}\right) & when\ a^u \leq x \leq a^l + \beta \\ 0 & otherwise \end{cases}$$

Assume that $\tilde{a}$ = (a^l, b^u, α, β) and $\tilde{b}$ =(b^l, b^u, α, β) be two trapezoidal fuzzy numbers and $x \in R$.
The arithmetic operations on fuzzy numbers are shown as follows.

Addition $\tilde{a}+\tilde{b}=(a^l+b^l, a^u+b^u, \alpha+\gamma, \beta+\theta)$
Subtraction $\tilde{a}-\tilde{b}=(a^l-b^u, a^u-b^l, \alpha+\theta, \beta+\gamma)$
Scalar multiplication:
if $x \geq 0$, $x\tilde{a} = (xa^l, xa^u, x\alpha, x\beta)$,
if $x \prec 0$, $x\tilde{a} = (xa^u, xa^l, -x\beta, -x\alpha)$

IV. RANKING FUNCTION

The ranking function is denoted by F(R), where R: F(R) $\rightarrow$ R, and F(R) is the set of fuzzy number defined on a real line.

a) Maleki Ranking function:

let $\tilde{a} = (a^l, b^u, \alpha, \beta)$ be fuzzy numbers then the ranking function is

$$R(\tilde{a}) = \int_0^1 (\inf \widetilde{a_\lambda} + \sup \widetilde{a_\lambda}) d\lambda$$

$$R(\tilde{a}) = a^l + b^u + \frac{1}{2}(\beta-\alpha), \text{ where } \alpha=\beta$$

b)Yager Ranking function:

Assume that $\tilde{a} = (a^l, b^u, \alpha, \beta)$ be fuzzy numbers then the ranking function is

$$R(\tilde{a}) = \frac{1}{2}[\int_0^1 [(a^l - \alpha L^{-1}(\lambda)] d\lambda + \int_0^1 [(a^u - \beta R^{-1}(\lambda)] d\lambda]$$

$$R(\tilde{a}) = (a^l + b^u - \frac{4}{5}\alpha + \frac{2}{3}\beta), \text{ where } \alpha=\beta$$

V.LINEAR PROGRAMMING PROBLEM

The linear programming problem is defined as follows:

Max or Min $Z = \sum_{j=1}^{n} c_j x_j$
subject to
$\sum_{j=1}^{n} a_{ij} x_j \leq b_i$ i=1,2,…m, j=1,2,3...,n
$x_j \geq 0$
where $c_j \in R^n, b_j \in R^m, a_{ij} \in R^{n*m}$

The parameter in the above model is crisp.

A. Fuzzy Linear Programming

In the above linear programming problem all the parameters are fuzzy numbers. Then the fuzzy linear programming problem is converted into a crisp linear programming problem where we consider two cases of fuzzy linear programming problem.

Case (i) : Fully fuzzy linear programming problem

Max $Z = \sum_{j=1}^{n} \tilde{c}_j x_j$
subject to
$\sum_{j=1}^{n} \widetilde{a_{ij}} x_j \leq \tilde{b}_i$ i=1,2,…m,   j=1,2,3...,n
$x_j \geq 0$
where $\tilde{c}_j, \tilde{b}_i$,and $\widetilde{a_{ij}}$ are trapezoidal fuzzy numbers.

Then the fuzzy linear programming is solved by using Maleki and Yager ranking functions.

a)Maleki Ranking function

Max $Z = \sum_{j=1}^{n} [\, c_j{}^l + c_j{}^u + \frac{1}{2}(\beta-\alpha)\,] x_j$
subject to
$\sum_{j=1}^{n} [\, a_{ij}{}^l + a_{ij}{}^u + \frac{1}{2}(\beta-\alpha)]\, x_j \leq$
$[\, b_i{}^l + b_i{}^u + \frac{1}{2}(\beta-\alpha)]$ i =1,2,…m, j=1,2,3...,n
$x_j \geq 0.$

b)Yager Ranking function

Max $Z = \sum_{j=1}^{0} [c_j{}^l + c_j{}^u - \frac{4}{5}\alpha + \frac{2}{3}\beta]\, x_j$
subject to
$\sum_{j=1}^{n} [a_{ij}{}^l + a_{ij}{}^u - \frac{4}{5}\alpha + \frac{2}{3}\beta] x_j \leq$
$\frac{1}{2}[b_i{}^l + b_i{}^u - \frac{4}{5}\alpha + \frac{2}{3}\beta]$
$x_j \geq 0$ $i = 0,1,2,\ldots m$, , j=1,2,3...,n

VI. NUMERICAL EXAMPLE

Max Z =(4,8,2,2) x_1+(1,3,1,1) x_2+(1,5,2,2) x_3
subject to,
(3,5,6,6) x_1+(6,8,4,4) x_2+(9,3,5,5) x_3 ≤(1,7,3,3)
(2,6,3,3) x_1+(4,9,2,2) x_2+(3,7,6,6) x_3≤(2,6,2,2)
(4,6,7,7) x_1+(5,7,6,6) x_2+(3,8,4,4) x_3≤(5,9,2,2)
$x_1, x_2, x_3 \geq 0$

Solution:

a)Maleki Ranking Function

Max Z = 12 x_1+4 x_2+6 x_3
subject to
8 x_1+14 x_2+ 7x_3 ≤ 8
8 x_1+13 x_2+10 x_3 ≤ 8
10 x_1+12 x_2+11 x_3 ≤ 14
$x_1, x_2, x_3 \geq 0$

Then the solution of the linear programming is
x_1 =1, x_2 =0,x_3 =0, Max Z= 12

b)Yager Ranking Function

Max Z = 5.86 x_1+1.93 x_2+2.86 x_3
subject to
7.2 x_1+13.46 x_2+6.33 x_3 ≤ 3.8
7.6 x_1+12.73 x_2+9.2 x_3 ≤ 3.866
9.06 x_1+11.2 x_2+10.46 x_3 ≤ 6.867
$x_1, x_2, x_3 \geq 0$

Then the solution of the linear programming is
x_1 =1.05, x_2 =0,x_3 =0, Max Z= 12.63

Case (ii): Multi objective fully fuzzy linear programming problem:

Consider the multi objective fully fuzzy linear programming problem as follows

$$\text{Max } Z_1 = \sum_{j=1}^{n} \tilde{c}_j x_j$$

$$\text{Max } Z_2 = \sum_{j=1}^{n} \tilde{d}_j x_j$$

subject to

$$\sum_{j=1}^{n} \widetilde{a_{ij}} x_j \le \tilde{b}_i \quad i=1,2,\dots,m, \quad j=1,2,3\dots,n$$

$$x_j \ge 0$$

where $\tilde{c}_j, \tilde{d}_j, \tilde{b}_i$,and $\widetilde{a_{ij}}$ are trapezoidal fuzzy numbers. Then the fuzzy linear programming is solved by using Maleki and Yager ranking functions.

a)Maleki Ranking function:

$$\text{Max } Z_1 = \sum_{j=1}^{n} [\ c_j{}^l + c_j{}^u + \tfrac{1}{2}(\beta - \alpha)\ ] x_j$$

$$\text{Max } Z_2 = \sum_{j=1}^{n} [\ d_j{}^l + d_j{}^u + \tfrac{1}{2}(\beta - \alpha)\ ] x_j$$

subject to

$$\sum_{j=1}^{n} [a^l{}_{ij} + a^u{}_{ij} + \frac{1}{2}(\beta - \alpha)] x_j \le [b_i^l + b_i^u + \frac{1}{2}(\beta - \alpha)]$$

$$x_j \ge 0. \qquad i=1,2,\dots,m, j=1,2,3\dots,n$$

b)Yager Ranking function:

$$\text{Max } Z_1 = \sum_{j=1}^{0} [c_j{}^l + c_j{}^u - \tfrac{4}{5}\alpha + \tfrac{2}{3}\beta]\, x_j$$

$$\text{Max } Z_2 = \sum_{j=1}^{0} [d_j{}^l + d_j{}^u - \tfrac{4}{5}\alpha + \tfrac{2}{3}\beta]\, x_j$$

subject to

$$\sum_{j=1}^{n} [a_{ij}{}^l + a_{ij}{}^u - \tfrac{4}{5}\alpha + \tfrac{2}{3}\beta] x_j \leq \tfrac{1}{2}[b_i{}^l + b_i{}^u - \tfrac{4}{5}\alpha + \tfrac{2}{3}\beta]$$

$$x_j \ge 0. \quad i=1,2,\dots,m, j=1,2,3\dots,n$$

VII. NUMERICAL EXAMPLE

$$\text{Max } Z_1 = (4,7,2,2,)x_1 + (1,4,6,6)x_2 + (2,8,1,1)x_3$$

$$\text{Max } Z_2 = (2,3,5,5)x_1 + (5,6,7,7)x_2 + (3,4,2,2)x_3$$

subject to

$$(1,4,2,2)x_1 + (1,3,4,4)x_2 + (2,2,6,6) \leq (4,8,6,6)$$

$$(3,5,5,5)x_1 + (2,8,2,2)x_2 + (4,7,1,1)x_3 \leq (5,5,3,3)$$

$$(5,7,5,5)x_1 + (3,6,1,1)x_2 + (5,6,4,4)x_3 \leq (2,4,3,3)$$

$$x_1, x_2, x_3 \ge 0$$

Solution:

a)Maleki Ranking function

$$\text{Max } Z_1 = 11x_1 + 5x_2 + 10x_3$$

$$\text{Max } Z_2 = 5x_1 + 11x_2 + 7x_3$$

subject to

$$5x_1 + 4x_2 + 4x_3 \leq 12$$

$$8x_1 + 10x_2 + 11x_3 \leq 10$$

$$12x_1 + 10x_2 + 11x_3 \leq 6$$

$$x_1, x_2, x_3 \geq 0$$

Then the solution of the linear programming is

$x_1=0.50$, $x_2=0$, $x_3=0$, Max $Z_1=5.50$ and

$x_1=0$, $x_2=0.67$, $x_3=0$, Max $Z_2=7.33$

b) Yager Ranking function:

$$\text{Max } Z_1 = 5.43x_1 + 2.1x_2 + 4.93x_3$$

$$\text{Max } Z_2 = 2.16x_1 + 5.03x_2 + 3.36x_3$$

subject to

$$2.36x_1 + 1.73x_2 + 1.6x_3 \leq 5.6$$

$$3.66x_1 + 4.86x_2 + 5.43x_3 \leq 4.8$$

$$5.66x_1 + 4.43x_2 + 5.23x_3 \leq 2.8$$

$$x_1, x_2, x_3 \geq 0$$

Then the solution of the linear programming is

$x_1=0.5$, $x_2=0$, $x_3=0$, Max $Z_1=2.69$ and

$x_1=0$, $x_2=0.63$, $x_3=0$, Max $Z_2=3.18$.

TABLE I
COMPARATIVE TABLE

Method	X_1	X_2	X_3	X_1	X_2	X_3	MaxZ$_1$	MaxZ$_2$
	I objective			II objective				
Maleki Ranking function	0.5	0	0	0	0.67	0	5.5	7.33
Yager Ranking function	0.5	0	0	0	0.63	0	2.69	3.18

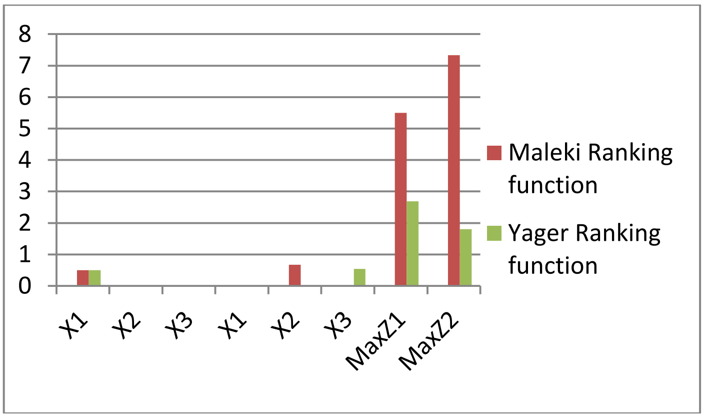

Fig.1 Comparitive illustration between Maleki and Yager Ranking function

III. CONCLUSIONS

The multi objective fully fuzzy linear programming problem is solved using Maleki and Yager linear ranking function. In the above cases we find that the values are more or less similar for all the cases of x_1, x_2 and x_3. On comparison of the maximum values in both the ranking methods it has been observed that there is variation and so Maleki method is better when compared to Yager. This can also be extended to problems of production planning, sales forecasting, inventory management and financial management

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