APPLICATION OF QUEUEING THEORY IN HEALTH SERVICES

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Abstract

Queueing theory is a mathematical study of waiting lines. Queues have become a part of our daily life. We have to wait for hours in queues to get our work done. As such great inefficiencies occur because of waiting. Operation Research is scientific discipline that can improve and plan a variety of waiting lines problems. In this paper, it proposed to study the application of queueing theory in health services. In past ReetuMehandiratta [1] and Samuel Formulation, Jeffery Hermann [2] have attempted to study applications of queueing theory in health sector. In what follows, we propose to enhance the work done so far. This paper is an attempt to analyze the theory (queueing) and instances of use of queueing theory in health care organizations. The paper summarizes a range of queueing theory results in the following areas. Traffic intensity, Average waiting time in queue, Average of time spent in system, Average queue length, Average number of individuals in the system. The paper also considers the waiting of patients in a local health centre as a single channel queueing system with Poisson arrivals and exponential service rate where arrivals are handled on first come first serve basis. Hence the m/m/1 queueing system is proposed.

Key words

Operations Research, Queueing Theory, Waiting Time, Poisson Distribution, Exponential Distribution.

I. INTRODUCTION

Operation research existed as a scientific discipline since 1930's/. It is discipline of applying appropriate analytical methods for decision making. Operation research has been studied in health care settings since 1952. One of the major uses of operational research in health care is in the form of queueing theory. Queues or queueing theory was first analyzed by A.K. EARLANG in 1913 in the context of telephone facilities. The existence of any nation is function of the survival of its citizens and in turn a function of adequate health care programs of its citizenry. Health, no doubt has a great influence on the economy. As a result, the government established hospitals, primary health centres, federal medical centres, university teaching hospitals and so on to improve the health care of India. In spite of all these efforts, it should be noted that there are still some avoidable problems which undetermined success of this sector. One of the most major problems is of waiting lines (queues) found in hospitals. Queueing is very volatile situation which causes unnecessary delay and reduce the efficiency of services. Apart from the time wasted, it also sometimes creates law and order problem. Queues in hospitals often have severe consequences. For instances delay in treatment of patients suffering from asthma, cardiac diseases often lead to complications and eventual death. In light of this, there is need for a critical evaluation of patient waiting time as well as reducing or eliminating it.

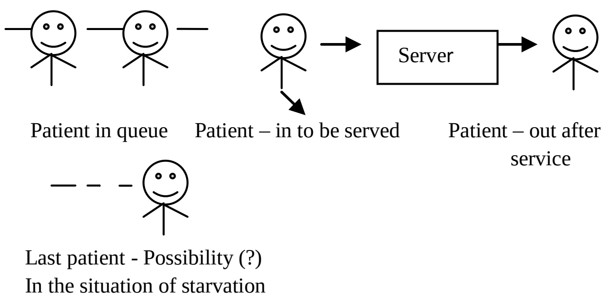
II. DESCRIPTION OF THE MODEL

SINGLE CHANNEL QUEUEING SYSTEM

Consider a single server retrial queueing system (m/m/1) in which customers arrive in a Poisson's process with arrival rate λ. These customers are identified as primary calls. We assume that the services in all phases are independent and identical and only one customer at a time is in the service mechanism.

When a customer enters at a time and the system is free, his/her service time starts at once and when the system is not free, the customer joins the queue. After completion of services, the customer is free from queue, if any and the service facility extends further. If the server is busy then the arriving customer goes to orbit and becomes a source of repeated calls. This pool of source of repeated calls may be viewed as a sort of queue. The time it takes to serve every customer is an exponential random variable with parameter μ.

Figure1: Single Channel Queueing Units.



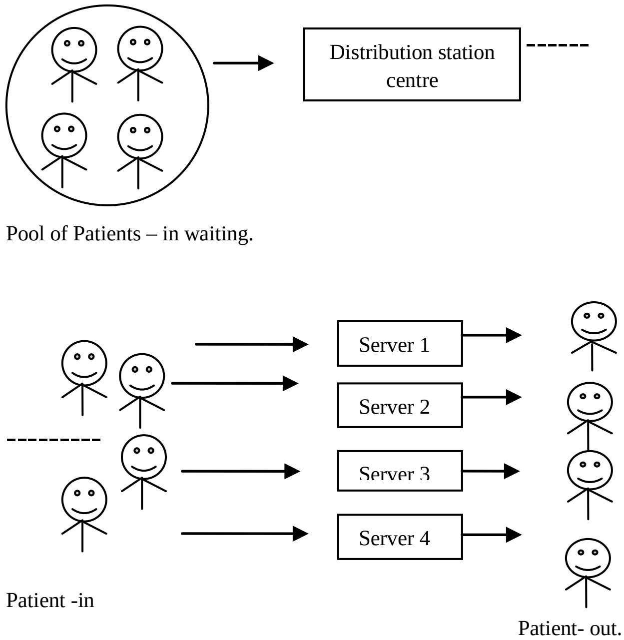
III. MULTICHANNEL QUEUEING SYSTEM

The multichannel queueing model is known in the Kendall's notation as the M/M/m model, where M signifies a Poisson distribution and m is number of parallel service channels in the system. This is commonly used to analyze the queueing problem. This model computes the average wait times and queue lengths, given arrival rate, number of servers and service rates. This particular model applies, in which there are multiple channels served by a single queue as at a bank teller or many airline tickets counters. The outputs of the model are as follows:

1. Expected waiting time per patient in the system (health centre).
2. Expected waiting time of patients in the queue.
3. Expected number of patients in the system (health centre).
4. Expected number of patients in the queue.

The exact calculation of these measures requires knowledge of the probability distribution of the arrival rate and service times. Moreover, successive inter- arrival times and services times are assumed to be statistically independent of each other. In this system, there are multiple servers with all sharing a common waiting line. A waiting line is created when all the servers are busy. As soon as one server becomes free, a customer (patient) is dispatched from the waiting line using the dispatching discipline in force.

Figure (2)—Multi Channel Queueing System.



IV. MODEL ASSUMPTION

This research is based on the following assumptions.

1. The finding obtained after investigation from one unit of the medical health centre should be valid in the other units.
2. The patients are almost well familiar with the organization system of the medical centre.
3. The arrival rate of the patients to queue and service rate are compatible to Poisson distribution or in other words the time interval between two consecutive arrivals and time service both follow exponential distribution.
4. The queueing discipline is such that the first patient goes to the server which is ready for the service.
5. In case of multichannel queueing system, it is assumed that none of the servers are unattended.

V. MODEL PARAMETERS

Traffic Intensity

It is obtained from dividing the average arrival rate λ (in time) to the average service rate μ

$$\rho = \lambda/\mu \qquad \text{-------------------------} \quad (1)$$

Whenever λ is larger, the demand will be more and the system will work harder and queue will be longer. On the contrary, whenever the λ is smaller, the queue will be shorter but in this case, the use of the system will be low. If the arrival time of customer to system were more than service rate λ>μ then ρ> 1, which means the system capacity is less than the arriving demand, therefore the queue length is increased. In this queueing system the average arrival rate is less than the average service rate i.e. λ<μ.

Average Waiting Time in Queue

The average waiting time in queue (before services is rendered) is equal to the average time which a customer waits in the queue for getting service.

Its formula is

$$\frac{\rho}{\mu(1-\rho)} = \frac{\lambda}{\mu(\mu-\lambda)} \qquad \text{------------------------------} \quad (2)$$

Average Time Spent in the System

The average time spent in a system (on queue and receiving service) is equal to the total time that a customer spends in a system which includes the waiting time and service time.

Its formula is

$$\frac{1}{\mu(1-\rho)} = \frac{1}{\mu-\lambda} \qquad \text{-------------------------------} \quad \mathbf{(3)}$$

Average Number of Individual in the System

The average number of individual in the system is equal to the average number of individual who are in the line or server.

It is defined as

$$\frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda} \qquad \text{--------------- -----------------} \quad (4)$$

Average Queue Length

The average queue length is composed of the average number of people who are waiting in the queue.

It is defined as

$$\frac{\lambda^2}{\mu(\mu-\lambda)} \qquad \text{----------------}\mathbf{(5)}$$

VI. ILLUSTRATION

The arrival time as well as the time service began and ended for 110 patients in the local health centre. A total of 23 days were used for the data collection. Here we find, total waiting time = 950 minutes. Total service time =805 minutes. We arrive at the following:-

1. The arrival rate λ= $\frac{\text{Total no. patients}}{\text{Total waiting time}} = \frac{110}{950}$ =0.1157
2. The service rate μ= $\frac{\text{Total no. patients}}{\text{Total service time}} = \frac{110}{805}$ ==0.1366
3. The average time in queue (before service is rendered) $= \frac{\lambda}{\mu(\mu-\lambda)} = \frac{0.1157}{0.1366(0.1366-0.1157)}$
4. The average time spent in the system $= \frac{1}{\mu-\lambda} = \frac{1}{0.1366-0.1157}$ = 47.84 minutes
5. Average number of individual in the system $= \frac{\mu}{\mu-\lambda} = \frac{0.1366}{0.1366-0.1157} = 5.53 \equiv 6$
6. Traffic Intensity = $\frac{\lambda}{\mu}$ = 0.8469
7. The probability of not queueing on the arrival = 1 – ρ = 0.16

VII. CONCLUSION

The Queueing Theory is an effective mathematical technique for the solving many critical problems of today's organization as well as society. Its application cover a wide range today's real life problems that include the health services in the countries like India. The poverty level and therefore health problems of Indian citizens have been crucial problems for the government at every level of the country.

In course of study it was observed that in the governmental as well as private health centres patients have to spare tremendous waiting time, sometimes leading to even death of some emergency patients or unnecessary increased medical complications. The hospitals are over- crowded without-door patients and to tackle all these patients there is mostly single- channel queueing units.

We have concentrated with the objective of reducing the traffic intensity as well as waiting time by dispersing the patients to multi- channel queueing units.

REFERENCES

1. *Mehandiratta,R.,2011, ''Application of Queueing Theory In Health Care.'' International Journal of Computing and Business Research 2 (2):2229-6166.*
2. *Fomundam,S.,Herrmann,J.W.,2007."A Survey of Queueing Theory Applications in Healthcare." ISR Technical Report 2007-24.*
3. *Adeleke, R.A., O.D.Ogunwale, O.Y.Halid.2009. "Application of Queueing Theory to Waiting Time of Out-Patients in Hospitals." Pacific Journal of Science and Technology.10(2):270-274.*
4. *Tabari, M.,Y.Gholipour and M.Seifi.Divkolaii, 2012. Application of the Queueing theory to Human Resource Management, 1211-1218.*
5. *Irfan, A., A.Bashir and A. Shakeel, 2007. Performance analysis of networks of queues under active queue management scheme.15:416-425.*
6. *Tabari,M., M.Seifi-Divikolaii and Y.Gholipour-Kanani,2012. Human Resource Management with using of Analyses Queue Approach. Indian Journal of Education and Information Management,1:79-87.*
7. *Applicatuonsof Queueing Theory in Health Care.*

