BIPOLAR FUZZY GRAPHS

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Abstract

In this paper, we define some basic concepts of bipolar fuzzy graphs. Some basic properties have been presented.

Key Words : Bipolar, fuzzy graphs, fuzzy graph cut level fuzzy graphs.

In 1994, Zhang initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are an extension of fuzzy sets whose range of membership degree is [-1, 1]. In bipolar fuzzy set, membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree [0,1] of an element indicates that the element somewhat satisfies the property, and the membership degree [-1,0] of an element indicates the element somewhat satisfies the implicit counter property.

Let X be nonempty set. A bipolar fuzzy set B on X is an object having the form $B=\{(x,\mu^{+}(x), \mu^{-}(x))|x\in X\}$, where $\mu^{+}: X\to[0,1]$ and $\mu^{-}:X \to[-1,0]$ are mappings.

If $\mu^{+}(x) \neq 0$ and $\mu^{-}(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for B. If $\mu^{+}(x) = 0$ and $\mu^{-}(x) \neq 0$, it is the situation that x does not satisfy the property of B but somewhat satisfies the counter property of B. It is possible for an element x to be such that $\mu^{+}(x) \neq 0$ and $\mu^{-}(x) \neq 0$ when membership function of the property overlaps that of its counter property over some portion of X. For the sake of simplicity, we shall use the symbol $B = (\mu^{+}, \mu^{-})$ for the bipolar fuzzy set $B = \{(x, \mu^{+}(x), \mu^{-}(x))|x\in X\}$.

Height [8] of a bipolar fuzzy set $B = \{(x, \mu^{+}(x), \mu^{-}(x))|x\in X\}$ of a nonempty set X is denoted by h(B) and defined as $h(B) = \max\{ \mu^{+}(x)|\ x\in X\}$. Depth [8] of a bipolar fuzzy set B of a nonempty set X is denoted by d(B) and defined as $d(B) = \min\{ \mu^{-}(x)|x\in X\}$. Let $= B_1\{(x, \mu^{+}_1(x), \mu^{-}_1 (x))|x\in X\}$ and $B_2\{(x, \mu^{+}_2(x), \mu^{-}_2(x))|x\in X\}$ be two bipolar fuzzy sets in X . $B_1 \subseteq B_2$ if $\mu^{+}_1(x) \leq \mu^{-}_2(x)$ for all $x \in X$ and $\mu^{-}_1(x) \geq \mu^{-}_2(x)$ for all $x \in X$. The support [8] of B is denoted by supp(B) and defined by supp(B) =

$\{x|\mu^+(x) \neq 0$ or $\mu^-(x) \neq 0\}$. The upper core [8] of B is denoted by $\bar{c}(B)$ and defined by $\bar{c}(B)\{x \mid \mu^-(x)=1\}$. Similarly, the lower core [8] of B is denoted by $\underline{c}(B)$ and defined by $\underline{c}(B)\{x \mid \mu^-(x)=-1\}$. Let $t_1 \in (0,1]$, $t_2 \in (-1,0]$ and B = (μ^+, μ^-) be a bipolar fuzzy set. $\{t_1, t_2\}$ cut level set [8] of B to be the crisp set $B_{t_2}^{t_1}\{x \in \sup p(B) \mid \mu^+(x) \geq t_1 \ and \ \mu^-(x) \leq t_2\}$.

For every two bipolar fuzzy sets A = (μ_A^+, μ_A^-) and B = (μ_B^+, μ_B^-) on X,

$$(A \cap B)(x) = (\min(\mu_A^+(x), \mu_B^+(x)), \max(\mu_A^-(x), \mu_B^-(x)).$$

$$(A \cup B)(x) = (\max(\mu_A^+(x), \mu_B^+(x)), \min(\mu_A^-(x), \mu_B^-(x)).$$

Definition 2 : A bipolar fuzzy graph H = (X, ξ) is simple if ξ has no repeated bipolar fuzzy edges and whenever A, B$\in\xi$ and A$\subseteq$B, then A = B.

Definition 3 : A bipolar fuzzy graph H = (X, ξ) is support simple if whenever A, B$\in\xi$ and A$\subseteq$B and supp(A) = supp(B), then A = B.

Definition 4 Let H_1 = (X_1, ξ_1) and H_1 = (X_2, ξ_2) be two bipolar fuzzy graphs. H_1 is called partial bipolar fuzzy graph of H_2 if $\xi_1 \subseteq \xi_2$.

Example 2 Let = $\{x_1, x_2, x_3, x_4, x_5\}$ be a finite set and = $\{B_1, B_2, B_3, B_4\}$ be the bipolar fuzzy set on subsets of X. Here B_1={(x_1,0.4,-0.3), (x_2,0.6,-0.2), (x_3,0.7,-0.4)}, B_2={(x_3,0.6,-0.5), (x_4,0.4,-0.7)}, B_3={(x_3,0.9,-0.6), (x_5,0.4,-0.2)}, B_4={(x_4,0.8,-0.7), (x_5,0.4,-0.1)}. The graph (X, ξ) is a simple and support simple bipolar fuzzy graph shown in Figure 2.

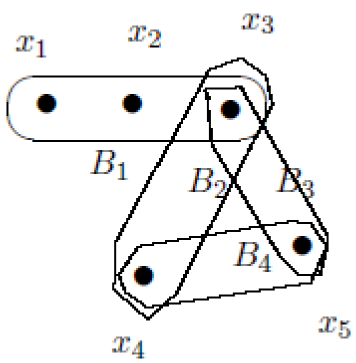

Figure 2: Example of simple and support simple bipolar fuzzy graph.

Definition 5 : Let X = $\{x_1, x_2, \ldots, x_n\}$ be a non empty finite set and B=$\{B_1, B_2, \ldots, B_k\}$ be bipolar sets of subsets of X. (α, β) - cut of bipolar fuzzy graph, H = (X, B), denoted by $H_{(\alpha,\beta)}$ is an ordered pair $H_{(\alpha,\beta)} = (X_{(\alpha,\beta)}, \xi_{(\alpha,\beta)})$where:

(1) $X_{(\alpha,\beta)} = X$

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(2) $\xi_{(\alpha,\beta)}=\{B_{j,(\alpha,\beta)} | B_{j,(\alpha,\beta)}=\{x_i \in B_j | \mu^+(x_i)\geq\alpha, \mu^-(x_i)\leq\beta\}$, i = 1,2,…,n, j=1,2,…,k}

(3) $B_{k+1(\alpha,\beta)} = \{x_i \notin Bj$, i=1,2,…n, j = 1,2,…,k}

(α, β) - cut of bipolar fuzzy graph is a crisp graph.

Definition 6 Let H = (X, ξ) be a bipolar fuzzy graph, and for $0 < \alpha \leq h(H)$, $d(H) \leq \beta < 0$, let $h_{(\alpha,\beta)} - (X_{(\alpha,\beta)}, \xi_{(\alpha,\beta)})$-level hypergraph of H. The sequence of real numbers $\{s_k, s_{k-1},\ldots,s_1,r_1,r_2,\ldots,r_n\}$ such that $d(H) = s_k < s_{k-1} < \ldots \leq s_1 < 0 < r_1 < r_2 < \ldots < r_n = h(H)$ which satisfies the following properties

(1) If $s_{i+1} \leq l < s_1$, $r_1 < k \leq r_{i+1}$, then $B_{(k,l)} = B_{(r_{i+1},s_{i+1})}$,

(2) $B_{(k,l)} = \hat{\phi} B_{(r_i,s_i)}$,

For a graph H, let fundamental sequence be $F(H) = \{s_k, s_{k-1},\ldots,s_1,r_1,r_2,\ldots,r_n\}$where $k \leq n$ be two positive integers. The core set of H is denoted by C(H) and defined by $C(H) = \{H_{(r_1,s_1)}, H_{(r_2,s_2)},\ldots, H_{(r_k,s_k)}\}$.

We now define dual bipolar fuzzy graph as follows.

Definition 7 Let H = (X, B) be a bipolar fuzzy graph where $X = \{x_1, x_2,\ldots,x_n\}$be a finite set and $B = \{B_1, B_2,\ldots,B_n\}$ be a bipolar fuzzy sets on subsets of X. The bipolar fuzzy graph $\overline{H} = (\overline{B}, \overline{X})$ is called dual bipolar fuzzy graph of H if

(1) $\overline{B} = \{b_1, b_2,\ldots b_n\}$ is set of vertices of $\overline{H}$ corresponding to $B_1, B_2,\ldots,B_n$ respectively.

(2) $\overline{X} = \{\overline{x}_1, \overline{x}_2,\ldots \overline{x}_n\}$ where $\overline{x}_j = \{(b_j, \mu_j^+(b_j), \mu_j^-(b_j)) | \mu_i^+(b_j) = \mu_j^+(x_i), \mu_i^-(b_j) = \mu_j^-(x_i)\}$

Definition 8 A bipolar fuzzy set B = (μ^+, μ^-) is called elementary bipolar fuzzy set if $\mu^+ : X \rightarrow [0,1]$, $\mu^- : X \rightarrow [0,1]$ are constant functions.

Definition 9 A bipolar fuzzy graph is called elementary bipolar fuzzy graph if all bipolar fuzzy edges are elementary.

Definition 10 Let H = (X, ξ) be a bipolar fuzzy graph and $C(H) = \{H_{(r_1,s_1)}, H_{(r_2,s_2)},\ldots, H_{(r_k,s_k)}\}$. H is said to be ordered if C(H) is ordered. The bipolar fuzzy graph is simply ordered if C(H) is simply ordered.

Definition 11 A bipolar fuzzy graph H = (X, ξ) is called $\{m^+, m^-\}$ tempered bipolar fuzzy graph of a crisp graph $H^* = (X, E)$ if there exists a bipolar fuzzy set $B = (m^+,m^-)$ such that $\xi = \{(\gamma_{E_i}^+, \gamma_{E_i}^-) | E_i \in E\}$ where

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$$\gamma_{E_i}^{+}(x) = \begin{cases} \min\{m^{+}(e) \mid e \in E_i\} & \text{if } x \in E_i \\ 0, & \text{if otherwise} \end{cases}$$

And

$$\gamma_{E_i}^{-}(x) = \begin{cases} \max\{m^{-}(e) \mid e \in E_i\} & \text{if } x \in E_i \\ 0, & \text{if otherwise} \end{cases}$$

Theorem 1 A bipolar fuzzy graph H = (X, ξ) is a $\{m^+, m^-\}$ tempered bipolar fuzzy graph of some crisp graph H^* then H is elementary, support simple and simply ordered.

Proof. Let H = (X, ξ) is a $\{m^+, m^-\}$ tempered bipolar fuzzy graph of some crisp graph H^*. As it is $\{m^+, m^-\}$ tempered, the positive membership values and negative membership values of bipolar fuzzy edges of H are constant. Hence it is elementary. Clearly if support of two bipolar fuzzy edges of the $\{m^+, m^-\}$ tempered bipolar fuzzy graph are equal then the bipolar fuzzy edges are equal. Hence it support simple. Let $C(H) = \{H_{(r_1,s_1)}, H_{(r_2,s_2)}, \ldots, H_{(r_k,s_k)}\}$ since H is elementary, it is ordered. Now we are to show that it is simple. Let $E \in H_{(r_{i+1},s_{i+1})} \setminus H_{(r_i,s_i)}$ then there exists $x^* \in E$ such that $\mu^+(x^*) = r_{i+1}$ and $\mu^-(x^*) = s_{i+1}$. Since $r_{i+1} < r_i$, $s_{i+1} > s_i$, it follows that $x^* \notin X_{(r_i,s_i)}$ and $E \not\subset X_{(r_i,s_i)}$. Hence H is simply ordered.

Bipolar fuzzy transversal of bipolar fuzzy graphs is defined below.

Definition 12 Let H = (X,ξ) be a bipolar fuzzy graph. A bipolar fuzzy transversal T = (τ^+, τ^-) of H is a bipolar fuzzy set defined on X with the property that $T_{(h(B),d(B))} \cap B_{(h(B),d(B))} \neq \phi$ for each $B \in \xi$. A minimal bipolar fuzzy transversal T for H is a bipolar fuzzy transversal of H with the property that if $T_1 \subset T$, then T_1 is not a bipolar fuzzy transversal of H.

We denote set of minimal bipolar fuzzy transversal as Tr(H). From the definition, it can be verified that Tr(H) ≠ ø.

1.8 Definition : Let G be an M-graph and A be M-fuzzy subgraph of G. Let Im $(\mu_A) = \{\alpha_i : \mu_A(x) = \alpha_i$ for every $x \in G\}$ and Im (νA) = $\{\beta_i : \mu_A(x) = \beta_i$ for every $x \in G\}$. Then $\{A_{<\alpha i,\beta i>}\}$ are the only level M-subgraph of A.

3.2 Theorem

Any M-sub-bipolar H of an M-bipolar G can be realized as a bi-level M-sub-bipolar of M-fuzzy sub-bipolar of G.

Proof

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Let G = ($G_1 \in G_2$, +, •) be an M-bipolar.

Let H = ($H_1 \in H_2$, +, •) be an M-sub-bipolar of G.

Define $\mu_{A1} : H_1 \to [0,1]$ and $\nu_{A1} : H_1 \to [0,1]$ by

$$\mu_{A1}(x) = \begin{cases} \alpha & for\, x \in H_1 \\ 0 & for\, x \notin H_1 \end{cases} \qquad \nu_{A1}(x) = \begin{cases} 0 & for\, x \in H_1 \\ \beta & for\, x \notin H_1, and\, define \end{cases}$$

$\mu_{A2} : H_2 \to [0 ,1]$ and $\nu_{A2} : H_2 \to [0 ,1]$ by

$$\mu_{A2}(x) = \begin{cases} \alpha & for\, x \in H_2 \\ 0 & for\, x \notin H_2 \end{cases} \qquad \nu_{A2}(x) = \begin{cases} 0 & for\, x \in H_2 \\ \beta & for\, x \notin H_2 \end{cases}$$

where $\alpha \in [0, \min\{\mu_{A1}(e_1), \mu_{A2}(e_2)\}]$ and $\beta \in [\max\{\nu_{A1}(e_1), \nu_{A2}(e_2)\}, 1]$.

Let x, y $\in$ G.

Suppose x, y $\in$ H, then

i. $x, y \in H_1 \Rightarrow x + y \in H_1$

$\mu_{A1}(x + y) = \alpha$, $\mu_{A1}(x) = \alpha$, $\mu_{A1}(y) = \alpha$ and

$\nu_{A1}(x + y) = 0$, $\nu_{A1}(x) = 0$, $\nu_{A1}(y) = 0$ then

$\mu_{A1}(x +y) \geq \min\{\mu_{A1}(x), \mu_{A1}(y)\}$

$\nu_{A1}(x + y) \leq \max\{\nu_{A1}(x), \nu_{A1}(y)\}$.

ii. $x, y \in H_2 \Rightarrow xy \in H_2$

$\mu_{A2}(xy) = \alpha$, $\mu A2(x) = \alpha$, $\mu A2(y) = \alpha$ and

$\nu_{A2}(xy) = 0$, $\nu_{A2}(x) = 0$, $\nu_{A2}(y) = 0$ then

$\mu_{A2}(xy) \geq \min\{\mu_{A2}(x), \mu_{A2}(y)\}$

$\nu_{A2}(xy) \leq \max\{\nu_{A2}(x), \nu_{A2}(y)\}$.

iii. $x \in H_1$ and $y \notin H_1 \Rightarrow x + y \notin H_1$

$\mu_{A1}(x + y) = 0$, $\mu_{A1}(x) = \alpha$, $\mu_{A1}(y) = 0$ and

$\nu_{A1}(x + y) = \beta$, $\nu_{A1}(x) = 0$, $\nu_{A1}(y) = \beta$ then

$\mu_{A1}(x +y) \geq \min\{\mu_{A1}(x), \mu_{A1}(y)\}$

$\nu_{A1}(x + y) \leq \max\{\nu_{A1}(x), \nu_{A1}(y)\}$.

iv. $x \in H_2$ and $y \notin H_2 \Rightarrow x\,y \notin H_2$

$\mu_{A2}(xy) = 0$, $\mu_{A2}(x) = \alpha$, $\mu_{A2}(y) = 0$ and

$\nu_{A2}(xy) = \beta$, $\nu_{A2}(x) = 0$, $\nu_{A2}(y) = \beta$ then

$\mu_{A2}(xy) \geq \min\{\mu_{A2}(x), \mu_{A2}(y)\}$

$\nu_{A2}(xy) \leq \max\{\nu_{A2}(x), \nu_{A2}(y)\}$.

Suppose x, y $\notin$ H, then

i. $x, y \notin H_1$, then $x + y \in H_1$ or $x + y \notin H_1$

μ_{A1} (x + y) = α or 0, μ_{A1} (x) = 0, μ_{A1} (y) = 0 and

ν_{A1} (x + y) = 0 or β, ν_{A1} (x) = β, ν_{A1}(y) = β, then

$\mu_{A1}(x + y) \geq \min\{\mu_{A1}(x), \mu_{A1}(y)\}$

$\nu_{A1}(x + y) \leq \max\{\nu_{A1}(x), \nu_{A1}(y)\}$.

ii. x, y $\notin H_2 \Rightarrow xy \in H_2$ or $xy \notin H_2$

μ_{A2} (xy) = α or 0, μ_{A2} (x) = 0, μ_{A2} (y) = 0 and

ν_{A2} (xy) = 0 or β, ν_{A2} (x) = β, ν_{A2} (y) = β, then

$\mu_{A2}(xy) \geq \min\{\mu_{A2}(x), \mu_{A2}(y)\}$

$\nu_{A2}(xy) \leq \max\{\nu_{A2}(x), \nu_{A2}(y)\}$.

Thus in all cases,

(A_1, +) is of G_1 and (A_2, •) is of G_2.

Clearly A = ($A_1 \cup A_2$, +, •) is fuzzy sub-bipolar of G,

Now, we have to prove that A is M-fuzzy sub-bigraph of G.

Suppose, $m \in M$ and $x \in H_1$, then $m + x \in H_1$.

Then, μ_{A1} (m + x) = α, μ_{A1} (x) = α, and

ν_{A1} (m + x) = 0, ν_{A1} (x) = 0, then

$\mu_{A1}(m + x) \geq \mu_{A1}(x)$,

$\nu_{A1}(m + x) \leq \nu_{A1}(x)$.

Suppose, $m \in M$ and $x \notin H_1$, then $m + x \in H_1$ or $m + x \notin H_1$.

Then, μ_{A1} (m + x) = α or 0, μ_{A1} (x) = 0, and

ν_{A1} (m + x) = 0 or β, ν_{A1} (x) = β, then

$\mu_{A1}(m + x) \geq \mu_{A1}(x)$,

$\nu_{A1}(m + x) \leq \nu_{A1}(x)$.

Clearly (A_1, +) is M-fuzzy sub-bipolar of G_1.

Suppose, $m \in M$ and $x \in H_2$, then $m + x \in H_2$.

Then, μ_{A2} (mx) = α, μ_{A2} (x) = α, and

ν_{A2} (mx) = 0, ν_{A2} (x) = 0, then

$\mu_{A2}(mx) \geq \mu_{A2}(x)$,

$\nu_{A2}(mx) \leq \nu_{A2}(x)$.

Suppose, $m \in M$ and $x \notin H_2$, then $m + x \in H_2$ or $m + x \notin H_2$.

Then, μ_{A2} (mx) = α or 0, μ_{A2} (x) = 0, and

ν_{A2} (mx) = 0 or β, ν_{A2} (x) = β, then

$\mu_{A2}(mx) \geq \mu_{A2}(x)$,

ν_{A2} (mx) $\leq \nu_{A2}$ (x).

Clearly (A_2, •) is an intuitionistic M-fuzzy sub-bipolar of G_2.

Clearly A = ($A_1 \cup A_2$, +, •) is M-fuzzy sub-bipolar of G, where

$\mu_A : G \rightarrow [0,1]$ and $\nu_A : G \rightarrow [0,1]$ are given by

$$\mu_A(x) = \begin{cases} \alpha & for\, x \in H \\ 0 & for\, x \notin H \end{cases} \qquad \nu_A(x) = \begin{cases} 0 & for\, x \in H \\ \beta & for\, x \notin H \end{cases}$$

For this M-fuzzy sub-bipolar, $A_{<\alpha,\beta>} = A_{1<\alpha,\beta>} \cup A_{2<\alpha,\beta>} = H$.

3.3 Theorem

Let G be an M-bipolar and A be M-fuzzy sub-bipolar of G. Two bilevel M-sub-bipolar $A_{<\alpha,\beta>}$, $A_{<\gamma,\delta>}$ with $\alpha < \gamma$ and $\delta < \beta$ of A are equal iff there is no x $\in$ G such that $\alpha \leq \mu_A(x) < \gamma$ and $\delta < \nu_A(x) \leq \beta$.

Theorem 1 Let G be a bipolar fuzzy graph where induced crisp graph G′ is an even cycle. Then G is bipolar fuzzy graph if and only if either m_2^+ and m_2^- and are constant functions or alternate edges have same positive membership values and negative membership values.

Proof. Let G = (A, B) be a regular bipolar fuzzy graph where A = (m^+_1, m^-_1) and A = (B^+_2, m^-_2) be two bipolar fuzzy sets on a non-empty finite set V and E$\subseteq$V×V respectively and underlying crisp graph G′ of G be an even cycle. If either m^+_2, m^-_2 are constant functions or alternate edges have same positive and negative membership values, then G is a bipolar fuzzy graph. Conversely, suppose G is a (k_1, k_2) bipolar fuzzy graph. Let n $e_1, e_2, \ldots, e_n$ be the edges of G′ in order. As in the theorem 3,

$$m_2^+(e_i) = \begin{cases} c_1, & if\, i\, is\, odd, \\ k_1 - c_1, & if\, i\, is\, even \end{cases}$$

$$m_2^-(e_i) = \begin{cases} c_2, & if\, i\, is\, odd, \\ k_2 - c_2, & if\, i\, is\, even \end{cases}$$

If $c_1 = k_1 - c_1$, then m^+_2 is constant. If $c_1 \neq k_1 - c_1$, then alternate edges have same positive and negative membership values. Similarly for m^-_2. Hence the results.

Theorem 2 The size of a (k_1, k_2) bipolar fuzzy graph is $\left(\frac{pk_1}{2}, \frac{pk_2}{2}\right)$ where p =|V|.

Proof. Let G = (A, B) be a bipolar fuzzy graph where A = (m^+_1, m^-_1) and (m^+_2, m^-_2) be two bipolar fuzzy sets on a non-empty finite set V and E$\subseteq$V×V respectively. The size of G is $S(G) = \left(\sum_{u \neq v} m_2^+(u,v), \sum_{u \neq v} m_2^-(u,v)\right)$. We have

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$\sum_{v\in V} d(v) = 2\left[\sum_{(u,v)\in E} m_2^+(u,v), \sum_{(u,v)\in E} m_2^-(u,v)\right] = 2S(G).$ So $2S(G) = \sum_{v\in V} d(v)$. i.e.

$2S(G) = \left(\sum_{v\in V} k_1, \sum_{v\in V} k_2\right).$

This gives 2S(G) = (pk_1, pk_2). Hence the result.

Theorem 3 If G is (k, k′) bipolar fuzzy graph, then

2S(G) +O(G) = (pk, pk′) where p =|V|.

Proof. Let G=(A,B) be a bipolar fuzzy graph where A = (m^+_1, m^-_1) and B = (m^+_2, m^-_2) be two bipolar fuzzy sets on a non-empty finite set V and V ×V respectively. Since G is a (k, k′) -totally regular fuzzy graph. So k = $td^+(v)$ = $d^+(v)$ + $m^+_1(v)$ and k′ = $td^-(v)$ = $d^-(v)$ + $m^-_1(v)$ for all v∈V. Therefore $\sum_{v\in V} k = \sum_{v\in V} d^+(v) + \sum_{v\in V} m_1^+(v)$ and $\sum_{v\in V} k' = \sum_{v\in V} d^-(v) + \sum_{v\in V} m_1^-(v)$. pk = $2S^+(G)$ and pk = $2S^-(G)$. So pk + pk′ = 2($S^+(G)$ + $S^-(G)$) + $O^+(G)$ + $O^-(G)$. Hence 2S(G) + O(G) = (pk, pk′).

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