New modification of first integral method

Mohamed.A.Abdoon[#1]

Math.Dept. Sudan University of science and technology (www.sust.com)

khourtom -sudan

moh.abdoon@gmail.com

Abstract: In this paper, the modified first integral method is use to find the actual solution of many nonlinear equation in simple way ,and anew technical to solving nonlinear partial differential equation.

keyword: first integral method, Exact solution , modified nonlinear equation

Introduction:

Many methods obtaining the exact solution of non linear equation ,some of the techniques are the bilinear transformation [1] ,the sine cosine method [2],F-expansion method [3], the first integral method was first proposed by Feng[4] to solving Burger-Korteweg- devries equation and so on, in this paper investigation a traveling wave solution for non linear partial differential equation ,study nonlinear phenomena ,in solving modified kdv-kp can be based on the theory of commutative algebra ,using the first integral method technique to solving modified kdv-kp equation .

First integral method:

The non linear partial differential equation form:

$$w(F,F_x,F_t,F_{xx},F_{xt},..........) \qquad (1)$$

Where $u(x,t)$ is the solution of (1) we use the transforms :

$$f(x,t)=f(\zeta),\zeta=\alpha x-\beta t \qquad (2)$$

we use the wave transforms :

$$\frac{\partial}{\partial t}(.)=-\beta\frac{\partial}{\partial\zeta}(.),\frac{\partial}{\partial x}(.)=\alpha\frac{\partial}{\partial\zeta}(.),\frac{\partial^2}{\partial t^2}(.)=\beta^2\frac{\partial^2}{\partial\zeta^2}(.),$$
$$\frac{\partial^2}{\partial x^2}(.)=\alpha^2\frac{\partial^2}{\partial\zeta^2}(.) \qquad (3)$$

The Eq (1) transforms the ordinary differential equations we obtain :

$$p(f,f,f,..............)=0 \qquad (4)$$

Anew independent variable:

$$x(\zeta)=f(\zeta),y(\zeta)=f_\zeta(\zeta) \qquad (5)$$

The system of ordinary differential equations:

$$x'(\zeta)=y(\zeta)$$
$$y'(\zeta)=F(x(\zeta),y(\zeta)) \qquad (6)$$

By the qualitative theory of differential equation [6],we find the integral of (6) under same condition , then the general solution of (6) can be obtained directly . However ,in general ,it is really difficult for us to realize this even for one first integral , because for a given plane autonomous system ,find its first integral will apply the Division theory to option first integral (6) , An exact solution of (1) obtained by solving this equation , Now let us recall the Division theory.

Division theorem:

Suppose that $P(x,y)$ and $Q(x,y)$ are polynomials of two variables x and y in $C[x,y]$. and $P(x,y)$ is irreducible in $C[x,y]$. if $Q(x,y)$ vanishes at all points of $P(x,y)$, then there exists a polynomial $G(X,Y)$ in $C[x,y]$ such that $Q(X,Y)=P(X,Y)G(X,Y)$.

where M is a positive integer, $1\le k\le M$, let

$u\to M$

$u^n\to nM$

$u'\to M+1$

$u''\to M+2$

.

$u^r\to M+r$

to determine the parameter M, we then collect all coefficients of powers of Y in the resulting equation where these coefficients have to vanish .

Having determined these parameters we obtain an analytic solution u(x,t) in a closed form .

this method may give periodic solution as well.

1. the Gardner eguation:

The standard Gardner equation ,or the combined kdv-mkdv equation , reads:

$$u_t+2auu_x-3bu^2u_x+u_{xxx}=0,a,b\succ 0 \qquad (7)$$

Using the wave variable $\zeta=x-ct$ and integrating the result will convert to the ODE:

$$-cu+au^2-bu^3+u''=0 \qquad (8)$$

using (5) we get :

$$X'=Y$$
$$Y'=bX^3-aX^2+cX \qquad (9)$$

According to the first integral method, we suppose that X and Y are nontrivial solution of (9) and

$$q(X,Y)=\sum_{i=o}^{M}a_i(X)Y^i=0 \qquad (10)$$

Balancing u^3 with u'' gives

3M=M+2

$$M=1 \qquad (11)$$

using (11) in (10) we get :

$$q=a_0+a_1Y=0 \qquad (12)$$

using Division Theorem, there exists a polynomial $g(x)+h(x)y$ in the complex domain c[X,Y] such that:

$$\frac{\partial q}{\partial \zeta}=\frac{\partial q}{\partial X}\frac{\partial x}{\partial \zeta}+\frac{\partial q}{\partial Y}\frac{\partial Y}{\partial \zeta}= \quad (13)$$

$$(g(x)+h(x)Y)\sum_{i=0}^{M}a_i(x)Y^i$$

using (12) in (13) and coefficients of Y on both sides we get:

$$a_1(x)=1 \quad (14)$$

$$a'_0=g(x) \quad (15)$$

$$[bx^3-ax^2+cx]=a_0(x)g(x) \quad (16)$$

since $a_1(x)=1$,chooseh(x)=0, $a_0(x)=A_2X^2+A_1X+A_0$ and $g(x)=2A_2X+A_1$ using in (14)-(16) we get:

$$A_0=0, A_1=\pm\sqrt{c}, A_2=\frac{b}{2}, c=\frac{4a^2}{9b^2} \quad (17)$$

using (17) in (12), we obtain:

$$Y=A_1X-A_2X^2 \quad (18)$$

combining (18) with (6),we obtain the exact solution to (7) and then the exact solution to the Gardner equation can be written as :

$$u_1(x,t)=\frac{-4a}{3b^2}[1\pm\frac{1}{2}\tanh[\frac{a}{3b}(\zeta+\zeta_0)] \quad (19)$$

$$u_2(x,t)=\frac{-4a}{3b^2}[1\pm\frac{1}{2}\coth[\frac{a}{3b}(\zeta+\zeta_0)] \quad (20)$$

where ζ_0 is integration constant. Thus the travelling wave solution to the Gardner equation can be written as :

$$u_1(x,t)=\frac{-4a}{3b^2}[1\pm\frac{1}{2}\tanh[\frac{a}{3b}(x-\frac{4a^2}{9b^2}t+\zeta_0)] \quad (21)$$

$$u_2(x,t)=\frac{-4a}{3b^2}[1\pm\frac{1}{2}\coth[\frac{a}{3b}(x-\frac{4a^2}{9b^2}+\zeta_0)] \quad (22)$$

2. Exact solution to the kdv system:

The kdv system given by :

$$u_t-u_{xxx}-2vu_x-uv_x=0$$

$$v_t-uu_x=0 \quad (23)$$

Use the wave transformation :

$$u(x,t)=f(\zeta), v(x,t)=g(x), \zeta=x-ct \quad (24)$$

Where k,l and λ are constants and $f(\zeta)$ is real function ,Substituting (24)in(1)we get:

$$-cf'-f''-2gf'-fg'=0 \quad (25)$$

$$-cg'-ff'=0 \quad (26)$$

integration(10) we can re write :

$$g=\alpha-\frac{1}{2c}f^2 \quad (27)$$

α is an integration constant , Now substitution (27)in (25) gives:

$$(2\alpha+c)f'-\frac{2}{c}f^2f'+f'''=0 \quad (28)$$

Integrating (28)we obtain :

$$f''=\beta-(2\alpha+c)f+\frac{2}{3c}f^3 \quad (29)$$

Where β is an integration constant , Now use new variables $X=f(\zeta)$ and $Y=f'(\zeta)$, Now Eq(29) changes into a system of ordinary differential equation :

$$X'=Y$$

$$Y'=\beta-(2\alpha+c)X+\frac{2}{3c}X^3 \quad (30)$$

Now Appling Division theorem , suppose that $X(\zeta)$ and $Y(\zeta)$ are nontrivial solution of (30):

$$q(x,y)=\sum_{i=0}^{m}a_i(x)y^i=0 \quad (31)$$

Is an irreducible Polynomial in the complex domain $C[X,Y]$ such that:

$$q[X(\zeta),Y(\zeta)]=\sum_{i=0}^{m}a_i(X(\zeta))Y^i(\zeta)=0 \quad (32)$$

$a_i(X)(i=0,1,2........,m)$ are polynomialand $a_m(X)\neq 0$,Eq(32) called first integral method ,there exist a polynomial $g(X)X+h(X)Y$ in the complex domain $C[x,y]$ such that :

$$\frac{dq}{d\zeta}=\frac{dq}{dX}.\frac{dq}{d\zeta}+\frac{dq}{dy}.\frac{dY}{d\zeta}=$$

$$(g(X)X+h(X)Y)\sum_{i=0}^{m}a_i(X)Y^i \quad (33)$$

Balancing u^3 with u'' gives

3M=M+2

$M=1$

by comparing with the coefficient $Y^i(i=1,0)$ on both sides of (33) we have :

$$a'_1=h(X)a_1(X)$$

$$a'_0=g(X)a_1(X)X+h(X)a_0(X) \quad (34)$$

$$a_1\left[\beta-(2\alpha+c)X+\frac{2}{3c}X^3\right]=g(X)a_0(X)X$$

Let $h(X)=0$ then $a_1(X)$ is constant choose $a_1(X)=1$, substitution in (34) then (34)we can write:

$$a_1(X)=1$$
$$a'_0(X)=g(X)X \qquad (35)$$
$$\left[\beta-(2\alpha+c)X+\frac{2}{3c}X^3\right]=a_0(X)g(X)X$$

If assume that :

$$a_0=A_2X^2+A_1X+A_0, g=A \qquad (36)$$

Substituting (36) in (35) we obtain :

Substituting (34)in (29) ,and drive a system of algebraic equations whose solution yield :

$$\beta=0, A_2=\sqrt{\frac{-(2\alpha+c)}{2}}, A_1=0,$$
$$A_0=\frac{4}{3c\sqrt{-(2\alpha+c)}}, A=2A_2 \qquad (37)$$

Setting (37) in (31) we obtain :

$$y+A_2X^2+A_0=0 \qquad (38)$$

Now, by combining (38) and (22) ,solving this equation and consider $X=f(\zeta)$ and $u(x,t)=f(\zeta)$

we get :

$$u(x,t)=\sqrt{\frac{A_0}{A_2}}\tan\left[-\sqrt{A_0A_2}(ct-x+\zeta_0)\right]$$
$$v(x,t)=\alpha-\frac{1}{2c}\left[\sqrt{\frac{A_0}{A_2}}\tan\left[-\sqrt{A_0A_2}(ct-x+\zeta_0)\right]\right]^2 \qquad (39)$$

ζ_0 is arbitrary constant .

Exat solution to the 2D-BKdV equation:

$$(u_t+\alpha uu_x+\beta u_{xx}+su_{xxx})_x+\gamma u_{yy}=0 \qquad (40)$$

where α,β,s and γ are real constants. assume that :

$$u(x,y,t)=u(\xi), \xi=hx+ly-wt \qquad (41)$$

where h,l,w are real constants. substitution of (41)in(40) yields :

$$-whu_{\xi\xi}+\alpha h^2(uu_\xi)_\xi+\beta h^3u_{\xi\xi\xi}+sh^4u_{\xi\xi\xi\xi}$$
$$+\gamma l^2u_{\xi\xi}=0 \qquad (42)$$

integration (42) twice with respect to ζ , then we have :

$$sh^4u_{\xi\xi}+\beta h^3u_\xi+\frac{\alpha}{2}h^2u^2+\gamma l^2u-whu=R \qquad (43)$$

where R is the second integration constant and the first one is take to zero,

$$u''(\xi)-ru'(\xi)-\alpha u^2(\xi)-bu(\xi)-d=0 \qquad (44)$$

where $r=\frac{\beta}{sh}, a=-\frac{\alpha}{2sh^2}, b=\frac{wh-\gamma l^2}{sh^4}$ and $d=\frac{R}{sh^4}$.

Now (44) changes into a system of ordinary differential equation :

$$X'=Y$$
$$Y'=ry+\alpha X^2+bX+d \qquad (45)$$

Now Appling Division theorem , suppose that $X(\zeta)$ and $Y(\zeta)$ are nontrivial solution of (45):

$$q(x,y)=\sum_{i=0}^{m}a_i(x)y^i=0 \qquad (46)$$

Is an irreducible Polynomial in the complex domain $C[X,Y]$ such that:

$$q[X(\zeta),Y(\zeta)]=\sum_{i=0}^{m}a_i(X(\zeta))Y^i(\zeta)=0 \qquad (47)$$

$a_i(X)(i=0,1,2..........,m)$ are polynomial and $a_m(X)\neq 0$,Eq(47) called first integral method ,there exist a polynomial $g(X)X+h(X)Y$ in the complex domain $C[x,y]$ such that :

$$\frac{dq}{d\zeta}=\frac{dq}{dX}.\frac{dq}{d\zeta}+\frac{dq}{dy}.\frac{dY}{d\zeta}=$$
$$(g(X)X+h(X)Y)\sum_{i=0}^{m}a_i(X)Y^i \qquad (48)$$

Balancing u^3 with u'' gives

2M=M+2

M=2

by comparing with the coefficient $Y^i(i=0,1,2)$ on both sides of (33) we have :

$$a_2'=\beta a_2$$
$$a_1'+2ra_2=\alpha a_2+\beta a_1$$
$$a_0'+ra_1+2a_2(ax^2+bx+d)=\alpha a_1+a_0\beta \qquad (49)$$
$$a_1[ax^2+bx+d]=\alpha a_0$$

Let $\beta=0$ then $a_2(X)$ is constant choose $a_2(X)=1$, substitution in (49) then (49)we can write:

$$a_2=1$$
$$a_1'+2r=\alpha$$
$$a_0'+ra_1+2(ax^2+bx+d)=\alpha a_1 \qquad (50)$$
$$a_1[ax^2+bx+d]=\alpha a_0$$

If assume that :

$$a_1=A_1X+A_0 \qquad (51)$$

Substituting (51) in (50) we obtain :

$$a_0=\frac{-2a}{3}x^3-bx^2+\frac{A_1(A_1+r)}{2}x^2-2dx+$$
$$A_0(A_1+r)x+D \qquad (52)$$

D is arbitrary integration constant .

$$A_1 = \frac{-4r}{5}, A_0 = -\frac{12r^3}{125a} - \frac{2br}{5a},$$

$$d = \frac{b^2}{4a} - \frac{9r^4}{625a}, \quad (53)$$

$$D = \frac{25}{48}(\frac{6r^2}{25} - b)(\frac{12r^2}{125a} + \frac{2b}{5a})^2$$

we assume that :

$$kb = \frac{6r^2}{25}, k \in R \text{ and } k \neq 0 \quad (54)$$

Substituting (54)in (53)in(47) ,and drive a system of algebraic equations whose solution yield :

$$y^2 - [\frac{4r}{5}x + \frac{2br}{5a}(k+1)]y - \frac{2a}{3}x^3 - bx^2$$
$$-\frac{2r^2}{25}x^2 - 2dx - \frac{2br^2}{25a}(k+1)x + D = 0 \quad (55)$$

from (55),y can be expressed in terms of x,i.e.,

$$y = \frac{2r}{5}x + \frac{br}{5a}(k+1) \pm$$
$$\sqrt{\frac{2a}{3}x^3 + (k+1)bx^2 + \frac{b^2}{2a}(k+1)^2 x + \frac{b^3}{12a^2}(k+1)^3} \quad (56)$$
$$= \frac{2r}{5}x + \frac{br}{5a}(k+1) \pm \sqrt{\frac{2}{3a^2}\left[ax + \frac{(k+1)b}{2}\right]^3}$$

combining(45)and(56),we have :

$$u(x,y,t) = -\frac{12\beta^2}{25\alpha s}.\left[\frac{e^{-\frac{\beta}{5sh}(hx+ly-wt)}}{e^{-\frac{\beta}{5sh}(hx+ly-wt)} + c}\right]^2 \quad (57)$$
$$+ \frac{wh - \gamma l^2}{\alpha h^2} + \frac{6\beta^2}{25\alpha s}$$

CONCLUTION:

The new modification first integral method , successfully for solving allot of nonlinear equation , and establish travelling wave solutions , which is based on the ring theory of commutative algebra , and us to solve complicated and tedious algebra calculation .

Refrences:

[1] R.Hirota , Direct method of finding exact solutions of nonlinear evolution equation , in :R.Bullongh , p.caudry (Eds).Backlund transformation , springer .Berlin 1980 ,p.115

[2]A.m,wazwaz ,the extended tanh method for handling nonlinear wave equation ,math .compact .modell. 40(2004)-349

[3]Y.B.zhou .m.l.wang Y.M.wang,periodic wave solution to coupled Kdv equation with variable coefficients , phys .let .A 308

[4]Z.s.Feng the first integer method to study the Burgers –Korteweg –devries equation J.PHYS .A 35(2002) 343-34

[5]Filiz tascan, Ahmed Bekir , travilling wave solution of the cahn-Allenequation byusing first integral method .APP.math.and.comp.207(2009)279-282

[6]T.R.Ding C.Z.Li , ordinary differential equations , peking university , 1996

[7] Hirota R. Direct method of Önding exact solutions of nonlinear evolution equations, In:

Bullough R, Caudrey P, editors. Backlund transformations. Berlin, Springer, 1980. p. 1157

[8] Gao, Yi-Tian, B. Tian, Generalized tanh method with symbolic computation and generalized

shallow water wave equation, Comput. Math. Appl. 33 (1997) no. 4, 115ñ118

[9] N.Taghizadeh,M.Mirzazadeh ,F.Farahrooz,exact soliton solution of the modified kdv-kp equation and the Burgers –kp equation by using the first integral method, Applied mathematics modeling 35(2011) 3991-3997

[10] Taghizadeh, N., Mirzazadeh, M., Farahrooz, F., Exact solutions of the nonlinear Schrodingerequation by the Örst integral method, J. Math. Anal. Appl. 374 (2011) no. 2 549ñ553

