On Prime Graphs

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Abstract:

In this paper we shall prove that the special butterfly graph, Franklin graph, and Herschel graph are prime graphs.

Key words:

Labeling, special butterfly graph, Franklin graph, Herschel graph, Goldner_Harary graph, Sousselier graph, prime graph.

1. Introduction

A labeling or valuation of a graph G is an assignment of labels to the vertices of G that induces for each edge xy a label depending on the vertex labels f(x) and f(y). For all terminology and notations we use[1]. Tout Dabborey and Howalla [2] introduced prime labeling of a graph. Let G be a graph with vertex set V. G is said to have a prime labeling if its vertices are labeled with distinct integers 1,2,…..|V| such that for each edge xy the labels assigned to x and y are relatively prime. Then G is said to be a prime graph. In [2] we shall see some graphs are prime graphs. Now we shall prove that following graphs are prime graphs.

II. The special Butterfly Graph [4]

Definition 1:

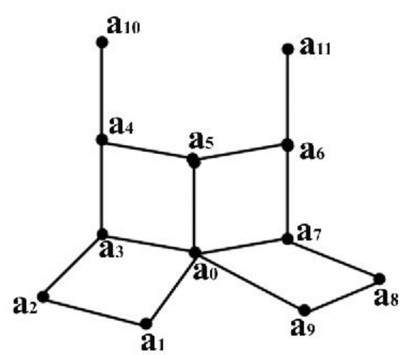
Consider the cycle C_{2n+2}, $n\geq 4$, let $a_0,a_1,\ldots,a_{2n+1}$ be the vertices of C_{2n+2}. Join $a_0\, a_{2i-1}$, $2\leq i\leq n$. Attach two pendent edges at a_n and a_{n+2}. Let the vertices attached with those edges be a_{2n+2}, a_{2n+3} respectively. Let us call the resulting figure as special butterfly graph and we shall denote it by BF_n [4].

When n = 4, we get the standard Butterfly graph and is denoted by BF_4.

The graph BF_n has $|V|$ = 2n+4 vertices and $|E|$ = 3n + 3 edges.

The graph BF_4 is as follows

Illustration: 1



Theorem 1:

The special butterfly graph BF_n is a prime graph for all n.

Proof:

The vertex set of BF_n is

$V(BF_n) = \{a_0, a_1, \ldots, a_{2n+1}, a_{2n+2}, a_{2n+3}\}$

The edge set of BF_n is

$E(BF_n)=\{(a_0a_{2i-1})/2\leq i\leq n\}\cup\{(a_i\, a_{i+1})/0\leq i\leq 2n\}$

$\cup\{ a_{2n+1}\, a_0\} \cup \{ a_n\, a_{2n+2}\} \cup \{ a_{n+2}\, a_{2n+3}\}$

Define $f : V(BF_n) \rightarrow \{ 1,2,\ldots, 2n+4\}$ as follows

Case (i) When n is odd

$f(a_i) = i+1 \quad 0\leq i\leq n-1$

$f(a_i) = i+2 \quad n\leq i\leq n+2$

$f(a_i) = i+3 \quad n+3\leq i\leq 2n+1$

$f(a_{2n+2}) = n+1$

$f(a_{2n+3}) = n+5$

clearly vertex labels are distinct

Now,

$\gcd(f(a_i), f(a_{i+1})) = 1$ (Since they are consecutive integers)

$\gcd(f(a_0), f(a_{2n+1})) = \gcd(1, f(a_{2n+1}))$

$= 1$

$\gcd(f(a_n), f(a_{2n+2})) = 1$ (since they are consecutive)

$\gcd(f(a_{n+2}), f(a_{2n+3})) = 1$ (since they are consecutive)

The labeling of x,y for every edge xy has gcd 1.

Hence f is a prime labeling.

Case (ii) When n is even

$f(a_i) = i+1 \quad 0\leq i\leq n$

$f(a_i) = i+3 \quad n+2\leq i\leq 2n+1$

$f(a_{2n+2}) = n+2$

$f(a_{n+1}) = n+3$

$f(a_{2n+3}) = n+4$

Clearly vertex labels are distinct.

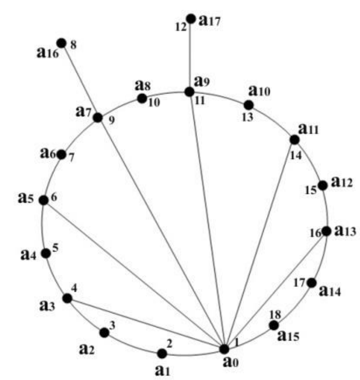
The labeling of x,y for every edge xy has gcd 1.

Thus f is a prime labeling.

Hence the special butterfly graph BF_n is a prime graph for all n.

Illustration: 2

BF_7:



A vertex switching of special Butterfly graph

Definition 2:

A vertex switching of a graph G is obtained by taking a vertex v of G, removing all the edges incident with v and adding edges joining v to every vertex which are not adjacent to v in G.

Theorem 2:

The graph G obtained by switching the vertex a_0 in a special butterfly graph BF_n is a prime graph.

Proof:

The vertex set of G is,

V(G) ={a_0, a_1, …. , a_{2n+1}, a_{2n+2}, a_{2n+3}}

The edge set of G is

E(G)={$(a_ia_{i+1})/1\leq i\leq 2n+1$}∪{$(a_0a_{2i})/1\leq i\leq n$}
∪{$a_{n+2}a_{2n+3}$}∪{a_na_{2n+2}}∪{a_0 a_{2n+2}}∪{ a_0 a_{2n+3}}

Define f: V (G) → {1,2,…., 2n+4} as follows

Case (i) when n is odd

$f(a_i) = i+1 \quad 0\leq i\leq n-1$

$f(a_i) = i+2 \quad n\leq i\leq n+2$

$f(a_i) = i+3 \quad n+3\leq i\leq 2n+1$

$f(a_{2n+2}) = n+1$

$f(a_{2n+3}) = n+5$

Clearly vertex labels are distinct

Then f admits prime labeling.

Case (ii) When n is even

$f(a_i) = i+1 \quad 0\leq i\leq n$

$f(a_i) = i+3 \quad n+2\leq i\leq 2n+1$

$f(a_{2n+2}) = n+2$

$f(a_{n+1}) = n+3$

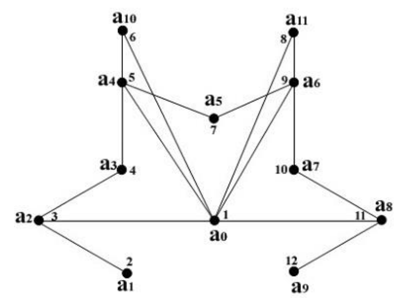
$f(a_{2n+3}) = n+4$

Clearly vertex labels are distinct.

Then f admits prime labeling.

Hence G is a prime graph.

Illustration: 3



Prime labeling switching the vertex a_0 in BF_4

III. Franklin graph

Definition 3:

The Franklin graph (FG) is a 3 – regular graph with 12 vertices and 18 edges.

Theorem 3:

The Franklin graph is a prime graph.

Proof:

Let FG be a Franklin graph

The vertex set of FG is

V(FG) = { v_i / $1\leq i\leq 12$}

The edge set of FG is

E(FG) ={v_i $v_{i+1}/1\leq i\leq 12, v_{13}=v_1$}

∪{$v_iv_{i+7}/1\leq i\leq 6$,i is odd}

∪{v_i $v_{i+5}/1 \leq i \leq 6$, i is even}

Define f: V (FG) → {1, 2, …, 12} as follows

$f(v_i) = i \quad 1\leq i\leq 12$

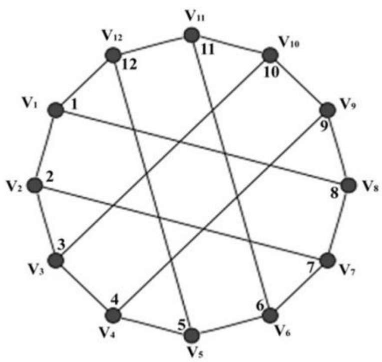
Clearly vertex labels are distinct

The labeling of x,y for every xy has gcd 1.

Thus f is a prime labeling.

Hence Franklin graph is a prime graph.

Illustration: 4



Theorem 4:

The graph FG$\odot K_1$ is a prime graph.

Proof:

Let $G^* = FG \odot K_1$

The vertex set of G^* is

$V(G) = \{v_i / 1 \le i \le 12\} \cup \{ v_i^1 / 1 \le i \le 12\}$

The edge set of G^* is

$E(G) = \{v_i v_{i+1} / 1 \le i \le 12, v_{13}=v_1\}$

$\cup \{v_i v_{i+7} / 1 \le i \le 6, \text{ i is odd}\}$

$\cup \{v_i v_{i+5} / 1 \le i \le 6, \text{ i is even}\}$

$\cup \{ v_i v_i^1 / 1 \le i \le 12\}$

Clearly G^* has 24 vertices and 30 edges.

Define f: V $(G^*) \to \{1,2,\ldots, 24\}$ as follows.

$f(v_i) = 2i-1 \qquad 1 \le i \le 12$

$f(v_i^1) = 2i \qquad 1 \le i \le 12$

Clearly vertex labels are distinct.

Then f admits prime labeling.

Hence G^* is a prime graph.

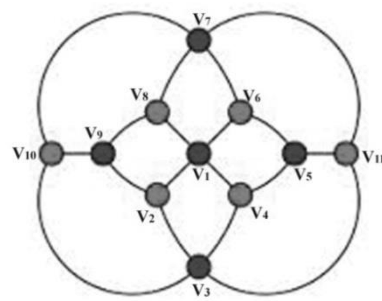
IV. Herschel graph

Definition 4:

The Herschel graph(HG) is a bipartite graph with 11 vertices and 18 edges, the smallest non-Hamiltonian polyhedral graph.

The Herschel graph is as follows.

Illustrations: 5



Theorem 5:

The Herchel graph HG is a prime graph.

Theorem 6:

The graph HG$\odot K_1$ is a prime graph.

Proof:

HG be a Herchel graph.

Let $H^* = HG \odot K_1$

The vertex set of H^* is

$V(H^*) = \{ v_i / 1 \le i \le 11\} \cup \{ v_i^1 / 1 \le i \le 11\}$

The edge set of H^* is

$E(H^*) = \{v_1v_{2i} / 1 \leq i \leq 4\}$

$\cup\{v_i v_{11} / i = 3,5,7\}\cup\{v_jv_{10} / j = 3, 7\}$

$\cup \{v_2 v_9\}\cup \{v_i v_{i+1} / 2\leq i\leq 9\}$

$\cup \{ v_i v_i^1 / 1\leq i\leq 11\}$

H^* has 22 vertices and 29 edges.

Define a labeling f: $V(H^*) \rightarrow \{1, 2, \ldots.. 22\}$ as follows.

$$f(v_i) = \begin{cases} 2i-1; 1 \leq i \leq 9 \\ 2i+1; i = 10 \\ 2i-3; i = 11 \end{cases}$$

$f(v_i^1) = 2i$

Clearly vertex labels are distinct.

The labeling of x,y for every edges xy has gcd 1.

Thus f is a prime labeling.

Hence H^* is a prime graph.

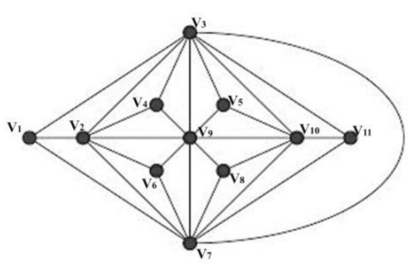
V. Goldner – Harary graph (GH)

Definition 5:

The Goldner – Harary graph is a simple undirected graph with 11 vertices and 27 edges.

The Goldner – Harary graph is as follows

Illustration: 6



Theorem 7:

The graph $G^1 = GH\odot K_1$ is a prime graph.

Proof:

The vertex set of G^1 is

$V(G) = \{v_1, v_2, \ldots.v_{11}, v_1^1, v_2^1, \ldots.v_{11}^1\}$

The edge set of G^1 is

$E(G^1) = \{ v_iv_9 / 2\leq i\leq 8\} \cup \{ v_iv_{i+1} / i=9,10\}$

$\cup\{ v_iv_{i+1} / 1\leq i\leq 3\}\cup\{ v_iv_{i+1} / 6\leq i\leq 7\}$

$\cup\{v_7v_i / i=1,2,3,10,11\}$

$\cup\{v_iv_{i+2}/i=1,2,3,8\}\cup\{v_3v_i/i=10,11\}$

$\cup\{v_2v_6\}\cup\{ v_5v_{10}\}\cup\{ v_iv_i^1 / 1\leq i\leq 11\}$

G^1 has 22 vertices and 38 edges.

Define a labeling $f:V(G^1)\rightarrow\{1,2,\ldots,22\}$as follows

$f(v_i) = 2i-1 \quad 1\leq i\leq 11$

$f(v_i^1) = 2i \quad 1\leq i\leq 11$

Clearly vertex labels are distinct.

The labeling of x ,y for every xy has gcd 1.

Thus f is a prime labeling.

Hence G^I is a prime graph.

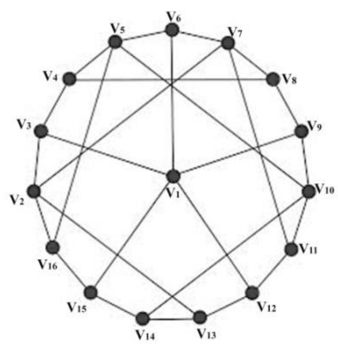
VI. Sousselier graph (SG)

Definition 6:

A Hamiltonian cycle is a graph cycle through a graph that vertex each node exactly once. A Hamiltonian graph is a graph possessing a Hamiltonian cycle. A graph that is not Hamiltonian is said to be non Hamiltonian.

A graph G is Hypohamiltonian if G is non Hamiltonian, but G – v is Hamiltonian for every v∈V. 16 vertices Hypohamiltonian graph is Sousselier graph. It has 27 edges.The Sousselier graph is as follows

Illustration: 7



Theorem 8:

The graph $S^* = SG \odot K_1$ is a prime graph.

Proof:

The vertex set of S^* is

$V(S^*) = \{v_1, v_2, \ldots\ldots, v_{16}, v_1^1, v_2^1, \ldots\ldots, v_{16}^1\}$

The edge set of S^* is

$E(S^*) = \{ v_i v_{i+1} / 2 \leq i \leq 16 \text{ and } v_{17} = v_2\}$

$\cup \{(v_i v_i^1)/ 1 \leq i \leq 16\} \cup \{(v_1 v_{3i})/ 1 \leq i \leq 5\}$

$\cup \{(v_i v_{2i}) \ i=4,5\} \cup \{(v_2 v_7), (v_2 v_{13}), (v_5 v_{16}),$

$(v_7 v_{11}), (v_{10} v_{14})\}$

S^* has 32 vertices and 43 edges.

Define a labeling $f: V(S^*) \rightarrow \{1,2,\ldots,32\}$ as follows

$f(v_i) = 2i - 1 \quad 1 \leq i \leq 16$

$f(v_i^1) = 2i \quad 1 \leq i \leq 16$

Clearly vertex labels are distinct.

Then f admits prime labeling.

Hence $S^* = SG \odot K_1$ is a prime graph.

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