FUZZY COST ANALYSIS OF $M^X/G_1\ G_2/1/MV$ QUEUING MODEL

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Abstract

The aim of this paper is to investigate the cost analysis of $M^X/G_1\ G_2/1$ Multiple vacation (MV) Queuing model in Fuzzy environment which helps to control the queues in different situation. A mathematical Parametric Non-linear Programming (NLP) method is used to construct the membership function of the system characteristic of a batch arrival multiple vacation policy queue in which arrival rate, service rate for vacation period, holding cost, setup cost, startup cost reward cost are fuzzy numbers. The α-cut and Zadeh's Extension Principle[19] are used to transform a fuzzy queue into a family of conventional crisp queues. By means of membership functions of the system characteristics, a set of parametric non linear program is developed to calculate the lower and upper bound of the system characteristics function at α. Thus the membership functions of the system characteristics are constructed. Numerical example is also illustrated to check the validity of the proposed model.

Key words

Batch arrival, multiple vacation policy, First essential service, second optional service, Fuzzy sets, α-cut, Membership function and Zadeh's extension principle.

I.INTRODUCTION

The first study of batch arrival queuing system with N policy was carried out by Lee and Srinivasan[9]. In their paper they have discussed the mean waiting time of an arbitrary customer and a procedure to find the stationary optimal policy under a linear cost structure. Later, many authors including Lee et al.,[10],[11] have analyzed the N policy of $M^X/G/1$ queuing models with servers having multiple and single vacation. In queuing models server's set up time corresponds to the preparatory work of the server before starting his service. Hur and Park [3] and Ke [8] are some of the authors who analyzed the N policy of $M^x/G/1$ queuing models with server's setup time. The batch arrival queuing system with double threshold policy, setup time and vacation are analyzed by Lee et al., [12] is the most general queuing system with threshold policies. Lee and park examined $M^X/G/1$ queuing model with early setup for production system and then developed a procedure to find the joint optimal thresholds which minimize a linear average cost. In everyday life there are queuing situation where the arriving customers require the first essential service and some may require the second optional service provided by the same server. Madan[14] has introduced the concept of second optional service, where the customers may depart from the system either with probability (1-r) or may immediately opt for second optional service with probability r. Choudhury and Paul[2] have extended the results of Madan. Later Madan and Choudhury [15] have studied the steady state analysis of the $M^x/(G_1,G_2)/1$ queue with restricted admissibility. Recently Julia Rose Mary et al., [5] introduced Bi-level threshold policy of $M^X/(G_1,G_2)/1/MV$ queue. In their model they obtained the stationary probability generating function of the queue length distribution through supplementary variable techniques and derived the total cost unit (TCU). The various performance measures were also calculated. Morerover in their paper, the inter arrival time, service times, setup time, vacation time are assumed to follow certain probability distributions with fixed parameters. In real life situations the parameter may only be characterized subjectively. Thus fuzzy analysis would be potentially much more useful and realistic than the commonly used crisp concepts. Li and Lee [13] investigated analytical results for two fuzzy queues using a general approach based on Zadeh's extension principle. Nagi and Lee [16] proposed a procedure using α cut and two variable simulations to analyze fuzzy queues. Using parametric programming Kao et al., [7] constructed the membership function of system characteristics for fuzzy queues. Chauan et al., [1] have obtained the membership function of system characteristics of a retrial queuing model with fuzzy arrival, retrial and service rate. Jeeva and Rathnakumari [4] analyzed a batch arrival single server, Bernoulli feedback queue with fuzzy vacations. Recently Ramesh and Kumara [17] also introduced a batch arrival queue with multiple servers and fuzzy parameters. With the help of these available literatures we analyze the optimum operating policy of batch arrival queue with second optional service and multiple vacation in fuzzy environment.

II.COST ANALYSIS of $M^X/G_1 G_2/1/MV$ QUEUE

Consider the model $M^X/ G_1 G_2/1/MV$ in which arrival stream forms a Poisson process and the actual number of customers in any arriving module is a random variable X. Let E(X) and $E(X^2)$ denote the probability generating function(PGF), first and second moment of random variable X respectively. With arrival rate λ, the server provides service to the customers with exponential service rate μ, First essential service (FES)S_1 to all the arriving customers. At the end of the FES, each customer may either choose the second optional service (SOS)S_2 or depart from the system. The server leaves the system for vacation of random length V_1 as soon as the system empties. After returning from the vacation if the server finds **m** (or) more customers in the system, he immediately starts a setup operation of random length D. The random variable V_1,V_2… are assumed to be independently and identically distributed with general representation V. The vacation duration V follows an exponential distribution with parameter η. E(D) , E(V),E(S_1),and E(S_2) are the I^{st} moment for the random variables D, V, S_1, S_2. For any queuing system cost and profit analysis constitute a very important aspect of its investigation. Hence for this model, we compute the total cost per unit by using the concept of cost computation given by Kanti swarap[6] and Taha[18]. According to this model the Total cost per unit time is given by

$$TCU = \frac{C_y}{E(Cycle)} + C_h L^R_{Sos(m,N)} + C_{set}P_{set} +$$
$$C_{dor}P_{dor} + C_{build}P_{build} + C_{busy}P_{busy} - C_v P_v \qquad (1)$$

where C_y, C_h, C_{set}, C_{dor}, C_{build}, C_{busy} and C_v are the startup cost per cycle, holding cost per customer, setup cost, standby cost, buildup cost, operating cost and reward cost per unit time.By substituting various performance measures in eqn(1) then the TCU becomes

$$TCU = \frac{1}{D_R(m,N)}\left[\begin{matrix} A^R_{Sos} + Z^R_{Sos}(m) + C_h + \\ (1-\rho_{Sos})(1/\lambda) \end{matrix}\right] + A'_{Sos}$$

where

$$A^R_{Sos} = \left[\begin{matrix} C_y(1-\rho_{Sos}) + C_{set}E(D)(1-\rho_{Sos}) + \\ C_h \dfrac{\lambda E(X)}{2} E(D^2) \end{matrix}\right]$$

$$Z^R_{Sos}(m) = \left\{ C_h \left[\begin{matrix} \lambda E(X)(E(D)E(V)) + \dfrac{E(V)}{2} + \\ E(V) - C_v E(V)(1-\rho_{Sos}) \end{matrix}\right]\right\}$$

$$A'_{Sos} = C_{busy}\rho_{Sos} + C_h L_{Sos}$$

$$D_R(m,N) = E(V) + E(D)(1/\lambda)$$

$$\rho_{Sos} = \lambda E(X)(E(S_1) + rE(S_2));$$

$$L_{Sos} = \left[\frac{\rho_{sos} + \lambda E(X(X-1)E(S_{Sos}) + (\lambda E(X))^2 E(S^2_{Sos})}{2(1-\rho_{sos})}\right]$$

$$E(S_{Sos}) = E(S_1) + rE(S_2);$$

$$E(S_{Sos}) = E(S_1) + rE(S_2);$$

$$E(S^2_{Sos}) = \left[E(S^2{}_1) + (rE(S_1)E(S_2) + rE(S^2_2)\right]$$

$$E(S_1) = 1/\mu = E(S_2)\,;\; E(S^2_1) = 2(1/\mu^2) = E(S^2_2);$$

$$E(D) = 1/\nu \;\; E(v) = 1/\eta\,;\;\; E(D^2) = 4/3\eta^2.$$

III.COST ANALYSIS of $M^X/G_1G2/1$ WITH FUZZY MULTIPLE VACATION PERIOD

We extend the above queuing system in fuzzy environment. Suppose the arrival rate λ, service rate μ, vacation parameter V, expected group size X, Startup cost per cycle C_h, setup cost C_{set}, operating cost C_{busy}, reward cost C_v are approximately known and can be represented as fuzzy set $\overline{\lambda}, \overline{\mu}, \overline{V}, \overline{X}, \overline{C_y}, \overline{C_h}, \overline{C_{set}}, \overline{C_{busy}}, \overline{C_{v.}}$ Using α cut for the various cost are represented by different levels of confidence. Let this interval of confidence be represented by [$x_{1\alpha}$, $x_{2\alpha}$]. Since the probability distribution for the α cuts can be represented by uniform distributions. We have

$P(x_\alpha) = 1/[x_{2\alpha} - x_{1\alpha}] \quad [x_{1\alpha} \le x_\alpha \le x_{2\alpha}]$

Then the mean and the second order moment of the distribution are obtained as $\frac{1}{2}[x_{2\alpha} + x_{1\alpha}]$ and $\frac{x_{2\alpha}^3 - x_{1\alpha}^3}{3(x_{2\alpha} - x_{1\alpha})}$. Further its variance is given by $\frac{1}{12}[x_{2\alpha} - x_{1\alpha}]^2$. Let

$\xi_{\bar{\lambda}}(y), \xi_{\bar{\mu}}(k), \xi_{\bar{\gamma}}(v), \xi_{\bar{\eta}}(t), \xi_{\overline{C_y}}(a), \xi_{\overline{C_{set}}}(p), \xi_{\overline{C_h}}(h), \xi_{\overline{C_v}}(w), \xi_{\bar{r}}(q), \xi_{\overline{C_{busy}}}(b)$ denote the membership function of $\bar{\lambda}, \bar{\mu}, \bar{\gamma}, \bar{\eta}, \overline{C_y}, \overline{C_{set}}, \overline{C_h}, \overline{C_v}, \bar{r}, \overline{C_{busy}}$. Then we have the following fuzzy sets as,

$$\bar{\lambda} = \{(y, \xi_{\bar{\lambda}}(y)) / y \in Y\} \quad \bar{\mu} = \{(k, \xi_{\bar{\mu}}(k)) / k \in K\},$$
$$\bar{\gamma} = \{(v, \xi_{\bar{\gamma}}(v)) / v \in V\} \quad \bar{\eta} = \{(t, \xi_{\bar{\eta}}(t)) / t \in T\},$$
$$\overline{C_y} = \{(a, \xi_{\overline{C_y}}(a)) / a \in A\}$$
$$\overline{C_{set}} = \{(p, \xi_{\overline{C_{set}}}(p)) / p \in P\}$$
$$\overline{C_h} = \{(h, \xi_{\overline{C_h}}(h)) / h \in H\}$$
$$\overline{C_v} = \{(w, \xi_{\overline{C_v}}(w)) / w \in W\}$$
$$\bar{r} = \{(q, \xi_{\bar{r}}(q)) / q \in Q\}$$
$$\overline{C_{busy}} = \{(b, \xi_{\overline{C_{busy}}}(b)) / b \in B\}$$

where Y,K,V,T,A,P,H,,W,Q,B are the crisp universal sets. Let f(y,k,v,t,a,p,h,w,q,b) denote the system characteristics of interest. Since $\bar{\lambda}, \bar{\mu}, \bar{\gamma}, \bar{\eta}, \overline{C_y}, \overline{C_{set}}, \overline{C_h}, \overline{C_v}, \bar{r}, \overline{C_{busy}}$ are fuzzy numbers, $f(\bar{\lambda}, \bar{\mu}, \bar{\gamma}, \bar{\eta}, \overline{C_y}, \overline{C_{set}}, \overline{C_h}, \overline{C_v}, \bar{r}, \overline{C_{busy}})$ is also a fuzzy number. By Zadeh's extension principle the membership function of the system characteristics $f(\bar{\lambda}, \bar{\mu}, \bar{\gamma}, \bar{\eta}, \overline{C_y}, \overline{C_{set}}, \overline{C_h}, \overline{C_v}, \bar{r}, \overline{C_{busy}})$

is defined as

$$\xi_{f(\bar{\lambda}, \bar{k}, \bar{\gamma}, \bar{\eta}, \overline{C_y}, \overline{C_{set}}, \overline{C_h}, \overline{C_v}, \bar{r}, \overline{C_{busy}})}(z) = \sup_{\substack{y \in Y, k \in K, v \in V, \\ t \in T, a \in A, \\ p \in P, h \in H, \\ w \in W, q \in Q, b \in B}} \min \left\{ \begin{array}{l} \xi_{\bar{\lambda}}(y), \xi_{\bar{\mu}}(k), \xi_{\bar{\gamma}}(v), \xi_{\bar{\eta}}(t), \\ \xi_{\overline{C_y}}(a), \xi_{\overline{C_{set}}}(p), \xi_{\overline{C_h}}(h), \\ \xi_{\overline{C_v}}(w), \xi_{\bar{r}}(q), \xi_{\overline{C_b}}(b) / \\ z(y, k, v, t, a, p, h, w, q, b) \end{array} \right\} \quad (2)$$

Also assume that if the system characteristics of interests is TCU (Total cost per unit).We have the membership function of TCU as

$$\xi_{\overline{TCU}}(z) = \sup_{\substack{y \in Y, k \in K, v \in V, \\ t \in T, a \in A, p \in P, h \in H, \\ w \in W, q \in Q, b \in B}} \min\{\xi_{\bar{\lambda}}(y), \xi_{\bar{\mu}}(k), \xi\bar{\gamma}(v), \xi_{\bar{\eta}}(t), \xi_{\overline{C_y}}(a), \xi_{\overline{C_{set}}}(p), \xi_{\overline{C_h}}(h), \xi_{\overline{C_v}}(w), \xi_{\bar{r}}(q), \xi_{\overline{C_b}}(b) / z = \frac{1}{D_R(m, N)} [A^R_{Sos} + Z^R_{Sos}(m) + h(1 - \rho_{Sos}) +] A'_{Sos}\} \quad (3)$$

The membership function in the eqn (3) is not in the usual form thus making it very difficult to imagine its shapes. For this we approach the problem by using the mathematical programming techniques. Parametric NLP's are developed to find α cuts of $f(\bar{\lambda}, \bar{\mu}, \bar{\gamma}, \overline{C_y}, \overline{C_{set}}, \overline{C_h}, \overline{C_v}, \bar{r}, \overline{C_{busy}})$ based on the extension principle.

IV.PARAMETRIC NON LINEAR PROGRAMMING

To construct the membership function of $\xi_{\overline{TCU}}(Z) = \alpha$, we have to derive the α cuts of $\bar{\lambda}, \bar{\mu}, \bar{\gamma}, \bar{\eta}, \overline{C_y}, \overline{C_{set}}, \overline{C_h}, \overline{C_v}, \bar{r}, \overline{C_{busy}}$ are represented as follows

$$\lambda(\alpha) = [y^L_\alpha \; y^U_\alpha] = \begin{bmatrix} \min\{y \in Y / \xi_{\bar{\lambda}}(y) \ge \alpha\}, \\ \max\{y \in Y / \xi_{\bar{\lambda}}(y) \ge \alpha\} \end{bmatrix} \quad (4a)$$

$$\mu(\alpha) = [k^L_\alpha \; k^U_\alpha] = \begin{bmatrix} \min\{k \in K / \xi_{\bar{\mu}}(k) \ge \alpha\}, \\ \max\{k \in K / \xi_{\bar{\mu}}(k) \ge \alpha\} \end{bmatrix} \quad (4b)$$

$$V(\alpha) = [v^L_\alpha \; v^U_\alpha] = \begin{bmatrix} \min\{v \in V / \xi_{\bar{\gamma}}(v) \ge \alpha\}, \\ \max\{v \in V / \xi_{\bar{\gamma}}(v) \ge \alpha\} \end{bmatrix} \quad (4c)$$

$$\eta(\alpha) = [t^L_\alpha \; t^U_\alpha] = \begin{bmatrix} \min\{t \in T / \xi_{\bar{\eta}}(t) \ge \alpha\}, \\ \max\{t \in T / \xi_{\bar{\eta}}(t) \ge \alpha\} \end{bmatrix} \quad (4d)$$

$$C_y(\alpha) = [a^L_\alpha \; a^U_\alpha] = \begin{bmatrix} \min\{a \in A / \xi_{\overline{C_y}}(a) \ge \alpha\}, \\ \max\{a \in A / \xi_{\overline{C_y}}(a) \ge \alpha\} \end{bmatrix} \quad (4e)$$

$$C_{set}(\alpha) = [p^L_\alpha \; p^U_\alpha] = \begin{bmatrix} \min\{p \in P / \xi_{\overline{C_{set}}}(p) \ge \alpha\}, \\ \max\{p \in P / \xi_{\overline{C_{set}}}(p) \ge \alpha\} \end{bmatrix} \quad (4f)$$

$$C_h(\alpha) = [h^L_\alpha \; h^U_\alpha] = \begin{bmatrix} \min\{h \in H / \xi_{\overline{C_h}}(h) \ge \alpha\}, \\ \max\{h \in H / \xi_{\overline{C_h}}(h) \ge \alpha\} \end{bmatrix} \quad (4g)$$

$$C_v(\alpha)=\left[w_\alpha^L\ w_\alpha^U\right]=\begin{bmatrix}\min\{w\in W/\xi_{\overline{C_v}}(w)\ge\alpha\},\\ \max\{w\in W/\xi_{\overline{C_v}}(w)\ge\alpha\}\end{bmatrix}\quad(4h)$$

$$r(\alpha)=\left[q_\alpha^L\ q_\alpha^U\right]=\begin{bmatrix}\min\{q\in Q/\xi_{\bar r}(q)\ge\alpha\},\\ \max\{q\in Q/\xi_{\bar r}(q)\ge\alpha\}\end{bmatrix}\quad(4i)$$

$$C_{busy}(\alpha)=\left[b_\alpha^L\ b_\alpha^U\right]=\begin{bmatrix}\min\{b\in B/\xi_{\overline{C}_{busy}}(b)\ge\alpha\},\\ \max\{b\in B/\xi_{\overline{C}_{busy}}(b)\ge\alpha\}\end{bmatrix}\quad(4j)$$

Further, the bounds of these intervals can be described as functions of α and can be obtained as

$$\begin{aligned}
y_\alpha^L &= \min\xi_{\bar\lambda}^{-1}(\alpha) & y_\alpha^U &= \max\xi_{\bar\lambda}^{-1}(\alpha)\\
k_\alpha^L &= \min\xi_{\bar\mu}^{-1}(\alpha) & k_\alpha^U &= \max\xi_{\bar\mu}^{-1}(\alpha)\\
v_\alpha^L &= \min\xi_{\bar\gamma}^{-1}(\alpha) & v_\alpha^U &= \max\xi_{\bar\gamma}^{-1}(\alpha)\\
t_\alpha^L &= \min\xi_{\bar\eta}^{-1}(\alpha) & t_\alpha^U &= \max\xi_{\bar\eta}^{-1}(\alpha)\\
a_\alpha^L &= \min\xi_{\overline{C_y}}^{-1}(\alpha) & a_\alpha^U &= \max\xi_{\overline{C_y}}^{-1}(\alpha)\\
p_\alpha^L &= \min\xi_{\overline{C_{set}}}^{-1}(\alpha) & p_\alpha^U &= \max\xi_{\overline{C_{set}}}^{-1}(\alpha)\\
h_\alpha^L &= \min\xi_{\overline{C_h}}^{-1}(\alpha) & h_\alpha^U &= \min\xi_{\overline{C_h}}^{-1}(\alpha)\\
w_\alpha^L &= \min\xi_{\overline{C_v}}^{-1}(\alpha) & w_\alpha^U &= \min\xi_{\overline{Cv}}^{-1}(\alpha)\\
q_\alpha^L &= \min\xi_{\bar r}^{-1}(\alpha) & q_\alpha^U &= \min\xi_{\bar r}^{-1}(\alpha)\\
b_\alpha^L &= \min\xi_{\overline{C_{busy}}}^{-1}(\alpha) & b_\alpha^U &= \min\xi_{\overline{C_{busy}}}^{-1}(\alpha)
\end{aligned}$$

Therefore by making use of the α cuts for TCU we construct the membership function of (3) which is parameterised by α. To derive the membership function of TCU it is suffice to find the left and right shape function of $\xi_{\overline{TCU}}(Z)$. This can be achieved by following the Zadeh's extension principle for $\xi_{\overline{TCU}}(Z)$ which is the minimum of

$$\xi_{\bar\lambda}(y),\xi_{\bar\mu}(k),\xi_{\bar V}(v),\xi_{\bar\eta}(t),\xi_{\overline{C_y}}(a),$$
$$\xi_{\overline{C_{set}}}(p),\xi_{\overline{C_h}}(h),\xi_{\overline{C_v}}(w),\xi_{\bar r}(q),\xi_{\overline{C_b}}(b).$$

Now to derive $\xi_{\overline{TCU}}(Z)=\alpha$, then atleast one of the cases to be hold which satisfy $\xi_{\overline{TCU}}(Z)=\alpha$. Thus,

Case(i)

$$\xi_{\bar\lambda}(y)=\alpha,\xi_{\bar\mu}(k)\ge\alpha,\xi_{\bar\gamma}(v)\ge\alpha,\xi_{\bar\eta}(t)\ge\alpha,$$
$$\xi_{\overline{C_y}}(a)\ge\alpha,\xi_{\overline{C_{set}}}(p)\ge\alpha,\xi_{\overline{C_h}}(h)\ge\alpha,$$
$$\xi_{\overline{C_v}}(w)\ge\alpha,\xi_{\bar r}(q)\ge\alpha,\xi_{\overline{C_b}}(b)\ge\alpha.$$

Case (ii)

$$\xi_{\bar\lambda}(y)\ge\alpha,\xi_{\bar\mu}(k)=\alpha,\xi_{\bar\gamma}(v)\ge\alpha,\xi_{\bar\eta}(t)\ge\alpha,$$
$$\xi_{\overline{C_y}}(a)\ge\alpha,\xi_{\overline{C_{set}}}(p)\ge\alpha,\xi_{\overline{C_h}}(h)\ge\alpha,$$
$$\xi_{\overline{C_v}}(w)\ge\alpha,\xi_{\bar r}(q)\ge\alpha,\xi_{\overline{C_b}}(b)\ge\alpha.$$

Case (iii)

$$\xi_{\bar\lambda}(y)\ge\alpha,\xi_{\bar\mu}(k)\ge\alpha,\xi_{\bar\gamma}(v)=\alpha,\xi_{\bar\eta}(t)\ge\alpha,$$
$$\xi_{\overline{C_y}}(a)\ge\alpha,\xi_{\overline{C_{set}}}(p)\ge\alpha,\xi_{\overline{C_h}}(h)\ge\alpha,$$
$$\xi_{\overline{C_v}}(w)\ge\alpha,\xi_{\bar r}(q)\ge\alpha,\xi_{\overline{C_b}}(b)\ge\alpha.$$

Case (iv)

$$\xi_{\bar\lambda}(y)\ge\alpha,\xi_{\bar\mu}(k)\ge\alpha,\xi_{\bar\gamma}(v)\ge\alpha,\xi_{\bar\eta}(t)=\alpha,$$
$$\xi_{\overline{C_y}}(a)\ge\alpha,\xi_{\overline{C_{set}}}(p)\ge\alpha,\xi_{\overline{C_h}}(h)\ge\alpha,$$
$$\xi_{\overline{C_v}}(w)\ge\alpha,\xi_{\bar r}(q)\ge\alpha,\xi_{\overline{C_b}}(b)\ge\alpha.$$

Case (v)

$$\xi_{\bar\lambda}(y)\ge\alpha,\xi_{\bar\mu}(k)\ge\alpha,\xi_{\bar\gamma}(v)\ge\alpha,\xi_{\bar\eta}(t)\ge\alpha,$$
$$\xi_{\overline{C_y}}(a)=\alpha,\xi_{\overline{C_{set}}}(p)\ge\alpha,\xi_{\overline{C_h}}(h)\ge\alpha,$$
$$\xi_{\overline{C_v}}(w)\ge\alpha,\xi_{\bar r}(q)\ge\alpha,\xi_{\overline{C_b}}(b)\ge\alpha.$$

Case (vi)

$$\xi_{\bar\lambda}(y)\ge\alpha,\xi_{\bar\mu}(k)\ge\alpha,\xi_{\bar V}(v)\ge\alpha,\xi_{\bar\eta}(t)\ge\alpha,$$
$$\xi_{\overline{C_y}}(a)\ge\alpha,\xi_{\overline{C_{set}}}(p)=\alpha,\xi_{\overline{C_h}}(h)\ge\alpha,$$
$$\xi_{\overline{C_v}}(w)\ge\alpha,\xi_{\bar r}(q)\ge\alpha,\xi_{\overline{C_b}}(b)\ge\alpha.$$

Case (vii)

$$\xi_{\bar\lambda}(y)\ge\alpha,\xi_{\bar\mu}(k)\ge\alpha,\xi_{\bar\gamma}(v)\ge\alpha,\xi_{\bar\eta}(t)\ge\alpha,$$
$$\xi_{\overline{C_y}}(a)\ge\alpha,\xi_{\overline{C_{set}}}(p)\ge\alpha,\xi_{\overline{C_h}}(h)=\alpha,$$
$$\xi_{\overline{C_v}}(w)\ge\alpha,\xi_{\bar r}(q)\ge\alpha,\xi_{\overline{C_b}}(b)\ge\alpha.$$

Case (viii)

$$\xi_{\bar\lambda}(y)\ge\alpha,\xi_{\bar\mu}(k)\ge\alpha,\xi_{\bar\gamma}(v)\ge\alpha,\xi_{\bar\eta}(t)\ge\alpha,$$
$$\xi_{\overline{C_y}}(a)\ge\alpha,\xi_{\overline{C_{set}}}(p)\ge\alpha,\xi_{\overline{C_h}}(h)\ge\alpha,$$
$$\xi_{\overline{C_v}}(w)=\alpha,\xi_{\bar r}(q)\ge\alpha,\xi_{\overline{C_b}}(b)\ge\alpha.$$

Case(ix)

$$\xi_{\bar\lambda}(y)\ge\alpha,\xi_{\bar\mu}(k)\ge\alpha,\xi_{\bar\gamma}(v)\ge\alpha,\xi_{\bar\eta}(t)\ge\alpha,$$
$$\xi_{\overline{C_y}}(a)\ge\alpha,\xi_{\overline{C_{set}}}(p)\ge\alpha,\xi_{\overline{C_h}}(h)\ge\alpha,$$
$$\xi_{\overline{C_v}}(w)\ge\alpha,\xi_{\bar r}(q)=\alpha,\xi_{\overline{C_b}}(b)\ge\alpha.$$

Case(x)

$$\xi_{\bar\lambda}(y)=\alpha,\xi_{\bar\mu}(k)\ge\alpha,\xi_{\bar\gamma}(v)\ge\alpha,\xi_{\bar\eta}(t)\ge\alpha,$$
$$\xi_{\bar X}(x)\ge\alpha,\xi_{\overline{C_y}}(a)\ge\alpha,\xi_{\overline{C_{set}}}(p)\ge\alpha,\xi_{\overline{C_h}}(h)\ge\alpha,$$
$$\xi_{\bar D}(u)\ge\alpha,\xi_{\overline{C_v}}(w)\ge\alpha,\xi_{\bar r}(q)\ge\alpha,\xi_{\overline{C_b}}(b)=\alpha.$$

This can be accomplished by using parametric NLP techniques. The NLP techniques to find the lower and upper bounds of α cut of $\xi_{\overline{TCU}}(Z)$ for case (i) as

$$[TCU]_\alpha^{L_1} = \min\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'} \quad (5a)$$

$$[TCU]_\alpha^{U_1} = \max\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'} \quad (5b)$$

For case (ii) as

$$[TCU]_\alpha^{L_2} = \min\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'} \quad (5c)$$

$$[TCU]_\alpha^{U_2} = \max\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'} \quad (5d)$$

For case (iii) as

$$[TCU]_\alpha^{L_3} = \min\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'} \quad (5e)$$

$$[TCU]_\alpha^{U_3} = \max\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'} \quad (5f)$$

For case (iv) as

$$[TCU]_\alpha^{L_4} = \min\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'} \quad (5g)$$

$$[TCU]_\alpha^{U_4} = \max\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'} \quad (5h)$$

For case (v) as

$$[TCU]_\alpha^{L_5} = \min\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'} \quad (5i)$$

$$[TCU]_\alpha^{U_5} = \max\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'} \quad (5j)$$

For case (vi) as

$$[TCU]_\alpha^{L_6} = \min\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'} \quad (5k)$$

$$[TCU]_\alpha^{U_6} = \max\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'} \quad (5l)$$

For case (vii) as

$$[TCU]_\alpha^{L_7} = \min\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'} \quad (5m)$$

$$[TCU]_\alpha^{U_7} = \max\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'} \quad (5n)$$

For case (viii) as

$$[TCU]_\alpha^{L_8} = \min\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'} \quad (5o)$$

$$[TCU]_\alpha^{U_8} = \max\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'} \quad (5p)$$

For case (ix) as

$$[TCU]_\alpha^{L_9} = \min\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'} \quad (5q)$$

$$[TCU]_\alpha^{U_9} = \max\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'} \quad (5r)$$

For case (x) as

$$[TCU]_{\alpha}^{L_{10}} = \min\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'} \qquad (5r)$$

$$[TCU]_{\alpha}^{U_{10}} = \max\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'} \qquad (5s)$$

As $\lambda(\alpha), \mu(\alpha), V(\alpha), \eta(\alpha), C_y(\alpha), C_{set}(\alpha), C_h(\alpha), C_v(\alpha), r(\alpha), C_{busy}(\alpha)$ are given in equation (5a-s)

$$y \in \lambda(\alpha), k \in \mu(\alpha), v \in V(\alpha), t \in \eta(\alpha), a \in A(\alpha), p \in C_{set}(\alpha), h \in C_h(\alpha), w \in C_v(\alpha), q \in r(\alpha), b \in C_{busy}(\alpha)$$

can be replaced by

$$y \in [y_\alpha^L\ y_\alpha^U], k \in [k_\alpha^L\ k_\alpha^U], v \in [v_\alpha^L\ v_\alpha^U], t \in [t_\alpha^L\ ty_\alpha^U], a \in [a_\alpha^L\ a_\alpha^U], p \in [p_\alpha^L\ p_\alpha^U], h \in [h_\alpha^L\ h_\alpha^U], w \in [w_\alpha^L\ w_\alpha^U], q \in [q_\alpha^L\ q_\alpha^U], b \in [b_\alpha^L\ b_\alpha^U]$$

which are given by the α cuts and in turn they form a nested structure with respect to α. Hence for given $0<\alpha_2<\alpha_1<1$ we have

$$[y_{\alpha_1}^L\ y_{\alpha_1}^U] \subseteq [y_{\alpha_2}^L\ y_{\alpha_2}^U], \quad [k_{\alpha_1}^L\ k_{\alpha_1}^U] \subseteq [k_{\alpha_2}^L\ k_{\alpha_2}^U]$$
$$[v_{\alpha_1}^L\ v_{\alpha_1}^U] \subseteq [v_{\alpha_2}^L\ v_{\alpha_2}^U], \quad [t_{\alpha_1}^L\ t_{\alpha_1}^U] \subseteq [t_{\alpha_2}^L\ t_{\alpha_2}^U]$$
$$[a_{\alpha_1}^L\ a_{\alpha_1}^U] \subseteq [a_{\alpha_2}^L\ a_{\alpha_2}^U] \quad [p_{\alpha_1}^L\ p_{\alpha_1}^U] \subseteq [p_{\alpha_2}^L\ p_{\alpha_2}^U],$$
$$[h_{\alpha_1}^L\ h_{\alpha_1}^U] \subseteq [h_{\alpha_2}^L\ h_{\alpha_2}^U], \quad [w_{\alpha_1}^L\ w_{\alpha_1}^U] \subseteq [w_{\alpha_2}^L\ w_{\alpha_2}^U]$$
$$[q_{\alpha_1}^L\ q_{\alpha_1}^U] \subseteq [q_{\alpha_2}^L\ q_{\alpha_2}^U], \quad [b_{\alpha_1}^L\ b_{\alpha_1}^U] \subseteq [b_{\alpha_2}^L\ b_{\alpha_2}^U]$$

Hence the lower bound of the $[TCU]_\alpha^L$ becomes

$$[TCU]_\alpha^L = \min\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'}\} \qquad (6a)$$

and the upper bound of the $[TCU]_\alpha^L$ becomes

$$[TCU]_\alpha^U = \max\{\frac{1}{D_R(m,N)}[A_{Sos}^R + Z_{Sos}^R(m) + h(\frac{1}{y}) + (1-\rho_{Sos})(\frac{1}{y})] + A_{Sos}^{'}\} \qquad (6b)$$

where

$$y_\alpha^L \le y \le y_\alpha^U, k_\alpha^L \le k \le k_\alpha^U, v_\alpha^L \le v \le v_\alpha^U, t_\alpha^L \le t \le t_\alpha^U, a_\alpha^L \le a \le a_\alpha^U, p_\alpha^L \le p \le p_\alpha^U, h_\alpha^L \le h \le h_\alpha^U, w_\alpha^L \le w \le w_\alpha^U, q_\alpha^L \le q \le q_\alpha^U, b_\alpha^L \le b \le b_\alpha^U$$

that is at least one of y,k,v,t,a,p,h,w,q,b must hit the boundaries of their α cut that satisfy $\xi_{\overline{TCU}}(Z) = \alpha$.

The crisp interval $[TCU]_\alpha^L, [TCU]_\alpha^U$ obtained from (6a)and(6b) represents α cuts of $[TCU]$. Further by applying the results of Zimmerman(2001) and convexity property, we obtain

$[TCU]_{\alpha_1}^L \ge [TCU]_{\alpha_2}^U$ *and* $[TCU]_{\alpha_1}^L \le [TCU]_{\alpha_2}^U$, where $0<\alpha_2<\alpha_1<1$. In both $[TCU]_\alpha^L and [TCU]_\alpha^U$ are invertible with respect to α then the left shape function $L(Z) = [(TCU)_\alpha^L]^{-1}$ and right shape function $R(Z) = [(TCU)_\alpha^U]^{-1}$ can be derived as

$$\xi_{\overline{TCU}}(Z) = \begin{cases} L(Z) & [TCU]_{\alpha=0}^L \le Z \le [TCU]_{\alpha=1}^L \\ 1 & [TCU]_{\alpha=1}^L \le Z \le [TCU]_{\alpha=1}^U \\ R(Z) & [TCU]_{\alpha=1}^U \le Z \le [TCU]_{\alpha=0}^U \end{cases}$$

In many cases the value of $\{[TCU]_\alpha^L [TCU]_\alpha^U / \alpha \in [0,1]\}$ cannot be solved analytically, consequently a closed form membership function of TCU cannot be obtained. The numerical solutions for $[TCU]_\alpha^L and [TCU]_\alpha^U$ at different levels of α can be collected that approximate the shape of L(Z) and R(Z) (ie),the set of intervals $\{[TCU]_\alpha^L [TCU]_\alpha^U / \alpha \in [0,1]\}$ will estimate the shapes.

V. NUMERICAL EXAMPLE

Consider a $M^X/(G_1G_2)/1/MV$ queuing system. The corresponding cost parameters such as the arrival rate, service rate μ, vacation parameter V, expected group size X, Startup cost per cycle C_y, holding cost C_h, setup cost C_{set}, operating cost C_{busy}, reward cost C_v are fuzzy numbers.

Let $\bar{\lambda} = [0.4 \quad 1.2 \quad 2.0 \quad 2.8]$,
$\bar{\mu} = [4.1 \quad 4.2 \quad 4.3 \quad 4.4]$
$\bar{\gamma} = [5 \quad 6 \quad 7 \quad 8]$, $\bar{\eta} = [0.2 \quad 0.8 \quad 1.6 \quad 2.4]$
$\overline{C_h} = [0.5 \quad 1 \quad 1.5 \quad 2]$,
$\overline{C_y} = [50 \quad 100 \quad 150 \quad 200]$,
$\overline{C_{set}} = [10 \quad 20 \quad 30 \quad 40]$, $\bar{r} = [.2 \quad .6 \quad 1 \quad 1.4]$,
$\overline{C_v} = [.6 \quad .8 \quad 1 \quad 1.2]$, $\overline{C_{busy}} = [5 \quad 10 \quad 15 \quad 20]$
we know that

$$TCU = \frac{1}{D_R(m,N)}\left[\frac{A^R_{Sos} + Z^R_{Sos}(m) + h +}{(1-\rho_{Sos})(1/y)}\right] + A^{'}_{Sos}$$

By considering E(x) = 0.01, E(x(x-1))=0.020 then the total cost per unit system in the fuzzy environment it becomes

$$A^R_{Sos} = \begin{bmatrix} a(1-\rho_{Sos}) + pE(D)(1-\rho_{Sos}) + \\ h\dfrac{yE(X)}{2}E(D^2) \end{bmatrix}$$

$$A^{'}_{Sos} = b\rho_{Sos} + hL_{Sos}$$

$$D_R(m,N) = E(v) + E(u)(1/y)$$
$$\rho_{Sos} = yE(X)(E(S_1) + qE(S_2))$$
$$L_{Sos} = \begin{bmatrix} \rho_{sos} + yE(X(X-1)E(S_{Sos}) + \\ \dfrac{(yE(X))^2 E(S^2_{Sos})}{2(1-\rho_{sos})} \end{bmatrix}$$
$$E(S_{Sos}) = E(S_1) + qE(S_2)$$
$$E(S^2_{Sos}) = \left[E(S^2{}_1) + (qE(S_1)E(S_2) + qE(S^2_2)\right];$$

$E(S^2_1) = 2(1/k^2) = E(S^2_2)$ $E(u^2) = 4/3t^2$ and y,k,v,t,a,p,h,w,q,b are the fuzzy variables corresponding to
$\bar{\lambda}, \bar{\mu}, \bar{\gamma}, \bar{\eta}, \overline{C_y}, \overline{C_{set}}, \overline{C_h}, \overline{C_{v.}}, \bar{r}, \overline{C_{busy}}$
respectively. Thus

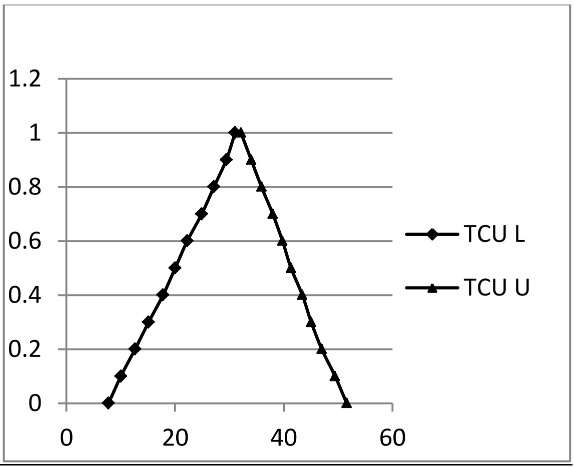
$$[y^L_\alpha \quad y^U_\alpha] = [0.4+\alpha \quad 2.8-\alpha]$$
$$[k^L_\alpha \quad k^U_\alpha] = [4.1+\alpha \quad 4.4-\alpha]$$
$$[v^L_\alpha \quad v^U_\alpha] = [5+\alpha \quad 8-\alpha]$$
$$[t^L_\alpha \quad t^U_\alpha] = [0.2+\alpha \quad 2.4-\alpha]$$
$$[h^L_\alpha \quad h^U_\alpha] = [0.5+\alpha \quad 2-\alpha]$$
$$[a^L_\alpha \quad a^U_\alpha] = [50+\alpha \quad 200-\alpha]$$
$$[p^L_\alpha \quad p^U_\alpha] = [10+\alpha \quad 40-\alpha]$$
$$[q^L_\alpha \quad q^U_\alpha] = [0.2+\alpha \quad 1.4-\alpha]$$
$$[w^L_\alpha \quad w^U_\alpha] = [0.6+\alpha \quad 1.2-\alpha]$$
$$[b^L_\alpha \quad b^U_\alpha] = [5+\alpha \quad 20-\alpha]$$

By substituting the above values, the effect of parameters on the total cost per unit of the system(TCU) is tabulated and its graphical representation is shown below.

Table1: The α cuts for the performance measure of TCU

α	TCU L	TCU U
0	7.7483	51.6002
0.1	10.0211	49.4601
0.2	12.6253	47.0035
0.3	15.1286	45.0601
0.4	17.7785	43.4347
0.5	20.0174	41.3475
0.6	22.2421	39.7715
0.7	24.9075	37.9854
0.8	27.1548	35.9062
0.9	29.4235	34.0219
1	31.0559	32.1269

Fig 1: The membership function for fuzzy TCU



Here we perform α cuts for fuzzy TCU at eleven distinct α levels of 0,0.1, 0.2…1.0. Crisp interval of fuzzy TCU in system for different possibilities of α level, is presented in table 1. Fig 1 depicts the rough shape of TCU constructed from α value. Further, we find that the above information is very useful for designing the fuzzy queuing system.

VI. CONCLUSION

The fuzzy queuing model has more applicability in the real environments than the crisp systems. This paper applies the concept of α cut and Zadeh's extension principle to fuzzy Bi level threshold policy of $M^X/(G1,G2)/1/MV$ queuing system and thereby deriving the membership function of the total cost per unit system for this model . We find that it is more meaningful to express TCU as a membership function rather than by crisp values (i.e) as fuzzy performance measures. The benefit and significance of such a fuzzy performance measure include maintaining the fuzziness of input information completely and the results can be used to represent the fuzzy system more accurately. Thus it can be concluded that the fuzzy cost based queuing systems are much more useful than the commonly used crisp queues.

REFERENCES

[1]Chauan KJ, Hung Hsim I and Hong LC (2007). *On retrial queuing model with fuzzy parameters. Physica* A: Statistical Mechanics and its Applications 374(1) 272-280.

[2] Choudhury G and Paul M (2006). *A batch arrival queue with a second optional service channel under N policy.* Stochastic Analysis and Application 24 1-21.

[3] Hur S and Park J (1999). *The effect of different arrival rates on N policy of M/G/1 with server setup.* Applied Mathematical Modeling 23 289-299.

[4] Jeeva M and E.Rathnakumari. *Fuzzy cost computation of M/M/1 and M/G/1 queuing models* ,British journal of mathematics and computer science,vol.4(1) pp.120-132,2013

[5] Julia Rose Mary K, Afthab Begum MI, and Jamila PM (2011). *Bi-level threshold policy of $M^X/(G1,G2)/1$ queue with early setup and single vacation.* International Journal of Operational Research 10(4) 469-492.

[6] Kanti Swarap and Gupta, *Operations research,* Fourteenth edition, Sultan Chand & sons, New Delhi, 2008

[7] Kao C, Li C and Chen S (1999). *Parametric programming to the analysis of fuzzy queues.* Fuzzy Sets and System 107 93-100.

[8] Ke JC (2001). *The control policy of $MX/G/1$ Queing system with startup and two vacation types.* Mathematical Methods of Operations Research 54(31) 471-490.

[9] Lee HS and Srinivasan MM (1989). *Control Policies for the $MX/G/1$ queuing system.* Management Sciences 35(6) 708-721.

[10] Lee HW, Lee SS and Park JO (1994 a). *Operating Characteristics of $MX/G/1$ queue with N Policy.* Queuing Systems 15 378-399.

[11] Lee HW, Lee SS and Park JO (1994 b). *Analysis of the $MX/G/1$ queue with N Policy and multiple vacations.* Journal of the Applied Probability 31 476-496.

[12] Lee HW, Park NL and Jeon J (2003). *Queue length analysis of a batch arrival queues under bi-level threshold control with early setup.* International Journal of System Science 34(3) 195-204.

[13] Li RJ and Lee ES (1989).*Analysis of fuzzy queues.* Computers and Mathematics with Application 17(7) 1143-1147.

[14] *Madan KC (2000). An M/G/1 queue with second optional service.* Queuing Systems 34 37-46.

[15] Madan KC and Choudhury G (2006).*Steady state analysis of an $X/(G1,G2)/1$ queue with restricted admissibility and random setup time.* Information and Management Science 17(2) 33-56.

[16] Nagi DS and Lee ES (1992). *Analysis and simulation of fuzzy queues.* Fuzzy Sets and Systems 46 321- 30.

[17] Ramesh.R. and Kumara G ghuru, *A batch-arrival queue with multiple servers and fuzzy parameters:* parametric programming approach, IJSR,vol 2,issue 9, 2013

[18] Taha HA. *Operations Research*, An introduction, Seventh edition, Prentice Hall, New Jersy;2003

[19] Zadeh, L.A, *Fuzzy sets*. Information and control, 8(3), pp.338-353,1965

[20] Zimmermann HF (2001). *Fuzzy Set Theory and its Application,* 4th edition (Kluwer Academic, Boston).

