Channel Capacity Analysis in MIMO-OFDM System using Water Filling Algorithm

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Abstract— **In this paper we have implemented water filling algorithm for allocating the power in order to increase the channel capacity of MIMO-OFDM system. From the considerations, channel is assumed as flat fading channel and the comparison is made for different MIMO-OFDM system using water filling algorithm with allocated power. The main idea is to observe the efficiency of channel capacity and outage probability of the system using water filling algorithm.**

Keywords*—Multi Input Multi Output (MIMO), Water filling algorithm, Capacity, Outage probability, Signal to Noise Ratio (SNR), Orthogonal Frequency Division Multiplexing (OFDM).***

I. INTRODUCTION

Large capacity gains in wireless systems can be achieved by using multiple antennas at the receiver and the transmitter [1] [2] [3]. These so called multiple-input multiple output (MIMO) systems are therefore of great interest to the wireless industry. One of the main key feature of MIMO-OFDM system is the possibility of multiplexing signals in space. With a MIMO-OFDM system, the data stream from a single user is demultiplexed into separate sub channels, each sub channel stream is then encoded into channel symbols. It is common to impose the same data rate on all transmitters, but adaptive modulation rate can also be utilized on each of the sub streams [1]. The signals are received by receive antennas.

MIMO-OFDM systems provide a number of advantages over SISO communication. Sensitivity to fading is reduced by the spatial diversity provided by multiple spatial paths. Under certain environmental conditions, the power requirements associated with high spectral efficiency communication can be significantly reduced. Here the spectral efficiency is defined as the total number of information bit per second per Hertz transmitted from one array to the other. As compared to SIMO system, MIMO-OFDM system need lesser transmit power to achieve the same capacity [4]. The capacity of a MIMO-OFDM system can further be increased if we know the channel parameters both at transmitter and at the receiver and assign extra power at the transmitter by allocating the power according to the water filling algorithm to all the channels. As we need to minimize the energy consumed by the circuit and wants to maximize the capacity and that is possible only if we use multiple MIMO-OFDM system.

This paper organized as follows. Section II gives, the brief explanation of MIMO-OFDM system and system model. Section III basic channel capacity and MIMO-OFDM channel capacity. Section IV gives brief introduction of water filling algorithm, Water filling capacity of MIMO-OFDM channel, section V results and discussion and section VI conclusion.

II. MIMO-OFDM SYSTEM

The MIMO-OFDM system achieve higher data rate, wide coverage, and increased reliability [5]. A core idea in MIMO-OFDM system is space time signal processing in which time is complemented with the spatial dimension inherent in the use of multiple spatially distributed antennas. As such MIMO-OFDM system can be viewed as an extension of the smart antennas, a popular technology using antenna arrays for improving wireless transmission dating back to several decades. MIMO-OFDM system is ability to turn multipath propagation, traditionally a pit fall of wireless transmission, into a benefit for the user. Thereby, enhancing wireless transmission over the MIMO-OFDM channel improves the channel capacity.

A. System model

Consider a MIMO-OFDM system with **Nr** receive and **Nt** transmit antenna .The system is represented as

$$\mathbf{y=Hx+n} \tag{1}$$

Where x is the (**N**tX1) transmit vector, y is the (**Nr**X1) receive vector, **H** is the **(Nr**X**Nt)** channel matrix and **n** is the (NrX1) additive white Gaussian noise (AWGN) vector at a given instant in time with its entry being independent and identically distributed zero-mean unit-variance Gaussian random variables. Generally the channel matrix is denoted by $\{H_{ij}\}$, this represents the complex gain of the channel between the j^{th} transmitter and the i^{th} receiver[6].

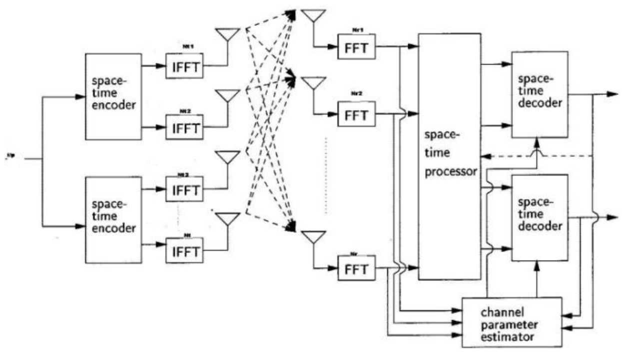

Figure 1. MIMO-OFDM communication system model

III. CHANNEL CAPACITY

Shannon defined the capacity of a channel as the maximum data rate at which data transmitted from transmitter, when passed through the channel, can be received at some receiver with negligible chance or error. Then the channel capacity is a convenient measure to analyze the potential gain of MIMO-OFDM system compared to SISO system. If the transmitted data are viewed as random variables, then the channel capacity referred to the maximum mutual information between them, then the channel capacity **C** can be expressed as

$$C = \max_{f(x)} I(x;y) \tag{2}$$

Where the maximization taken over all possible probability distributions f(x) of x, for a narrow band channels with AWGN, Shannon derived the normalized capacity (capacity per unit BW) to be

$$C = \log_2(1+\rho)\,\text{bps/Hz} \tag{3}$$

A. MIMO-OFDM channel capacity

The capacity of MIMO-OFDM channels is an extension of mutual information formula, and then the capacity of random MIMO-OFDM channel with power constraint P_T can be expressed as

$$c = E_H\left[\max_{P(s):tr(\phi)P\le P_T} I(x;y)\right] \tag{4}$$

Where $\phi = E\left[xx^{\dagger}\right]^2$ is the covariance matrix of the transmit signal vector x. Acquiring channel knowledge at the transmitter is in general very difficult in practical system. When the transmitter has no channel knowledge, it is optimal to evenly distribute the available power ρ among the transmit antenna. The MIMO-OFDM channel capacity with $\rho = \frac{P_T}{\sigma^2}$ can be written as

$$C = \left[\log_2 \det\left(\frac{\rho}{Nt} HH^{\dagger}\right) + I_{Nr}\right]\text{bps/Hz} \tag{5}$$

This is for any (N_t,Nr) MIMO-OFDM system. Where ρ is the average signal to noise ratio (SNR) per receive antenna, in above equation "det" means determinant, I_{NrXNt} means NrXNt identity matrix and "†" means transpose conjugate.

B. Outage probability

Another measure of channel capacity that is frequently used in outage capacity. With outage channel capacity the channel capacity is associated to an outage probability [7]. Capacity treated as random variable depends on the channel instantaneous reasons and remains constants. If the channel capacity falls below the outage capacity there is no possibility that the transmitted block of information can be decoded with no errors, which error coding scheme employed. The outage probability is

$$P_{out} = P\left(\log\det(I_{Nr} + HQH^{\dagger}) < R\right) \tag{6}$$

Where $Q = E\left[HH^{\dagger}\right]$ is covariance, R is information rate to be transmitted. It is conjectured that P_{out} is minimized by using a uniform power allocation over a subset of the transmit antennas.

IV. WATER FILLING ALGORITHM

Water filling is the solution of several optimization problems related to channel capacity. The well known water filling algorithm solves the problem of maximizing the mutual information between the input and output of a channel. This capacity achieving solution has seems to be pouring water over a surface given by the inverse of the sub channels gains hence the name water filling or water pouring[8].

C. Water filling capacity of MIMO-OFDM channel

The sub channels are unable to accessible when the channel knowledge is absent at the transmitter. So the power allocation in the entire sub channel is logical under this scenario. When the channel state information is available at transmitter, the water filling method is used to optimize the transmitted signal power.

The principle of water filling algorithm is a large power is assigned to better sub channel and vice versa. The aim of this algorithm is to allocate power across the channels so as to enhance the total capacity. By the knowledge of channel matrix H it reflects the channel strength and the amount of power allocated to each channel. This power allocation is subject to the constraint that the sum of the power is equal to the total power, P_T available at transmitter [9].

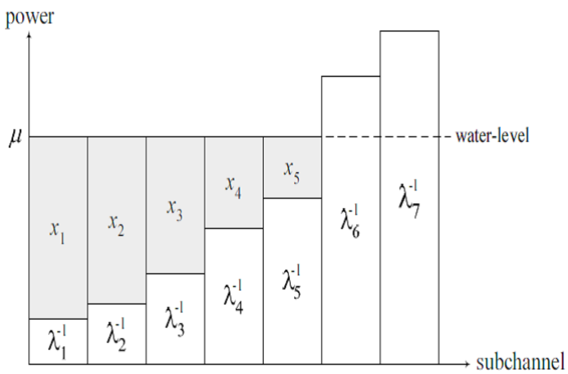

Figure 2. Water filling in parallel sub channels

From the figure$\lambda_1,\lambda_2......,\lambda_M$ with λ_i as the positive square root of i^{th} eigen value and i=1 to M non zero λ values [3] and M=min{Nr,Nt}. Figure 2 shows the water filling procedure into the sub channels of the MIMO-OFDM channels [10].

So we know the total power available, P_T and gains of parallel sub channels then the optimum power allocated to the i^{th} sub channel is

$$P_i = \left(\mu - \frac{1}{\lambda_i^2}\right)^+ \text{, and i=1,2,3........,M.} \qquad (7)$$

Where $(x)^+ = \max(0,x)$ and μ is the "water level". used to determine the power assigned to each of the sub channel of the MIMO-OFDM channel. M chosen to meet the power constraint so that

$$\sum_{i=1}^{M} P_i = P_T \qquad (8)$$

If the P_i positive then the power is allocated to the i^{th} sub channel otherwise the sub channel is unused i=1,2,...M: where M is the number of sub channels that have survived after checking the above conditions and are to be used for transmission of signal. Now the channel capacity of MIMO-OFDM system with water filling algorithm can be expressed as [3]

$$C_{wf} = \sum_{i=1}^{M} \log_2\left[1 + \left(\frac{P_i}{\sigma^2}\right) * \lambda_i^2\right] \qquad (9)$$

If the channel is known at the transmitter, the capacity can be enhanced by using the good channels i.e. those with the highest gain by applying an unequal power distribution. Hence capacity gain is (C_{wf} –C).

V Results and Discussion

The MIMO-OFDM channel capacity has been simulated for different number of transmitters and receiver antennas using water filling algorithm for allocation of optimum power ,$P_{T=1}$ to the parallel sub channels. The mean capacity or Ergodic capacity [11], water filling gain in bps/Hz and outage probability of the MIMO-OFDM channel (NtXNr) is plotted against SNR in dB.

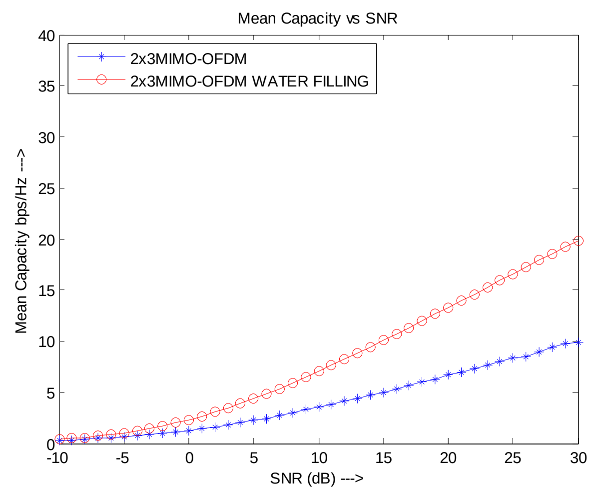

Figure 3. Mean capacity versus SNR in dB for 2x3 MIMO-OFDM system with and without using water filling algorithm

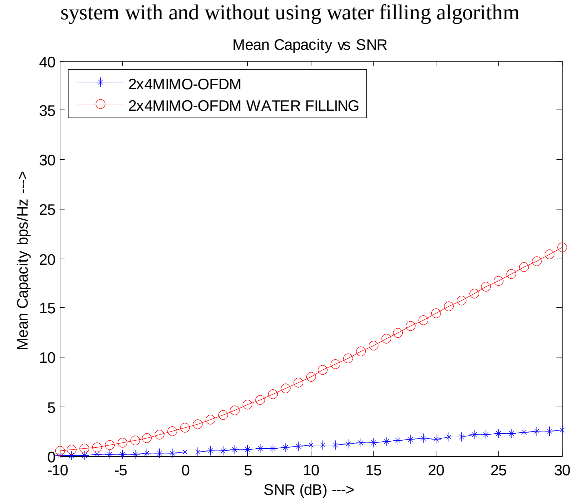

Figure 4. Mean capacity versus SNR in dB for 2x4 MIMO-OFDM system with and without using water filling algorithm

Figures 3, 4 and 5 shows that Capacity of MIMO-OFDM Systems with and without water filling algorithm results increasing capacity for 2x3, 2x4, 4x4 systems.

The outage probabilities of MIMO-OFDM system with and without water filling algorithm are reduced by increasing the number of antennas. The outage probabilities of MIMO-OFDM system with and without water filling algorithm for 2x3, 2x4, 4x4 systems are shown in the figures 6, 7, 8.

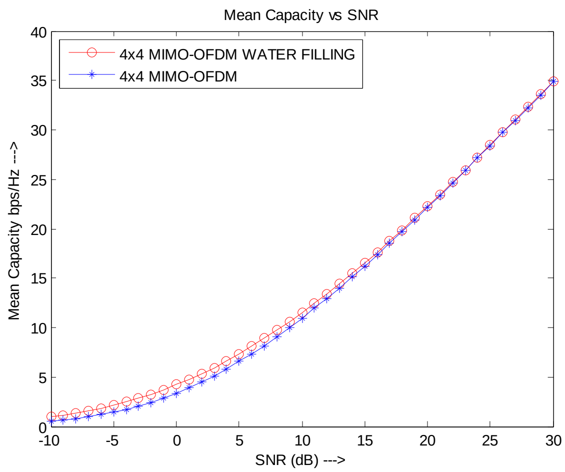

Figure 5. Mean capacity versus SNR in dB for 4x4 MIMO-OFDM system with and without using water filling algorithm.

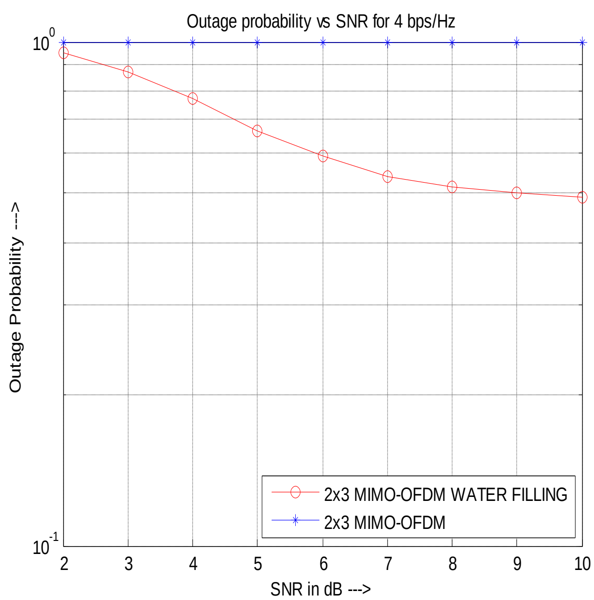

Figure 6. Outage probability versus SNR for 4bps/Hz for 2x3 MIMO-OFDM system

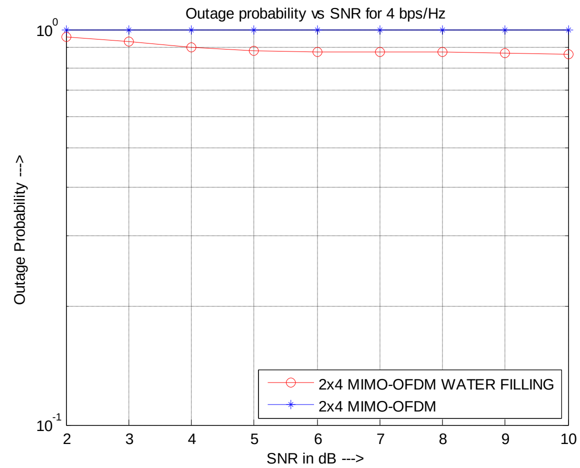

Figure 7. Outage probability versus SNR for 4bps/Hz for 2x4 MIMO-OFDM system.

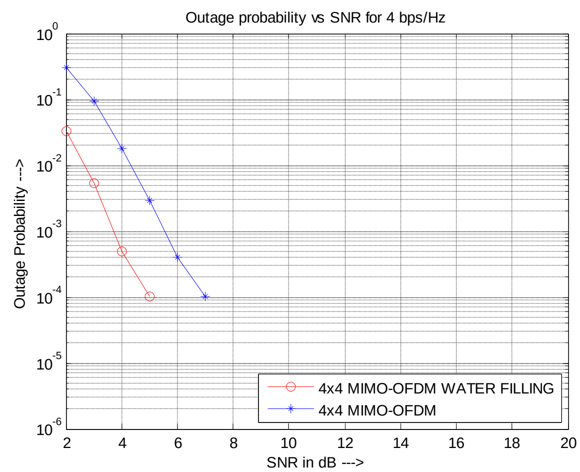

Figure 8. Outage probability versus SNR for 4bps/Hz for 4x4MIMO-OFDM system

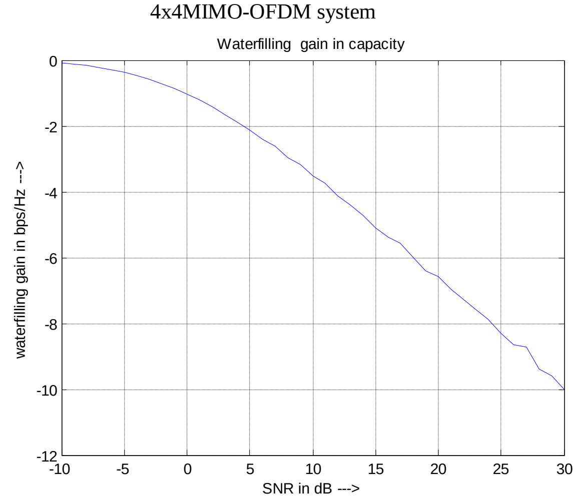

Figure 9. Water filling gain in capacity for 2x3 MIMO-OFDM system.

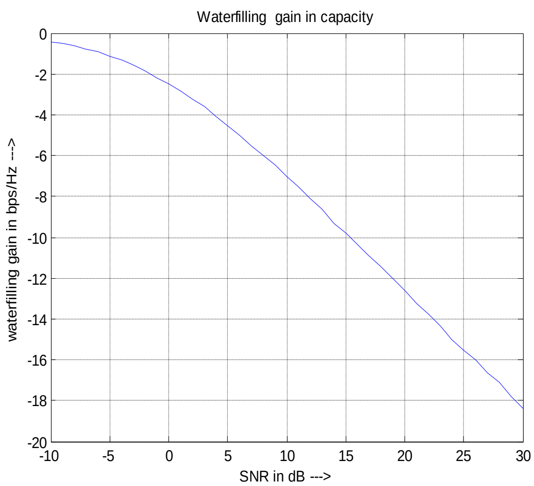

Figure 10. water filling gain in capacity for 2x4 MIMO-OFDM system

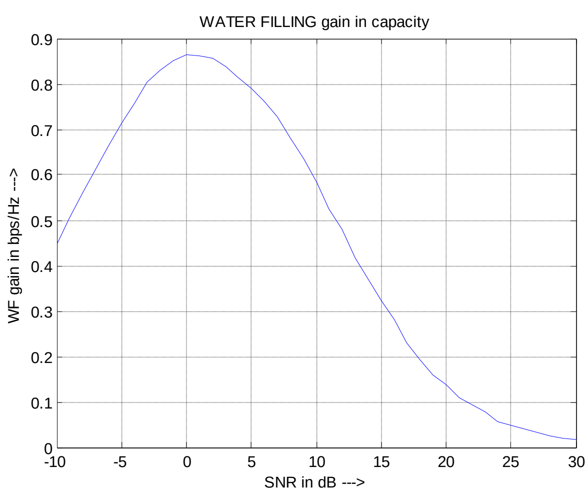

Figure 11. Water filling gain in capacity for 4x4 MIMO-OFDM system

The water filling gain capacity of MIMO-OFDM for 2x3, 2x4, 4x4 systems are shown in the figures 9, 10, 11.

The water filling gain capacity is reduced by increasing the number of antennas hence it is observed that there is an improvement in the channel capacity and outage probability by using water filling algorithm.

V. Conclusion

In this paper, a brief introduction on the capacity of MIMO-OFDM channel has been given and the water filling algorithm was implemented in this work. The use of multiple antennas on both the transmitter and receiver side of a communication link have shown to greatly improve the spectral efficiency of both fixed and wireless system. We have developed an understanding and described the mean capacity allocation in wireless cellular network based on the proposed water filling power allocation in order to enhance the capacity of MIMO-OFDM system with different channel assumptions. We observed maximum power is allocated to the channel having greater gain. The graphs of capacity versus SNR shows that the capacity of the MIMO-OFDM channels increases as the number of antennas used at the transmitter and receiver increases. The water filling gain in capacity decreases as increasing the number of transmitter and receiver. The variation of outage probability of the MIMO-OFDM system also discussed.

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