

Performance Analysis of Channel Estimation Based on MMSE Equalizer in OFDM System

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Abstract – To overcome the large number of distortions caused by channel fading, the estimation of the channel is necessary in OFDM system. Various algorithms have been introduced for channel estimation such as least square estimation (LS), MMSE etc. Most of the channel estimation techniques are performed in frequency domain. The conventional method of channel estimation based on pilot channel arrangement process without equalizer has low BER performance. Thus, equalizer is used to reduce the effect of inter symbol interference (ISI). In this paper, we compare the performance of the OFDM system with MMSE equalizer and the system without equalizer in term of BER vs. SNR. The comparison of LSE and MMSE channel estimation algorithm is also done in OFDM system in terms of MSE vs. SNR. The performance of the OFDM system with MMSE equalizer is observed by varying the system parameter. The simulation results shows MMSE channel estimation algorithm is better than LSE algorithm. It also shows that the performance of OFDM system is improved by using MMSE equalizer in comparison with the system without it.

Keywords - OFDM; channel estimation; MMSE equalizer; Cyclic prefix (CP); Pilot channel

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has become a popular technique for transmission of signals over wireless channels. OFDM converts frequency – selective channel into a parallel collection of frequency flat sub channels [1]. In OFDM system, the available spectrums are divided into many orthogonal sub-channels which are used for data transmission. The low symbol rate makes the use of the guard interval between symbols affordable to eliminate ISI [2] and utilizes echoes and time spreading to achieve diversity gain i.e. SNR improvement.

Many channel estimation algorithm have been developed such as least square estimation (LS) [3]. The LS estimation is the simplest channel estimation and having lower complexity.

However, it has larger mean square error (MSE) and easily influenced by noise and inter-carrier interference. In [4], Minimum mean square error (MMSE) algorithm is introduced LMMSE algorithm [5] is a simplified algorithm of Minimum Mean Square Error (MMSE). In [6], kang et al proposed a pilot based channel estimation. In, Non-sampled space multipath channel, the channel impulse response will leak all the energy in time domain [7]. MMSE equalizer designs the filter to minimize $E[|e|^2]$, where 'e' is the error signal, which is the filter output minus the transmitted signal.

In this paper, we focused on the equalizer process as well as pilot based channel estimation method without equalizer method. Equalizer is meant to work in such a way that BER should be low and SNR should be high [8]. Equalizer is used when the signal is received at the receiver after the channel estimation process and after equalizing the signal then only it passes to the demodulation to get the output in binary form. In communication systems, all channel estimation and signal equalization is done on so-called baseband representation of signal and channel. In Pilot channel method, we basically focused on the channel vector coefficient approaches. Basically, two types of pilot channel estimation method occur; Block type and comb type pilot channel estimation [9].

The outline of the paper is as follows: In Section 2, the OFDM system model has been introduced. In section 3, we analyze the architecture of the OFDM system. Similarly, in section 4, we introduced the LS estimation process and MMSE estimation process. In section 5, simulation result is presented and finally in section 6, the desired conclusions are provided.

2. OFDM SYSTEM MODEL

In multipath fading channel, the channel impulse response in time domain could be expressed as

$$h(t) = \sum_{l=0}^{L-1} a_l \delta(t - \tau_l Ts) \quad (i)$$

where L is the number of multipath, a_l is the complex time-varying channel coefficient of the l -th path, Ts is sampling interval, and $\tau_l Ts$ is the delay of the l -th path. When τ_l is an integer, the multipath channel is sample-spaced channel. Or the multipath channel is non-sample-spaced channels [10]. From (i) the channel impulse response after sampling the frequency response of $h(t)$ is expressed as [11]:

$$h(n) = \frac{1}{N} \sum_{l=1}^L a_l e^{-j\frac{\pi}{N}(n+(N-1)\tau_l)} \frac{\sin \pi \tau_l}{\sin \frac{\pi}{N}(\tau_l - n)} \quad (ii)$$

Thus the corresponding frequency response is expressed as:

$$H(k) = \sum_{l=1}^L h_n e^{-j2\pi \frac{kl}{n}} \quad (iii)$$

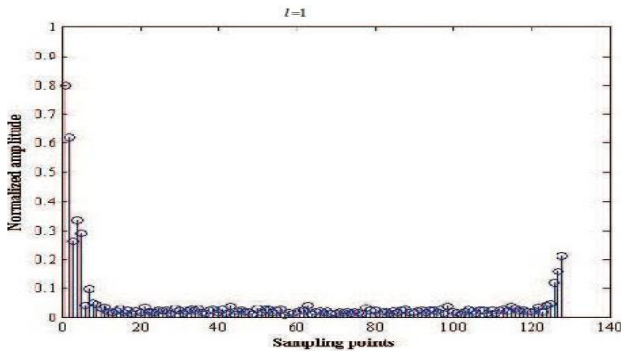


Fig 1:-The channel impulse response in non-Sample channel.

3 SYSTEM ARCHITECTURE

A standard OFDM system is shown in Fig.2 [12]. The information symbols are grouped into blocks and Inverse Fast Fourier Transform (IFFT) is performed on each block then a proper cyclic prefix (CP) extension is added after the modulation of the signal and then transmitted. At the receiver, Fast Fourier Transform (FFT) is performed on each received OFDM symbol after the CP is removed. The received signal at the input of DFT is given by

$$y_k[n] = H_k[n] x_k[n] + w_k[n]; k=1, \dots, N; -\infty < n < \infty \quad (iv)$$

Where $x_k[n]$ is the k th information symbol of the n th OFDM symbol, $H_k[n]$ is the gain of the k th sub channel during the n th OFDM symbol, $w_k[n]$ is the noise, and N is the total number of sub carriers.

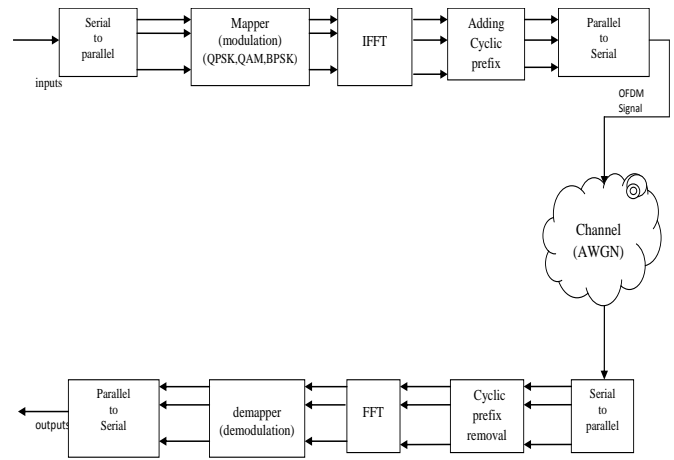


Fig 2:- OFDM Block Diagram

At the sending end, the data streams are modulated by inverse Fast Fourier transform (IFFT) and a cyclic prefix is added for every OFDM symbol to eliminate ISI caused by multi-path fading channel. The received signal can be expressed:-

$$y_j(k) = \sum_{i=1}^{N_T} x_i(k) H_{i,j}(k) + w_j(k) \quad 0 \leq k \leq N-1 \quad (v)$$

Where k is the k -th sub carrier $Y_j(k)$ is the signal of the j -th receive antenna in frequency domain, $x_i(k)$ is the signal of the i -th transmitter antenna, $H_{i,j}(k)$ is the discrete response of the channel on subcarrier k between the i -th transmitter antenna and the j -th receive antenna, and $w_j(k)$ is the complex Gaussian noise with zero-mean and variance $\frac{N_0}{2}$.

Basically, pilots are used to track the phase error if present after frequency correction. Insertion of pilot symbols mean to synchronize and estimation of the channel. More the number of pilots, more the channel capacity. Normally, in IEEE 802.11 OFDM standard, 64 sub carriers are used. Out of which 48 are for data, 4 for pilot and remaining for Zero padding.

4 CHANNEL ESTIMATION ALGORITHM

A. LS Channel Estimation

LS based channel estimation is the simplest channel estimation assuming \hat{H}_{LS} as the estimate of the channel impulse response H . Estimation via LS based process in frequency domain on subcarrier (K) can be obtained as :-

$$\hat{H}_{LS}(K) = \frac{Y(k)}{X(k)} = H(k) + \frac{W(k)}{X(k)} \quad 0 \leq k \leq N-1 \quad (vi)$$

Without using any knowledge of the statistics of the channels, the LS estimators are calculated with very low complexity, but they suffer from a high mean-square error.

B. DFT Based Estimation

In OFDM system, the length of the channel impulse response L is usually less than the length of the cyclic prefix Lg . Conventional DFT-based algorithm just takes advantage of this feature. It transforms the in-frequency channel estimation into in-time channel estimation, considers the part which is larger than Lg as noise, and then treats that part as zero in order to eliminate the impact of the noise.

The algorithm can be described in the following procedures [11].

Step 1: Calculate the LS estimate $\hat{H}_{LS}(k)$ in the usual LS manner.

Step 2: Convert $\hat{H}_{LS}(k)$ to time domain:

$$\hat{h}_{LS}(n) = \text{IFFT}[\hat{H}_{LS}(k)] = h(n) + \tilde{w}(n) \quad 0 \leq n \leq N-1 \quad (\text{vii})$$

Where $\tilde{w}(n) = \text{IFFT}[w(k)/x(k)]$

Step 3: Eliminate the impact of noise in time domain

$$\hat{h}_{DFT}(n) = \begin{cases} \hat{h}_{LS}(n) & 0 \leq n \leq Lg - 1 \\ 0 & Lg \leq n \leq N - 1 \end{cases} \quad (\text{viii})$$

Step 4: Convert time domain response to frequency response by Discrete Fourier transform.

$$\hat{H}_{DFT}(k) = \text{FFT}[\hat{h}_{DFT}(n)] \quad 0 \leq k \leq N-1 \quad (\text{ix})$$

C. MMSE Based Estimation Process

The pilot based process uses MMSE estimator that employs the second-order statistics of the channel condition to minimize the mean-square error and it is denoted by R_{gg} , R_{HH} and R_{YY} the auto covariance matrix of \hat{g} , \hat{H} and \hat{Y} , respectively, and by R_{gy} the cross covariance matrix between \hat{g} and \hat{Y} .

If 'X' is transmitted over a channel 'h' such that $y = X h$ [13]

Error is given as : $e = y' - y$ (x)

Where y' is estimated output.

Mean Square Error = $\text{mean}\{(y' - y)^2\} = E\{(y' - y)^2\}$

where 'E' is operator for expected value.

The estimated channel H_{MMSE} can be found out by the equation

$$H_{MMSE} = F * (R_{gy} * R_{yy}^{-1} * Y) \quad (\text{xi})$$

where F is a noise matrix

$$R_{gy} = R_{gg} * F' * X'$$

$R_{YY} = X * F * R_{gg} * F' * X' + \text{variance of noise} * \text{Identity Matrix}$. Steps involved for calculating the MSE performance is :

- 1) Generate the channel matrix.
- 2) Convert the channel matrix to frequency domain.
- 3) Calculate the MMSE estimator R_{gg} .
- 4) Evaluation of the Auto-covariance matrix of G- R_{gg}

The MMSE estimator yields much better performance than LS estimators, especially under the low SNR scenarios. A major drawback of the MMSE estimator is its high computational complexity, especially as matrix inversions are needed each time the data 'X' changes.

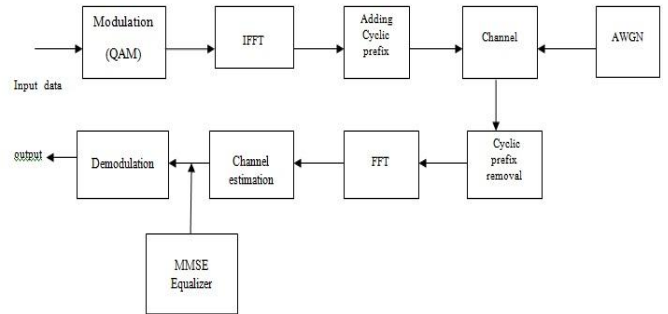


Fig 3: - Estimation of the channel in OFDM system based on MMSE equalizer.

When the equalizer [14] is used in the OFDM system, it provides better BER performance rather than previous. Here, using the QAM modulation technique, the MMSE equalizer is used for the channel estimation to make BER performance better as well as to reduce the complexity than the OFDM system without using the equalizer.

4. SIMULATION RESULT

After the simulation process, we have observed the performance of BER vs.SNR with and without using the MMSE equalizer in OFDM system. In the simulation process, we have used the different number of sub-carrier depending upon the data transmission in the OFDM system. Here we have used NDSC (Number of data sub- carrier) = 64.Likewise, the symbols are modulated by QAM as well as BPSK modulation. Since it is based on the AWGN noise channel, therefore, inside the equalizer process, we have used pre-defined values for the channel

coefficients. We can also see the variation of the channel coefficients values through the graph.

From Fig (4), we can see MMSE estimation is quite better than LSE estimation algorithm. In the conventional method (without equalizer), At SNR = 22, the BER performance is nearly 10^{-3} from fig. (5). But from fig (6), the BER performance is improved to 10^{-5} depending upon the variation of the channel coefficients (h).

Similarly, the variation of the number of data subcarriers and FFT size are also shown in fig (7) & fig (8) respectively.

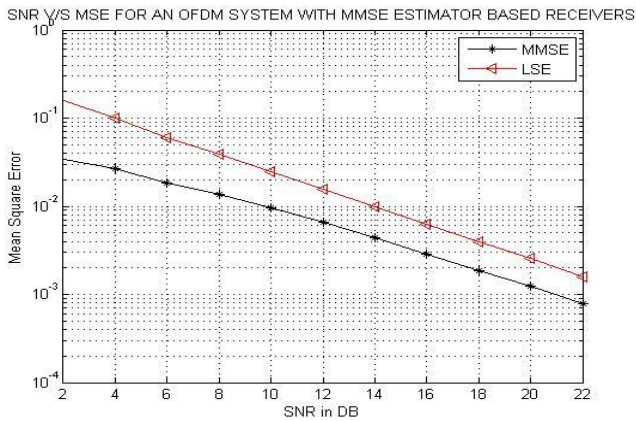


Fig 4:- MSE vs.SNR performance in OFDM system

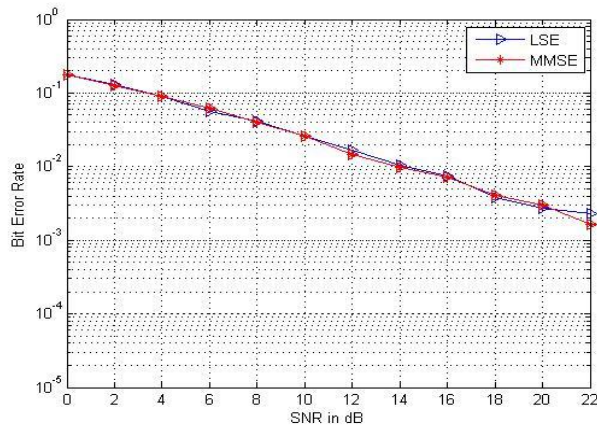


Fig 5:- BER performance in OFDM system without equalizer.

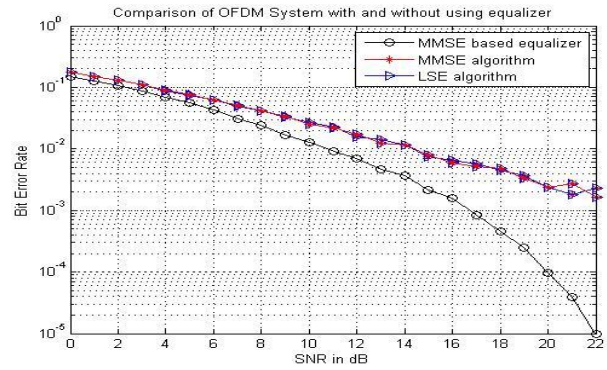


Fig 6:- BER performance in OFDM system with MMSE equalizer

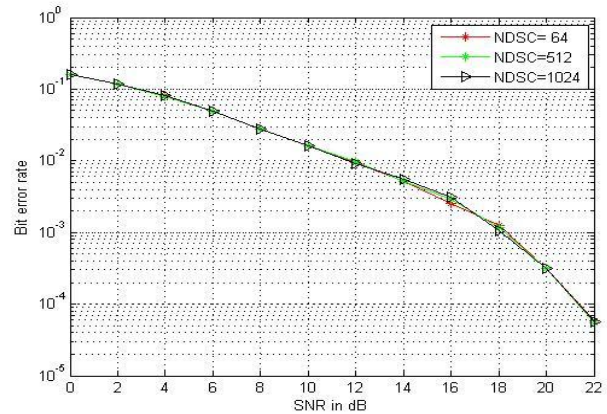


Fig 7:- Variations of the Number of Data Subcarriers (NDSC) based on MMSE equalizer in OFDM system.

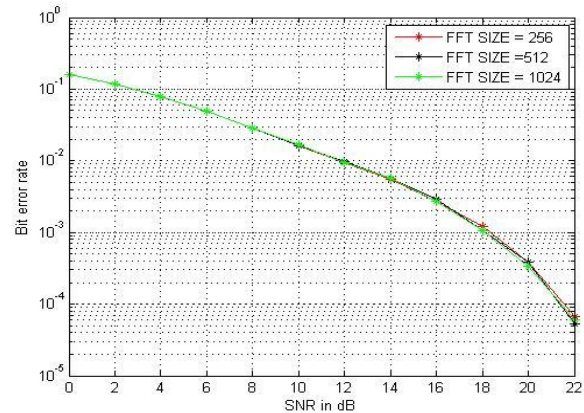


Fig 8:- Variations of the Number of FFT size based on MMSE equalizer in OFDM system.

6. CONCLUSIONS

The proposed method uses the MMSE equalizer in OFDM system in order to get the large gain. The simulation result shows that the OFDM system that uses MMSE equalizer is far better than the system without using it. The simulation result shows that due to effectiveness of MMSE equalizer in OFDM system BER vs. SNR has been improved from 10^{-3} to 10^{-5} . The simulation results also shows that variation of FFT/IFFT size has very less significant effect on improved system. The improved method achieves a satisfying tradeoff between complexity and performance.

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