ANALYSIS OF A TWO-UNIT COLD STANDBY SYSTEM WITH INSTRUCTIONS AND THE ACCIDENTAL EFFECTS

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ABSTRACT

In the present paper, a two-unit cold standby system with instructions and the accidental effects is examined. On the failure of the operative unit, the unit is undertaken for repair by an assistant of an expert repairman who, during his repair time, may meet with an accident which may cause major or minor injury to him. On minor injury, the expert comes to give instructions to his assistant and then after getting the instructions, the assistant repairman does the repair perfectly without meeting with any accident. On the major injury, the expert himself undertakes the unit for further repair. Various measures of the system effectiveness are obtained by making use of semi-Markov processes and regenerative point technique. Study through graphs is also made.

INTRODUCTION

Two-unit standby system have widely been discussed by a large number of researchers in the field of reliability. The system have been analysed under various situations. One of the situations is that the repairman while repairing a failed unit may meet with an accident. Also there may be situation when we have an expert repairman who cannot meet with an accident while repairing. But the expert repairman may be costly and hence he may not be called every time and hence his assistant repairman is called first to repair the failed unit with the known fact that he may meet with an accident while repairing. The expert repairman is called only if urgently required i.e. when the losses are of serious nature.

Thus, in the present paper, we introduce the concept of accident together with two types of repairman in a two-unit cold standby system – one expert repairman and the other assistant. On the failure of a unit, the expert sends his assistant repairman immediately to repair the failed unit. During his repair time, an accident may take place which may cause minor or major injury to the assistant. On the minor injury, the expert repairman comes to the system and gives instructions to the assistant repairman for repairing the rest of the repair of the failed unit. The assistant repairman after getting instructions from the expert does the rest of repair perfectly without meeting further accident. In case of major injury, the expert repairman comes to the system and he himself undertakes the unit for further repair. He also undertakes the failed unit himself in case both the units fail. The expert repairman repairs the unit without meeting with any accident. The expert repairman after repairing a failed unit goes back and returns only if both the units fail or the assistant repairman gets major injury with an accident. The other assumptions are as usual.

System is analysed by making use of semi-Markov process and regenerative point technique and determine the expression for various measures of system effectiveness such as mean time to system failure, steady state availability, total fraction of busy time of expert repairman and of assistant repairman at steady state, total number of visits for the expert repairman per unit time. Using the above measures, the profit is calculated. Also determine the MTSF and the profit for a particular case where the repair time, instruction time follow exponential distribution. The graphs are also plotted for the particular case to study the behaviour of the MTSF and the profit with respect to various rates.

NOTATIONS

O	operative unit
Cs	cold standby
Fra	failed unit under repair of the assistant repairman before the occurrence of accident.
Fre	failed unit under repair of the expert repairman
Fri	failed unit waiting for repair while instructions are going on
Fraa/	the unit is under repair of the assistant repairman after getting instructions
Fw	failed unit waiting for repair
FRa	the repair of the failed unit by the assistant repairman before the occurrence of an accident is continuing from the

	previous state.
FRe	repair of the failed unit by expert repairman is continuing from previous state
FRI	the waiting for repair is continuing from the previous state when the expert repairman is giving instructions to his assistant.
FRaa	repair of the failed unit is continuing from the previous state by the assistant repairman after getting instructions from the expert
a_1	probability that accident does not occur
a_2	probability that accident occurs and the assistant repairman gets minor injury i.e. $= (1 - a_1)p_1$, where p_1 is the probability of minor accident.
a_3	probability that accident occurs and the assistant repairman gets major injury i.e. $= (1 - a_1)p_2$, where p_2 is the probability of major accident
λ	constant failure rate of operative unit
$i(t), I(t)$	p.d.f. and c.d.f. of instruction time
$g_1(t), G_1(t)$	p.d.f. and c.d.f. of repair time of assistant repairman before the occurrence of accident
$g_2(t), G_2(t)$	p.d.f. and c.d.f. of repair time of assistant repairman after the occurrence of accident
$g_e(t), G_e(t)$	p.d.f. and c.d.f. of repair time of expert repairman.

TRANSITION PROBABILITY AND MEAN SOJOURN TIMES

The state transition diagram is shown as in Fig. 1. The epochs of entry into states 0, 1, 2, 3, 5, 8 and 10 are regeneration points and hence these states are regenerative states. The states 4, 6, 7, 8, 9 and 10 are down states.

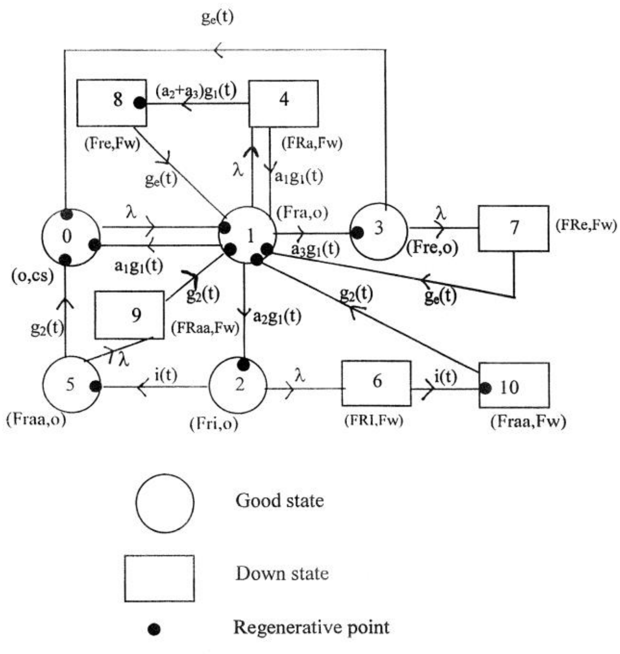


Figure 1

The transition probabilities are:

$$dQ_{01}(t) = \lambda e^{-\lambda t} dt$$

$$dQ_{10}(t) = a_1 e^{-\lambda t} g_1(t) dt$$

$$dQ_{12}(t) = a_2 e^{-\lambda t} g_1(t) dt$$

$$dQ_{13}(t) = a_3 e^{-\lambda t} g_1(t) dt$$

$$dQ_{14}(t) = \lambda e^{-\lambda t} \overline{G_1}(t) dt$$

$$dQ_{11}^{(4)} = a_1 \left(\lambda e^{-\lambda t} \copyright 1\right) g_1(t) dt$$

$$dQ_{18}^{(4)}(t) = (a_2 + a_3)\left(\lambda e^{-\lambda t} \copyright 1\right) g_1(t) dt$$

$$dQ_{25}(t) = e^{-\lambda t} i(t) dt$$

$$dQ_{26}(t) = \lambda e^{-\lambda t} \overline{I(t)} dt$$

$$dQ_{2,10}^{(6)}(t) = \left(\lambda e^{-\lambda t} \copyright 1\right) i(t) dt$$

$$dQ_{30}(t) = e^{-\lambda t} g_e(t) dt$$

$$dQ_{37}(t) = \lambda e^{-\lambda t} \overline{G_e}(t) dt$$

$$dQ_{31}^{(7)}(t) = \left(\lambda e^{-\lambda t} \copyright 1\right) g_e(t) dt$$

$$dQ_{50}(t) = e^{-\lambda t} g_2(t) dt$$

$$dQ_{59}(t) = \lambda e^{-\lambda t} \overline{G_2}(t) dt$$

$$dQ_{51}^{(9)}(t) = \left(\lambda e^{-\lambda t} \copyright 1\right) g_2(t) dt$$

$$dQ_{81}(t) = g_e(t) dt$$

$$dQ_{10,1}(t) = g_2(t) dt$$

The non-zero elements p_{ij} of the transition probability matrix for the system are found out as $p_{ij} = \lim_{s \to 0} q_{ij}^*(s)$.

The mean sojourn time (μ_i) in state *i* are

$$\mu_0 = 1/\lambda, \qquad \mu_1 = \frac{1 - g_1^*(\lambda)}{\lambda}$$

$$\mu_2 = \frac{1 - i^*(\lambda)}{\lambda}, \qquad \mu_3 = \frac{1 - g_e^*(\lambda)}{\lambda}$$

$$\mu_5 = \frac{1 - g_2^*(\lambda)}{\lambda}$$

$$\mu_8 = -g_e^{*}{}'(0), \qquad \mu_{10} = -g_2^{*}{}'(0)$$

The unconditional mean time taken by the system to transit for any regenerative state j, when it is counted from epoch of entrance into that state i, is mathematically stated as

$$m_{ij} = \int t dQ_{ij}(t) = -\lim_{s \to 0} q_{ij}^{*}{}'(s)$$

Thus,

$$m_{01} = \mu_0$$

$$m_{10} + m_{12} + m_{13} + m_{14} = \mu_1$$

$$m_{10} + m_{12} + m_{13} + m_{11}^{(4)} + m_{18}^{(4)} = -g_1^{*}{}'(0) = k_1(say)$$

$$m_{25} + m_{26} = \mu_2$$

$$m_{25} + m_{2,10}^{(6)} = -i^{*}{}'(0) = k_2(say)$$

$$m_{30} + m_{37} = \mu_3$$

$$m_{30} + m_{31}^{(7)} = \mu_8 = m_{81}$$

$$m_{50} + m_{59} = \mu_5$$

$$m_{50} + m_{51}^{(9)} = \mu_{10} = m_{10,1}$$

MEAN TIME TO SYSTEM FAILURE

To determine the MTSF of the system, we regard the failed states of the system absorbing. By probability arguments, we obtain the following recursive relations for $\phi_i(t)$:

$$\phi_0(t) = Q_{10}(t)(s)\phi_1(t)$$

$$\phi_1 = Q_{10}(t)(s)\phi_0(t) + Q_{12}(t)(s)\phi_2(t)$$
$$+ Q_{13}(t)(s)\phi_3(t) + Q_{14}(t)(s)\phi_4(t)$$

$$\phi_2(t) = Q_{25}(t)(s)\phi_5(t) + Q_{26}(t)$$

$$\phi_3(t) = Q_{30}(t)(s)\phi_0(t) + Q_{37}(t)$$

$$\phi_5(t) = Q_{50}(t)(s)\phi_0(t) + Q_{59}(t)$$

Taking L.S.T. of these equations and solving for $\phi_0^{**}(s)$. The mean time to system failure (MTSF) when the system starts from the state 0, is

$$T_0 = \lim_{s \to 0} \frac{1 - \phi_0^{**}(s)}{s} = \frac{N}{D}$$

where

$$N = \mu_0 + \mu_1 + \mu_2 p_{13} + \mu_3 p_{13} + p_{12} p_{25} \mu_5$$

and

$$D = 1 - p_{10} - p_{13} p_{30} - p_{12} p_{25} p_{50}$$

AVAILABILITY ANALYSIS

Using the arguments of the theory of regenerative processes, the availability $A_i(t)$ is seen to satisfy the following recursive relations:

$$A_0(t) = M_0(t) + q_{01}(t) © A_1(t)$$

$$A_1(t) = M_1(t) + q_{10}(t) © A_0(t) + q_{12}(t)$$
$$© A_2(t) + q_{13} © A_3(t)$$
$$+ q_{11}^{(4)}(t) © A_1(t) + q_{18}^{(4)}(t) © A_8(t)$$

$$A_2(t) = M_2(t) + q_{25}(t) © A_5(t) + q_{2,10}^{(6)}(t)$$
$$© A_{10}(t)$$

$$A_3(t) = M_3(t) + q_{30}(t) © A_0(t) + q_{31}^{(7)}(t)$$
$$© A_1(t)$$

$$A_5(t) = M_5(t) + q_{50}(t) © A_0(t) + q_{51}^{(9)}(t)$$
$$© A_1(t)$$

$$A_8(t) = q_{81}(t) © A_1(t)$$

$$A_{10}(t) = q_{10,1}(t) © A_1(t)$$

where

$$M_0(t) = e^{-\lambda t},$$

$$M_2(t) = e^{-\lambda t}\overline{I(t)},$$

$$M_3(t) = e^{-\lambda t}\overline{G_e}(t), \qquad M_4(t) = e^{-\lambda t}\overline{G_1(t)},$$

$$M_5(t) = e^{-\lambda l}\overline{G_2(t)}$$

Taking Laplace transforms of above equations and solving for $A_0^{**}(s)$, the steady state availability of the system is given by

$$A_0 = \lim_{s \to 0}(sA_0^*(s)) = N_1 / D_1$$

where

$$N_1 = \mu_0(p_{10} + p_{13} p_{10} + p_{12} p_{25} p_{50}) + \mu_1 + \mu_2 p_{12}$$
$$+ \mu_3 p_{13} + \mu_5 p_{12} p_{25}$$

and

$$D_1 = \mu_0\left(p_{10} + p_{13}p_{10} + p_{12}p_{25}p_{50}\right) + k_1 + k_2 p_{12} + \mu_8\left(p_{13} + p_{18}^{(4)}\right) + \mu_{10}p_{12}$$

Other measures of the system effectiveness have been obtained in the similar fashion;

The total fraction of time for which the system is under repair.

$$B_0^{(e)} = N_2 / D_1$$

The total fraction of the time for which the assistant is busy in repairing the failed unit.

$$B_0^{(a)} = N_3 / D_1$$

The expected number of visits per unit time

$$V_0 = N_4 / D_1$$

where

$$N_2 = k_2 p_{12} + \mu_8\left(p_{13} + p_{18}^{(4)}\right)$$

$$N_3 = k_1 + p_{12}\mu_{10}$$

$$N_4 = p_{12} + p_{13} + p_{18}^{(4)}$$

COST-BENEFIT ANALYSIS

The expected total profit incurred to the system in steady-state is given by

$$P = C_0 A_0 - C_1 B_0^{(e)} - C_2 B_0^{(a)} - C_3 V_0$$

where

C_0 = revenue per unit up time of the system

C_1 = cost per unit time for which the expert repairman is busy in repairing the failed unit and in giving instructions to the assistant repairman

C_2 = Cost per unit time for which the assistant repairman is busy in repairing the failed unit

C_3 = cost per visit of the expert

GRAPHICAL STUDY

Let us assume that the repair times and instruction times and exponentially distributed as under:

$$g_1(t) = \alpha e^{-\alpha t}, \qquad g_2(t) = \beta e^{-\beta t}$$

$$i(t) = \gamma e^{-\gamma t}, \qquad g_e(t) = \delta e^{-\delta t}$$

Figs. 2 and 3 show the behaviour of the MTSF and the profit respectively with respect to failure rate (λ) for different values of repair rate (α). From the graphs, it can be seen that the MTSF and the profit both decrease as the failure rate increases. However, the values of MTSF and the profit increase with the increase in the values of repair rate (α).

The behaviour of the MTSF and the profit with respect to probability of major accident (P_2) for different values of repair rate (δ) is depicted as in Figs. 4 and 5 respectively. It is concluded from the graphs that as the probability of major accident increases, the MTSF increases whereas profit decreases. This contradiction is rare and has occurred in this case because as the probability of major accident increases, the system goes under the repair of the expert as a result of which the operation time increase and hence the MTSF increases since the cost factor is not taken into consideration for determining the MTSF. As the system is undertaken by the expert in this case, the cost involves and hence the profit decreases as p_2 increases. However, the MTSF and the profit both increase with the increase in repair rate (δ) if cost is fixed.

Figs. 6 and 7 represents the behaviour of the MTSF and the profit respectively with respect to repair rate (β) for different values of instruction rate (γ). It is clear from the graphs that as the repair (β) increases, the MTSF as well as the profit increase, keeping the other parameters as fixed. Also the values of the MTSF and the profit increase with the increase in instruction rate, keeping the other parameters as fixed.

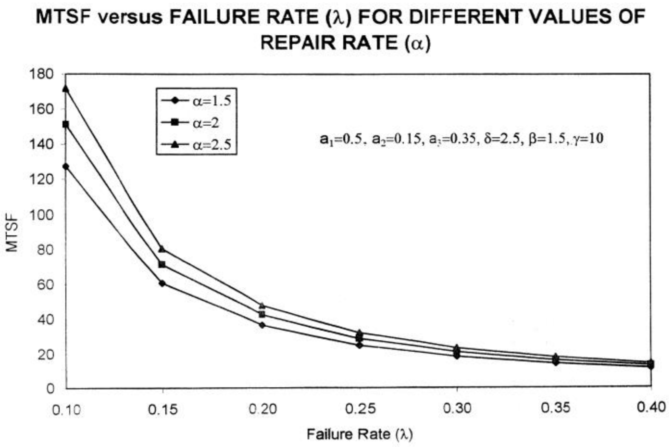
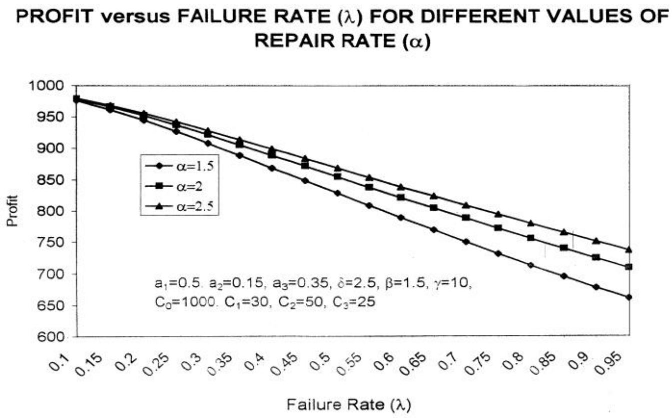


Figure - 2



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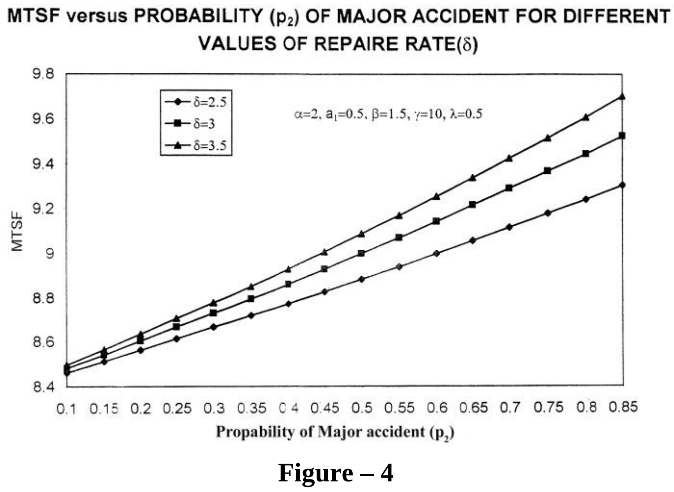

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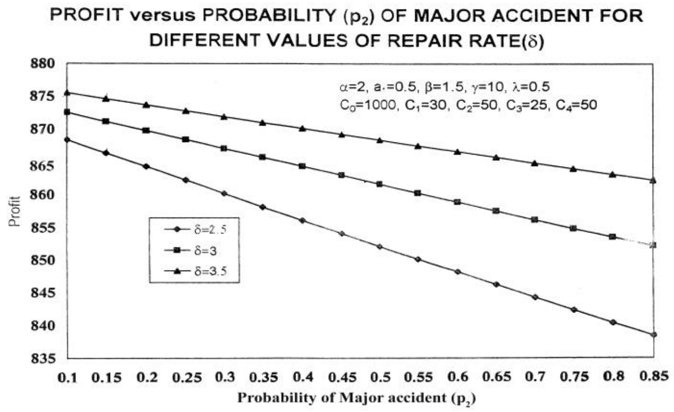

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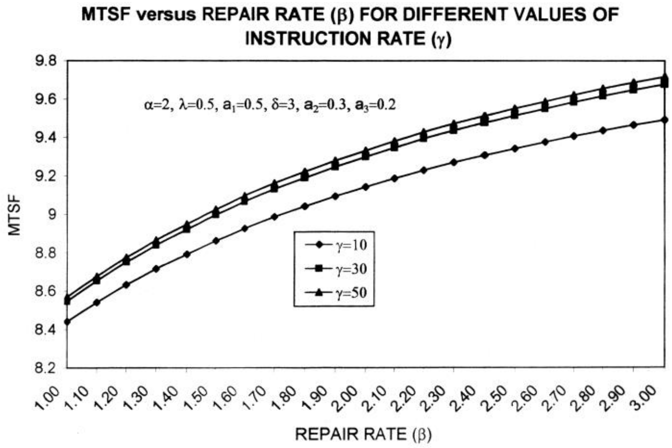

Figure – 6

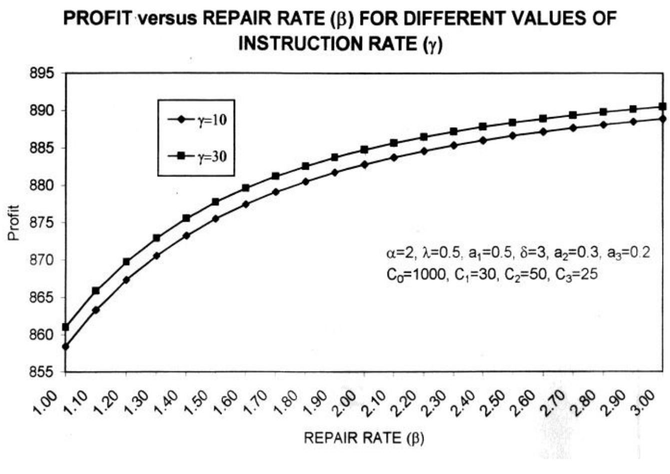

Figure - 7

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