Optimal Performance Measures of Retrial Queuing Model with Fuzzy Parameters Using Robust Ranking Technique

R.Ramesh[#1] , S.Kumara ghuru[#2]

[#1] *Department of Mathematics, Arignar Anna Government Arts College, Musiri, Tamilnadu, India.*

rameshsanju123@gmail.com

[#2] *Department of Mathematics, Chikkanna Government Arts College, Tiruppur, Tamilnadu, India.*

kumarunmath@yahoo.co.in

Abstract **-In this paper we propose a procedure to find the various performance measures in terms of crisp values for retrial queuing model where the arrival rate, service rate and retrial rate are fuzzy numbers. Here the inter arrival time, service time and retrial rate are Triangular and also Trapezoidal fuzzy numbers. Our idea is to convert the fuzzy inter arrival rate, service rate and retrial rates into crisp values by applying robust ranking technique. Then apply the crisp values in the classical queuing performance measure formulas. Ranking fuzzy numbers plays a huge role in decision making under fuzzy environment. This ranking technique is most reliable method, simple to apply and can be used for all types of queuing problems. A numerical example is solved successfully for both triangular and trapezoidal fuzzy numbers.**

Key words- **Fuzzy retrial queues, Fuzzy sets, Fuzzy ranking, Membership Functions.**

1 . INTRODUCTION

Retrial queues describe operation of many telecommunication networks e.g. the local and wide area networks with the random multiple access protocols, call centers etc. There has been rapid growth in the literature on the queuing systems with repeated attempts which are characterized by the following feature: When an arriving customer finds that all servers are busy and no waiting position is available the customers joins a virtual pool of blocked customer called orbit. The detailed overviews of the related references with retrial queues can be found in the recent book of Falin and Templeton[5] and the survey papers, Artalejo[1,2].Most of the related studies are based on traditional queuing theory, is that the inter arrival times and service times are assumed to follow certain probability distribution. However, in practice their cases that these parameters may be obtained subjectively from Yo .J.B. , Tsujimura.Y Gon.M, Yamazaki.G [13]. Zadeh [10] in 1965 first introduced Fuzzy set as a mathematical way of representing impreciseness or indistinctness in everyday life.

The fuzzy queues are much more realistic than commonly used crisp queues [3,4,12]. V.G. Kulkarni, H.M. Liang,and J.H. Dshalalow gave a number of applications of retrial queues in science and engineering can be found in Ref. [6]. Ranking techniques have been analyzed by such researchers like F.Choobinesh and H.Li[7], R.R.Yager[8], S.H.Chen[9], R.Nagarajan andA.Solairaju[11].

In this paper we develop a method that is able to provide performance measures in terms of crisp values for retrial queuing model with fuzzified exponential arrival rate (i.e. the expected number of arrivals per time period) and service rate (i.e the expected number of services per time period) . Here Robust ranking technique has been used to reach crisp values.

II . PRELIMINARIES

Definition 1 : A fuzzy set is determined by a membership function mapping elements of a domain space or universe of discourse Z to the unit interval [0,1]. (i,e) $\tilde{A}$ = {(z, $\mu_{\tilde{A}}$ (z)); z Є Z}.

Here $\mu_{\tilde{A}}$: Z →[0,1] is a mapping called the degree of membership function of the fuzzy set $\tilde{A}$ and $\mu_{\tilde{A}}$ (z)is called the membership value of z Є Z in the fuzzy set $\tilde{A}$. These membership grades are often represented by real numbers ranging from [0,1].

Definition 2 : (*Triangular fuzzy number*):

For a triangular fuzzy number $\tilde{A}$(z), it can be represented by $\tilde{A}$(a_1,a_2,a_3;1) with membership function μ(z) given by

$$\mu(z)=\begin{cases}\dfrac{z-a_1}{a_2-a_1}, & a_1 \le z \le a_2\\ 1 \quad , & z=a_2\\ \dfrac{a_3-z}{a_3-a_2}, & a_2 \le z \le a_3\\ 0, otherwise\end{cases} \qquad (1)$$

Definition 3 : (*Trapezoidal fuzzy number*):

For a trapezoidal fuzzy number $\tilde{A}$(z), it can be represented by $\tilde{A}$(a_1,a_2,a_3,a_4;1) with membership function μ(z) given by

$$\mu(z)=\begin{cases}\dfrac{z-a_1}{a_2-a_1}, & a_1 \le z \le a_2\\ 1 \quad , & a_2 \le z \le a_3\\ \dfrac{a_4-z}{a_4-a_3}, & a_3 \le z \le a_4\\ 0, otherwise\end{cases} \qquad (2)$$

Definition 4 : (α-cut of a fuzzy number)

The α-cut of a fuzzy number $\tilde{A}$(z) is defined as $\tilde{A}$(α) = { z :μ(z) ≥ α, α Є [0,1]}

Addition of two triangular fuzzy numbers is (a_1,a_2,a_3)+(b_1,b_2,b_3) =(a_1+ b_1 , a_2+ b_2, a_3+ b_3).

Addition of two Trapezoidal fuzzy numbers is (a_1,a_2,a_3,a_4)+(b_1,b_2,b_3,b_4)=(a_1+b_1 ,a_2+b_2,a_3+b_3,a_4+b_4)

III . RETRIAL QUEUING MODEL WITH FUZZY PARAMETERS

We consider an FM/FM/1/1-(FR) queuing system in which customers arrive at a service facility from outside at rate $\tilde{\lambda}$, where $\tilde{\lambda}$ is a fuzzy number. An arriving customer enters the service facility if the facility is not occupied. Otherwise he/she enters the orbit and attempts service after an uncertain amount of time, called retrial time. Unless otherwise mentioned, the orbit capacity is assumed to be infinite. The successive retrial times are independent and identically distributed according to an exponential distribution with fuzzy retrial rate θ. Arriving customers at the server form a single waiting line and are served in the order of their arrivals. The service time provided by a single server is exponentially distributed with fuzzy rate $\tilde{\mu}$. In addition, various stochastic processes involved in the system are independent of each other.

In this model the arrival rate $\tilde{\lambda}$, retrial rate $\tilde{\theta}$, and service rate $\tilde{\mu}$ are approximately known and can be represented by convex fuzzy sets. Let $\phi_{\tilde{\lambda}}(x)$, $\phi_{\tilde{\theta}}(v)$ and $\phi_{\tilde{\mu}}(y)$ denote the membership functions of $\tilde{\lambda}$, $\tilde{\theta}$ and $\tilde{\mu}$. Then we have the following fuzzy sets:

$$\tilde{\lambda}=\left\{(x,\phi_{\tilde{\lambda}}(x))\,\middle|\,x\in X\right\} \qquad (3)$$

$$\tilde{\theta}=\left\{(v,\phi_{\tilde{\theta}}(v))\,\middle|\,v\in V\right\} \qquad (4)$$

$$\tilde{\mu}=\left\{(y,\phi_{\tilde{\mu}}(y))\,\middle|\,y\in Y\right\} \qquad (5)$$

where X, V and Y are the crisp universal sets of the arrival, retrial and service rates, respectively. Let f(x, v, y) denote the system characteristic of interest. Since $\tilde{\lambda}$, $\tilde{\theta}$ and $\tilde{\mu}$ are fuzzy numbers,

f ($\tilde{\lambda}$, $\tilde{\theta}$, $\tilde{\mu}$) is also a fuzzy number.

Assume that the system characteristic of interest is the expected number of customers and the expected waiting time in the orbit. From the knowledge of retrial queuing theory [2,5], if x/y<1 we have the expected waiting time and the expected number of customers in the orbit for a crisp retrial queuing system are given by

$$E(W) = \frac{x}{(y - x)}\left(\frac{1}{x}+\frac{1}{v}\right) \quad (6)$$

$$E(N) = \frac{x^2}{(y - x)}\left(\frac{1}{x}+\frac{1}{v}\right) \quad (7)$$

A . Robust Ranking Technique – Algorithm

To solve the problem we defuzzify the fuzzy numbers into crisp ones by a fuzzy number ranking method. Robust ranking technique [11] which satisfies compensation, linearity, and additive properties and provides results which are consistent with human intuition. Let a convex fuzzy number $\tilde{z}$, The Robust Ranking Index is defined by

$$R(\tilde{z}) = \int_0^1 \frac{(z_\alpha^L + z_\alpha^U)}{2} d\alpha \quad (8)$$

Where (z_α^L, z_α^U) is the α –level cut of the fuzzy number $\tilde{z}$. In this paper we use this method for ranking the fuzzy numbers. The Robust ranking index R($\tilde{z}$) gives the representative value of the fuzzy number $\tilde{z}$. It satisfies the linearity and additive property.

IV . Numerical Example

In a packet-switching network, we considered a computer network in which there are a group of host computers connected to interface message processors. Messages arrive at the host computer following a Poisson stream. If the host computer wishes to transmit the message to another host computer, it must send the message and the final address to the interface message processor to which it is associated. If the processor is free the message is accepted; otherwise, the message comes back to the host computer and is stored in a buffer to be retransmitted some time later. The buffer in the host computer, the interface processor and the retransmission policy correspond to the orbit, the server and the retrial discipline, respectively, in the queuing terminology. Clearly, a FM/FM/1/1-(FR) queuing model can model the above system.

A . For Triangular fuzzy number

Suppose the arrival, retrial , and service rates are triangular fuzzy numbers represented by

$\tilde{\lambda}$ = [15,16,18] , $\tilde{\theta}$ = [1,8,22] , $\tilde{\mu}$ = [19,20,22]

whose intervals of confidence are [15+α , 18-2α], [1+7α , 22-14α] and [19+ α , 22-2α] respectively. Concerned with system efficiency, the management wants to calculate the optimal performance measures of the system characteristics, including the expected waiting time and the number of customers in the orbit. Now we evaluate R(1,8,22) by applying Robust ranking method. According to (1) the membership function of the triangular fuzzy number (1,8,22) is

$$\mu(z) = \begin{cases} \frac{z-1}{7}, & 1 \le z \le 8 \\ 1 \quad , & z = 8 \\ \frac{22-z}{14}, & 8 \le z \le 22 \\ 0, otherwise & \end{cases}$$

The α-cut of the fuzzy number (1,8,22) is $(z_\alpha^L,\ z_\alpha^U)$ =(1+7α , 22-14α) and according to (8)

The Robust Ranking Index of $\tilde{\theta}$ is

$$R(\tilde{\theta}) = R(1,8,22) = \frac{1}{2}\int_0^1 (23 - 7\alpha) d\alpha$$

$$= \frac{1}{2}\left[23\alpha - \frac{7\alpha^2}{2}\right]_0^1 = 9.75$$

Proceeding similarly we get

$$R(\tilde{\lambda}) = R(15,16,18) = \frac{1}{2}\int_0^1 (33 - \alpha) d\alpha$$

$$= \frac{1}{2}\left[33\alpha - \frac{\alpha^2}{2}\right]_0^1 = 16.25$$

and $$R(\tilde{\mu}) = R(19,20,22) = \frac{1}{2}\int_0^1 (41 - \alpha) d\alpha$$

$$= \frac{1}{2}\left[41\alpha - \frac{\alpha^2}{2}\right]_0^1 = 20.25$$

By (6) and (7)

The expected waiting time in the orbit

$$E(W) = \frac{x}{(y - x)}\left(\frac{1}{x}+\frac{1}{v}\right)$$

$$= \frac{16.25}{(20.25 - 16.25)}\left(\frac{1}{16.25}+\frac{1}{9.75}\right)$$

$$= 0.661$$

The expected number of customers in the orbit

$$E(N) = \frac{x^2}{(y - x)}\left(\frac{1}{x}+\frac{1}{v}\right)$$

$$= \frac{16.25^2}{(20.25 - 16.25)}\left(\frac{1}{16.25}+\frac{1}{9.75}\right)$$

= 10.74

B . For Trapezoidal fuzzy number

Suppose the arrival , retrial , and service rates are trapezoidal fuzzy numbers represented by $\tilde{\lambda}$ = [15,16,17,18] , $\tilde{\theta}$ = [1,8,15,22] , $\tilde{\mu}$ = [19,20,21,22] whose intervals of confidence are [15+α ,18- α], [1+7α , 22-7α] and [19+ α , 22- α] respectively. Concerned with system efficiency, the management wants to calculate the optimal performance measures of the system characteristics, including the expected waiting time and the number of customers in the orbit.

Now we evaluate R(1,8,15,22) by applying Robust ranking method. According to (2) the membership function of the trapezoidal fuzzy number (1,8,15,22) is

$$\mu(z)=\begin{cases}\frac{z-1}{7}, & 1\le z\le 8\\ 1 \quad , & 8\le z\le 15\\ \frac{22-z}{7}, & 15\le z\le 22\\ 0, otherwise & \end{cases}$$

The α-cut of the fuzzy number (1,8,15,22) is $(z_\alpha^L,\ z_\alpha^U)$ =(1+7α , 22-7α) and according to (8)

The Robust Ranking Index of $\tilde{\theta}$ is

$$R(\tilde{\theta}) = R(1,8,15,22) = \frac{1}{2}\int_0^1 23\,d\alpha = 11.5 ,$$

Proceeding similarly we get

$$R(\tilde{\lambda}) = R(15,16,17,18) = \frac{1}{2}\int_0^1 33\,d\alpha = 16.5 \text{ , and}$$

$$R(\tilde{\mu}) = R(19,20,21,22) = \frac{1}{2}\int_0^1 41\,d\alpha = 20.5$$

By (6) and (7)

The expected waiting time in the orbit

$$E(W) = \frac{x}{(y - x)}\left(\frac{1}{x}+\frac{1}{v}\right)$$

$$= \frac{16.5}{(20.5 - 16.5)}\left(\frac{1}{16.5}+\frac{1}{11.5}\right)$$

= 0.602

The expected number of customers in the orbit

$$E(N) = \frac{x^2}{(y - x)}\left(\frac{1}{x}+\frac{1}{v}\right)$$

$$= \frac{16.5^2}{(20.5 - 16.5)}\left(\frac{1}{16.5}+\frac{1}{11.5}\right)$$

= 9.933

V . Conclusion

In this paper, Fuzzy set theory has been applied to retrial queues. Retrial queuing models have been used in operations and service mechanism for evaluating system performance. Moreover, the fuzzy problem has been transformed into crisp problem using Robust ranking indices. Since the performance measures such as the waiting time in the orbit and the expected number of customers in the orbit are crisp values, the manager can take the best and optimum decisions. We conclude that the solution of fuzzy problems can be obtained by Robust ranking method very effectively. The approach proposed in this paper provides practical information for system manager and practitioners.

REFERENCES

[1]. Artalejo J.R. (1999) Accessible bibliography on retrial queues, Mathematical and Computer modeling 30,1-6.

[2].Artalejo J.R (1999) A classified bibliography of research on retrial queues: Progress in 1990-1999,Top, 7, 187-211.

[3].Buckley J.J (1990) Elementary queuing theory based on possibility theory, Fuzzy sets and systems 37,43-52.

[4].Buckley J.J , Feuring .T ,Hasaki .Y (2001) Fuzzy queuing theory resisted, Int.J. Uncertain Fuzzy 9 527-537.

[5].G.I. Falin, J.G.C. Templeton, Retrial Queues, Chapman and Hall, London, 1997.

[6].V.G. Kulkarni, H.M. Liang, Retrial queues revisited, in: J.H. Dshalalow (Ed.), Frontiers in Queuing: Models and Applications in Science and Engineering, CRC Press, Inc., 1997, pp. 19–34.

[7].F.Choobinesh and H.Li, “ An index for ordering fuzzy numbers,” Fuzzy sets ans Systems, vol 54, pp.287-294,1993.

[8].R.R.Yager, “ A procedure for ordering fuzzy subsets of the unit interval, Information Sciences, Vol.24,pp.143-161,1981

[9].S.H.Chen, " Ranking fuzzy numbers with maximizing set and minimizing set", Fuzzy Sets and Systems, Vol.17,pp.113-129,1985
[10]. Zadeh L.A, Fuzzy sets, Information and control.Vol.8 pp.338-353,1965
[11].R.NagarajanandA.Solairaju"Computing Improved Fuzzy Optimal Hungarian Assignment Problems with Fuzzy Costs under RobustRanking Techniques,"Vol.6-no.13,pp.6-13,2010
[12].A.Kaufmann (1975) Introduction to the theory of fuzzy subsets, Vol I, Academic press, Newyork.
[13].Yo .J.B. , Tsujimura.Y Gon.M, Yamazaki . G (1996) Performance evaluation of network models Based on fuzzy theory system, 8, 393-408.