

# Noise Level Estimation Deblurring using Directional Filters

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**Abstract**— State-of-the-art single image deblurring techniques are sensitive to image noise. Even a small amount of noise, which is inevitable in low-light conditions, can degrade the Quality of blur kernel estimation dramatically. The recent approach of Tai and Lin tries to iteratively denoise and deblur a blurry and noisy image. However, as we show in this work, directly applying image denoising methods often partially damages the blur information that is extracted from the input image, leading to biased kernel estimation. We propose a new method for handling noise in blind image deconvolution based on new theoretical and practical insights. Our key observation is that applying a directional low-pass filter to the input image greatly reduces the noise level, while preserving the blur information in the orthogonal direction to the filter. Based on this observation, our method applies a series of directional filters at different orientations to the input image, and estimates an accurate Radon transform of the blur kernel from each filtered image. Finally, we reconstruct the blur kernel using inverse Radon transform.

**Keywords**— noise level estimation, weak blurred image, image gradient, PCA

## I. INTRODUCTION

Taking handheld photos in low-light conditions is challenging. Since less light is available, longer exposure times are needed – and without a tripod, camera shake is likely to happen and produce blurry pictures. Increasing the camera light sensitivity, i.e., using a higher ISO setting, can reduce the exposure time, which helps. But it comes at the cost of higher noise levels. Further, this is often not enough, and exposure time remains too long for handheld photography, and many photos end up being blurry and noisy. Although many techniques have been proposed recently to deal with camera shake, most of them assume low noise levels. In this work, we do not make this assumption and aim to restore a sharp image from a blurry and noisy input. This work was performed when the first author interned at Adobe Research. Many single image blind deconvolution methods have been recently proposed. Although they generally work well when the input image is noise-free, their performance degrades rapidly when the noise level increases. Specifically, the blur kernel estimation step in previous deblurring approaches is often too fragile to reliably estimate the blur kernel when the image is contaminated with noise, as shown in

Fig. 1. Even assuming that an accurate blur kernel can be estimated, the amplified image noise and ringing artifacts generated from the non-blind deconvolution also significantly degrade the results. To handle noisy inputs in single image deblurring, Tai and Lin first apply an existing denoising package as preprocessing, and then estimate the blur kernel and the target image from the denoised result. This process iterates a few times to produce the final result. However, applying existing denoising methods is likely to damage, at least partially, the detailed blur information that one can extract from the input image, thereby leading to biased kernel estimation. In Sec. II, we illustrate that standard denoising methods, from bilateral filtering to more advanced approaches such as Non-Local Means and BM3D, have negative impacts on the accuracy of kernel estimation. In this paper, we propose a new approach for estimating an accurate blur kernel from a noisy blurry image. Our approach still involves denoising and deblurring steps. However, we carefully design the denoising filters and deblurring procedures in such a way that the estimated kernel is not affected by the denoising filters. That is, we shall see that, unlike existing approaches, we can theoretically guarantee that our approach does not introduce any bias in the estimated kernel. Our approach is derived from the key observation that if a directional low-pass linear filter is applied to the input image, it can reduce the noise level greatly, while the frequency content, including essential blur information, along the orthogonal direction is not affected. We use this property to estimate 1D projections of the desired blur kernel to the orthogonal directions of these filters. These projections, also known as the Radon transform, will not be affected by applying directional low-pass filters to the input image,

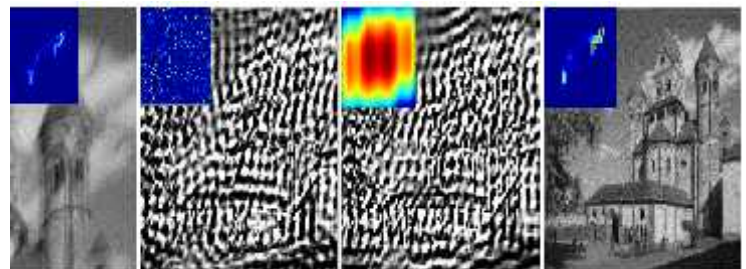


Figure 1.

Previous deblurring methods are sensitive to image noise. (a) Synthetic input image with 5% noise and the ground truth kernel(overlaid). It is cropped to better show blur and noise. (b) Estimated kernel and latent image by Cho and Lee. (c) Results by Levin et al. (d) Results of our method.

We apply a series of directional low-pass filters at different Orientations, and estimate a slice of kernel projection from each image. This yields an accurate estimate of the Radon transform. Finally, we reconstruct the blur kernel using the inverse Radon transform. Once a good kernel is obtained, we incorporate denoising filtering into the final deconvolution process to suppress noise and obtain a high-quality latent image. Results on synthetic and real noisy data show that our method is more robust and achieves better results than previous approaches.

**A. Side effects of denoising as preprocessing**

Before introducing our approach, we first analyze the negative impact of employing denoising as preprocessing on kernel estimation. In single image deblurring, a blurry and noisy input image  $b$  is usually modeled as

$$b = l * k + n,$$

where  $l$ ,  $k$  and  $n$  represent the latent sharp image, blur kernel, and additive noise, respectively,  $*$  is the convolution operator. Solving  $l$  and  $k$  from input  $b$  is a severely ill posed problem, and the additional noise  $n$  makes this problem even more challenging.

Assuming that  $l$  is known, a common approach to solve for  $k$

$$k = \operatorname{argmin}_k \{ \|b - l * k\|^2 + p(k) \},$$

where  $p(k)$  is the additional regularization term that imposes smoothness and/or sparsity prior on  $k$ . Without considering the regularization term, this becomes a least squares problem and the optimal  $k$  can be found by solving the following linear system:

$$L^T L k = L^T b - L^T (b' + n),$$

(3) where  $k$  and  $b$  are the corresponding vector forms of  $k$  and  $b$ , respectively, and  $L$  is the matrix form of  $\Gamma$ . We also introduce the noise-free blurry image

$b' = b - n$ . We estimate the relative error of  $k$  with respect to the noise in  $b$  using the condition number of the linear system, that

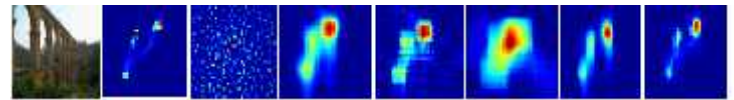
$$\frac{e(k)}{e(b)} = \frac{\| (L^T L)^{-1} L^T n \| / \| (L^T L)^{-1} L^T b' \|}{\| L^T n \| / \| L^T b' \|} \leq \| (L^T L) \| \cdot \| (L^T L)^{-1} \| = \kappa(L^T L),$$

where  $e(k)$  and  $e(b)$  are relative errors in  $k$  and  $b$ , respectively. Thus, the noise  $n$  in the input image will be amplified at most by the condition number  $\kappa(L^T L)$  for kernel estimation, where  $L^T L$  is often called the deconvolution matrix and has a block-circulant-with-circulant-block (BCCB) structure. Eq. shows that the upper bound on the error in the estimated kernel is proportional to the amplitude of the noise in input image. Building on this result, one can attempt to apply sophisticated denoising filter to the blurry image to reduce the noise amplitude, hoping that this will

improve the kernel estimate. However, denoising filters also alter the profile of edges, e.g.,. This information is critical to accurate kernel estimation, and as we shall see, the benefits of the noise reduction are often outweighed by artifacts caused by the profile alteration. To illustrate it, we first look at a simple noise reduction method, Gaussian smoothing. Convolution with a Gaussian  $G_g$  decreases the noise level. However, the kernel estimation then becomes:

$$k_g = \operatorname{argmin}_{k_g} \| b * G_g - l * k_g \|^2 \\ \operatorname{argmin}_{k_g} \| (l * k - n) * G_g - l * k_g \|^2 \\ \approx \operatorname{argmin}_{k_g} \| l * (k * G_g - k_g) \|^2 \quad k * G_g, \quad (5)$$

where  $k$  is the blur kernel for the original input image and  $k_g$  is the optimal solution after Gaussian denoising. Eq. shows that the estimated kernel  $k_g$  is a blurred version of the actual kernel  $k$ . Further, since  $G_g$  is a low-pass filter, the high frequencies of  $k$  are lost and recovering them from  $k_g$  would be very difficult, if possible at all. This result comes from the initial noise reduction and is independent



**Figure 2.**

The side effects of employing different denoising methods as preprocessing step in single image deblurring. (a) the synthetic input image with 5% noise. (b) the ground truth kernel. (c) the blur kernel estimated without applying any denoising method to the input image (a). (e)-(g) the estimated blur kernels after applying different denoising filters. (h) the kernel estimated by our method of the kernel estimation method except for the noise reduction.

**II. APPLYING DIRECTIONAL FILTERS**

We now show that directional low-pass filters can be applied to an image without affecting its Radon transform, while decreasing its noise level. We consider the directional low-pass filter

$$I(p) * f_\theta = \frac{1}{c} \int_{-\infty}^{\infty} w(t) I(p + t u_\theta) dt,$$

where  $I$  is an image,  $p$  is a pixel location,  $t$  is the spatial distance from one pixel to  $p$ ,  $c$  is the normalization factor defined as

$$c = \int_{-\infty}^{\infty} w(t) dt, \text{ and } u_\theta = (\cos \theta, \sin \theta)^T$$

is a unit vector of direction  $\theta$ . The profile of the filter is determined by  $w(t)$ , for which we use a Gaussian function: where  $\sigma$  controls the strength of the filter. Filtering the image affects the estimated kernel. With the same argument as for Eq., the kernel that we estimate from the filtered image is:

$$b_{\theta} = b * f_{\theta}$$

$$k_{\theta} = k * f_{\theta}$$

Similarly to filtering with a 2D Gaussian  $G_{\theta}$ , applying  $f_{\theta}$  averages pixels and reduces the noise level. Since  $f_{\theta}$  filters only along the direction  $\theta$ , it has nearly no influence on the blur information in the orthogonal direction. We exploit this property to estimate the projection of the original kernel  $k$  along the direction  $\theta$ . The projection can be formulated as Radon transform, which is the collection of integrals of a signal (i.e.,  $k$ ) along projection lines.

$$\rho = x \sin(\theta) + y \cos(\theta)$$

$$R_{\theta}(k) = \int \int k(x, y) \delta(\rho - x \sin(\theta) - y \cos(\theta)) dx dy,$$

The particular value on Radon transform corresponding to one projection line where  $k(x, y)$  indicates the value at the coordinate  $(x, y)$  on

kernel  $k$ .  $\theta$  and  $\rho$  are the angle and offset of the projection line, respectively. Thus, the projection of kernel  $k$  along the projection direction  $\theta$  is:

$$R_{\theta}(k_{\theta}) = R_{\theta}(k * f_{\theta}) = R_{\theta}(k) * R_{\theta}(f_{\theta}) = R_{\theta}(k),$$

where  $R_{\theta}$  is the Radon transform operator to the direction  $\theta$ , and  $\theta = \theta + \pi/2$ . It is a linear operator, and one can verify that  $R_{\theta}(f_{\theta})$  is a 1D delta function, given the definition of  $f_{\theta}$ . Eq. shows  $f_{\theta}$  has no impact on the Radon transform of the blur kernel to the orthogonal direction of the filter. This is the foundation of the proposed approach. An example is shown in Fig. 3.

### III. THE ALGORITHM

We now explain how we recover the sharp image, with the kernel estimation first, and then the deconvolution step.

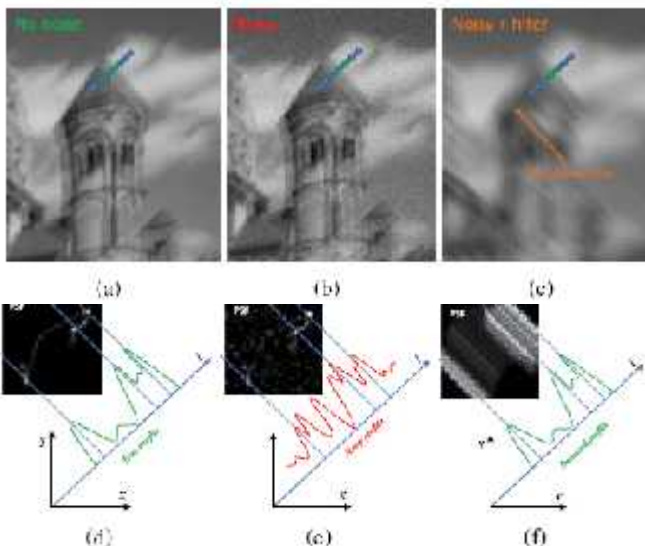


Figure 3. Directional denoising mechanism in single image deblurring.

(a)(b)(c) are the synthetic image before adding noise, after adding noise, and after applying a directional filter ( $\theta = 30^\circ$ ),

respectively. (d)(e)(f) are the corresponding estimated blur kernels and their Radon transforms in the same direction. Note that the estimated kernel in (f) is largely damaged by the directional filter, but its Radon transform is the same as the one in (d).

### Experimental results

We implemented our method in Matlab on an Intel Core i5 CPU with 8GB of RAM. We apply directional filters along 36 regularly sampled orientations, that is, one sample every  $5^\circ$ . The computation time is a few minutes for a one-megapixel image. For all the experiments, we set the

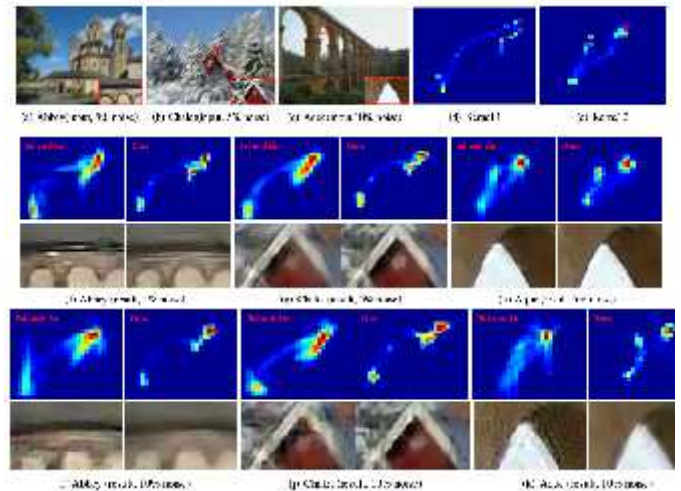


Figure 4.

Comparing Tai and Lin's method and our method on synthetic data. Three input blurry image examples with different levels of noise are shown in (a),(b),(c). (d) and (e) are the ground truth blur kernels from Levin et al. (d) is used for the examples "Abbey" and "Chalet", and (e) is used for the example "Aque". (f-k) show the estimated kernels and the latent images of Tai and Lin's method and our method with 5% noise and 10% noise. Due to the space limit only the areas highlighted by the bounding boxes in (a-c) are shown. Full size images for comparison are in the supplementary material. extent  $\rho$  of the directional filter to 30 pixels. We also set  $w_1 = 0.05$  and  $w_2 = 1$  (Eq. 11), and  $w_3 = 0.05$  (Eq. 12).

### IV. PROPOSED ALGORITHM

#### Noise Level Estimation Based on PCA

After decomposing the image into overlapping patches, we can write the image model as  $y_i = z_i + n_i$  where  $z_i$  is the original image patch with the  $i$ -th pixel at its center written in a vectorized format and  $y_i$  is the observed vectorized patch corrupted by i.i.d zero-mean Gaussian noise vector  $n_i$ . The goal of noise level estimation is to calculate the unknown standard deviation  $\sigma_n$  given only the observed noisy image. The image patches can be regarded as data in Euclidean space. Let us consider the variance of the data projected onto a certain axis. We can define the direction of the axis using the unit vector  $u$ . Assuming that the signal and the noise are uncorrelated, the variance of the projected data on direction  $u$  can be expressed as:  $V(u^T y_i) = V(u^T z_i) + \sigma_n^2$  where  $V$



$\sigma_i^2$  represents the variance of the dataset  $\{y_i\}$ ,  $\sigma_n$  is the standard deviation of the Gaussian noise. We define the minimum variance direction  $u_{\min}$  as  $u_{\min} = \arg \min_u V(u^T z) = \arg \min_u V(u^T y)$ .

Following the same manner of the maximum variance formulation in [1], the minimum variance direction is calculable using the PCA. The minimum variance direction is the eigenvector associated to the minimum eigenvalue of the covariance matrix defined as  $\hat{\gamma} = \frac{1}{N} \sum_{i=1}^N (y_i - \mu)(y_i - \mu)^T$ , (4) where  $N$  is the data number and  $\mu$  is the average of the dataset  $\{y_i\}$ . The variance of the data projected onto the minimum variance direction equals the minimum eigenvalue of the covariance matrix. Then we can derive the equation

$$\sigma_{\min}^2(\hat{\gamma}) = \sigma_{\min}^2(z) + \sigma_n^2$$

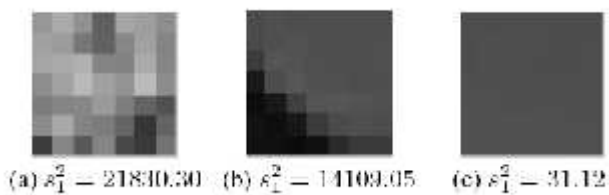


Figure 5.

of the covariance matrices the noisy patches as Eq. However, the decomposition problem is an ill-posed problem because the minimum eigenvalue of the covariance matrix of the noise-free patches is unknown. Although this decomposition problem is an ill-posed problem in general, we can estimate the noise level if we can select weak textured patches from the noisy images as described below.

The weak textured patches are known to span only low dimensional subspace. The minimum eigenvalue of the covariance matrix of such weak textured patches is approximately zero. Then, the noise level can be estimated simply as  $\sigma_n^2 = \sigma_{\min}^2(\hat{\gamma}_0)$ , (6) where  $\hat{\gamma}_0$  is the covariance matrix of the selected weak textured patches.

Consequently, we can estimate the noise level easily if we can select the weak textured patches from the noisy image. However, the weak textured image patch selection also presents a challenging problem, as discussed in the next subsection.

### V. SYNTHETIC DATA

We first conducted experiments on images that we convolved with a known blur kernel and to which we added noise in a controlled fashion. This allows us to report quantitative measures in addition to visual results.

Comparisons with Tai and Lin's method Tai and Lin's method is the most related work to ours since it also seeks to handle noisy images. This section focuses on comparing this method with our approach. We first ran comparison on synthetic images Fig, where the latent sharp images were blurred using two blur kernels provided by Levin et al. We then added Gaussian noise with zero mean and standard deviations of 0.05 and 0.1 for a [0,1] intensity range. Tai and Lin kindly provided the results for their method.

The comparison shows that visually overestimated blur kernels are closer to the ground truth, and our estimated latent images contain more details and less ringing artifacts. We also evaluate the results quantitatively

by computing the Peak Signal-to-Noise Ratio (PSNR) and Structural Similarity (SSIM) (Table 1).

Comparisons with other methods We also conducted experiments to explore how noise affects the performance of other state-of-the-art single-image blind deconvolution methods. Using the "Aque" image and the blur kernel shown in Fig, we generated 10 input images with noise from 1% to 10%. We then applied different blind deconvolution methods to these test images, and measure the PSNR PSNR SSIM

		PSNR		SSIM	
		Noise	5%	10%	5%
Abbey	Tai	22.43	21.05	.8122	.7242
	Ours	<b>22.73</b>	<b>21.61</b>	<b>.8150</b>	<b>.7270</b>
Chalet	Tai	19.79	18.95	.8244	.7162
	Ours	<b>22.80</b>	<b>19.35</b>	<b>.8273</b>	<b>.7200</b>
Aque	Tai	26.58	24.53	.8206	.7415
	Ours	<b>28.46</b>	<b>25.58</b>	<b>.8512</b>	<b>.7469</b>

Table 1.

The comparison experiments of our method and Tai and Lin [17] on synthetic blurry images with different amount of noises. The performances are evaluated by PSNR and SSIM, comparing the generated latent images with the ground truth.

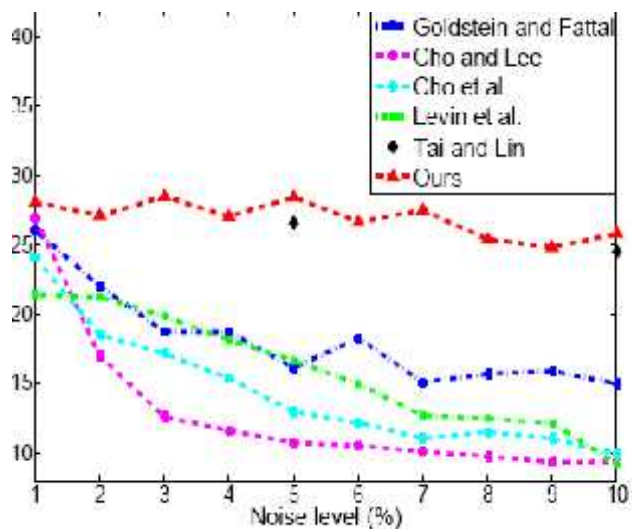


Figure 6.

The PSNR curves of various blind deconvolution algorithms, including Goldstein and Fattal, Cho and Lee, Cho et al., Levin et al., and our method, on the 10 synthetic test images with noise level from 1% to 10%, generated by the "Aque" image and the kernel shown in Fig. The two data points of Tai and Lin's method are shown as black diamonds, which are provided by the authors. While the PSNR values are

closer to ours, the visual difference is still significant; our approach produces cleaner images. All images are included in the supplementary material. Figure 4 shows the accuracy of each method. The accuracies of previous methods degrade rapidly when the noise level increases. On the contrary, our method is more robust, i.e., it works more reliably in the presence of noise, and achieves satisfactory results even when the input noise level is high. This figure also includes two data points of the Tai and Lin's method provided by the authors themselves.

4.2. Results on real examples We first compared our method and Tai and Lin's method on real-world images shown in their original paper and the results are shown in Fig. 5. The results of other state-of-the-art methods can be found in [1]. Our estimated kernels are sharper than Tai and Lin's. The close-ups show that our method recovers more high-frequency details. For the boundaries of objects, our results have less noticeable ringing artifacts. Overall, our approach produces visually more satisfying results. We further show our results on real-world photographs that were captured under common low-light conditions with a Nikon D90 DSLR camera and a 18-105mm lens. We compare our results with those of other state-of-the-art methods, including Goldstein and Fattal, Cho and Lee, Cho et al., Levin et al. The results show that our recovered latent images exhibit less artifacts, such as noise and ringing, and contain more high-frequency details at the same time. These observations are consistent across all test images. We provide additional examples in supplemental material.

## VI. CONCLUSION

We have shown that most state-of-the-art image deblurring techniques are sensitive to image noise. In this paper, we propose a new single image blind deconvolution method that is more robust to noise than previous approaches. Our method uses directional filters to reduce the noise while keeping the blur information in their orthogonal direction intact. By applying a series of such directional filters, we showed how to recover correct 1D projections of the kernel in all directions, which we use to estimate an accurate blur kernel using the inverse Radon transform. We also introduced

a noise-tolerant non-blind deconvolution technique that generates high-quality final results. The effectiveness of the proposed approach is demonstrated on several comparisons on synthetic and real data.

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