

# Discrete Wavelet Transform (DWT) and Singular Value decomposition (SVD) applied to Digital Image

Mrs.S.Jothi Lakshmi, M.E., \*<sup>1</sup>,  
 Dept. of Computer Science and Engineering  
 PSNA College of Engineering and Technology  
 Dindigul, Tanil Nadu, India.  
[jothiman@gmail.com](mailto:jothiman@gmail.com)

Dr.VE.Jayanthi, M.E.,P.hd., \*<sup>2</sup>,  
 Department of Electronics and communication Engineering  
 PSNA College of Engineering and Technology  
 Dindigul, Tanil Nadu, India.  
[jayanthi.ramu@gmail.com](mailto:jayanthi.ramu@gmail.com)

**Abstract** - The main objective of developing an image-watermarking technique is to improve undetectability and satisfy the robustness requirements. To achieve this objective, the discrete wavelet transform (DWT) and singular value decomposition (SVD) is used along with a hybrid image-watermarking scheme. In our approach, we use two images, one is the original image to be embedded and the other one is a watermark image. The watermark image is splitted into two halves w1 and w2. The original image is divided in to DWT sub bands which have singular values and different frequency. The sub bands with low-high frequency and high-low frequency are marked as X and Y respectively and the two halves W<sup>1</sup> and W<sup>2</sup> of the watermark image is embedded on the elements of singular values of cover image's DWT sub bands. The same approach is repeated for each DWT sub band of the original image. This approach can be applied for 128 x 128, 128 x 256, 256 x 256 matrix. Here, the watermark image is not embedded directly on to the wavelet coefficients but rather than on the elements of singular values of cover image's DWT sub bands. Experimental results are given to demonstrate that the proposed approach is capable of withstanding a variety of image-processing attacks.

**Index Terms** - Undetectability, discrete wavelet transform (DWT), singular value decomposition (SVD), digital watermarking

## INTRODUCTION

The basic idea in the DWT for a one dimensional signal is the following. A signal is split into two parts, usually high frequencies and low frequencies. The edge components of the signal are largely to the high frequency part. The low and high frequency part is split again into two parts of high and low frequencies [4]. This process is continued an arbitrary number of times, which is usually determined by the application at hand.

LL		LL	LH
		HL	HH
LL	LH	HH	
HL	HH		

LL	LH
HL	HH

### A. SVD-Based Watermarking

The objective of this project is to apply linear algebra “Singular Value Decomposition (SVD) “to mid-level image processing, especially to area of image compression and recognition. The method is factoring a matrix A into three new matrices X, S, and Y, in such way that  $T A = XSY$ . Where X and Y are orthogonal matrices and S is a diagonal matrix. The experiments are conducted under different term k of singular value, and the outer product expansion of image matrix A for image compression this project also demonstrates how to use SVD approach for image

processing in area of Face Recognition (FR). In this project, we assume a matrix  $A$  with  $m$  lines and  $n$  columns,  $m \times n$ , this assumption is made for convenience only, all the result will also hold if  $n \times m$ . MATLAB is used as a platform of programming and experimenting the project, since MATLAB provides a high-performance for integrating various computations, visualization and programming concepts. The singular value decomposition (SVD) one of the most useful tools of linear algebra, is a fractional and approximation technique which effectively reduces any matrix into smaller invertible and square matrix. The SVD provides where other linear approximation techniques fail. With almost every tool and techniques discussed in linear algebra, there is a delimiter, "provide that the matrix is both invertible and square." The singular value decomposition not only approximate this special case scenario, but it will also work its magic on every other possible scenario. The SVD works wonderfully with both under and over-determined matrices.

SVD based watermarking is present in past literature. Quan and Qingsong [2] embedded the singular values of the watermark to the singular values of the mid band DCT coefficients. Tong et al. [3] proposed a blind digital image watermarking scheme based on SVD and fast Independent Component Analysis (ICA) algorithm. Aslantas [4] presented algorithm using SVD with genetic algorithm to get transparent and robust watermarking. Qi et al. [5] presented a blind adaptive quantization index modulation and SVD based watermarking scheme to embed watermark bits in the approximate subband of wavelet domain. This method is robust against JPEG compression attack.

A semi-blind reference watermarking scheme based on DWT-SVD for copyright protection and authenticity is presented by Bhatnagar and Raman [6]. Al-Haj and Manasrah [7] generated an imperceptible and robust digital image watermarking algorithm based on cascaded DWT and SVD. Senthil and Bhaskaran [8] proposed a work, which is based on wavelet transformation methods that embeds and extracts the watermark in digital images with the considerations of perceptual transparency and robustness against geometric attacks. Most of the previously suggested schemes are based on DWT, SVD independently or DWT-SVD in combination. Author's also developed a bit plane method to improve robustness [9] and wavelet with subband threshold computing method [10] to improve fidelity of watermarked image.

If watermark is embedded into a singular value of selected subband imperceptibility, robustness and perfect reconstruction is achieved. SVD is a linear algebraic numerical technique and used to form diagonalizable matrices in numerical analysis. Singular value matrix forms the characteristic equation, and the degree of characteristic equation is equal to the dimension of column or row of the image. If the watermark is embedded into singular values of an image, it has very good stability and does not have degradation because singular values represent algebraic image properties which are intrinsic and not visual [1]. A new way to enhance the characteristics i. e. robustness and fidelity in the digital image watermarking is achieved by using unique characteristics of LWT-SVD. The flow chart for decomposition of image into frequency domain using LWT is as shown in Fig. 1. Reconstruction is exact opposite to the decomposition.

### B. Properties of SVD

There are many properties and attributes of SVD, here we just present parts of the properties that we used in this project.

1. The singular value  $s_1, s_2, \dots, s_n$  are unique, however, the matrices  $U$  and  $V$  are not unique
2. Since  $A^T A = V S^T S V^T$ , so  $V$  diagonalizes  $A^T A$ , it follows that the  $v_j$  are the eigenvector of  $A^T A$ .
3. Since  $A A^T = U S S^T U^T$ , so it follows that  $U$  diagonalizes  $A A^T$  and that the  $u_i$ 's are the eigenvectors of  $A A^T$ .
4. If  $A$  has rank of  $r$  then  $v_1, v_2, \dots, v_r$  form an orthonormal basis for range space of  $A^T$ ,  $R(A^T)$ , and  $u_1, u_2, \dots, u_r$  form an orthonormal basis for range space  $A$ ,  $R(A)$ .
5. The rank of matrix  $A$  is equal to the number of its nonzero singular values.

### C. Discrete Wavelet Transform

The Discrete Wavelet Transform (DWT) has shown considerable promise in image processing applications, such as the JPEG2000 still image compression standard and image denoising. A hardware DWT core could be integrated into digital camera or scanner to perform image processing inside the device. Research on DWT architectures and hardware implementations includes efficient filter/subsample-based architectures as well as implementations based on the lifting scheme.

Mathematical transforms are used in signal processing in order to extract what is known as “hidden” information. One of these mathematical tools is the Discrete Wavelet Transform (DWT), which has been increasingly employed in non-destructive testing and, more specifically, in image processing.

In a VLSI model is simulated to estimate hardware performance. A scalable architecture for performing many different DWTs using the lifting scheme is proposed in .A comparison of both a convolution-based and a high-throughput pipelined lifting scheme-based DWT architecture is presented in. DWT using the lifting scheme and techniques for improving performance at the cost of increased power consumption and vice-versa. provide unique scalable architectures using the line-based lifting scheme method to reduce memory requirements. Flexible hardware architecture for performing the DWT on a digital image is presented in this paper. That include smaller memory requirements, fixed-point arithmetic, instead of more costly floating-point, and fewer arithmetic computations. In addition, the architecture is flexible in that it can be configured to perform different variations of the DWT. For example, the JPEG2000 still image compression standard The proposed architecture allows the hardware for either of these two DWTs to be configured automatically. The illumination information is embedded in the LH and HL sub bands only. The edges are concentrated in the other sub bands (i.e., LL, and HH). Hence, separating the high frequency sub bands and applying the illumination enhancement in the LH and HL sub bands will protect the edge information from possible degradation. After reconstructing the final image by using DWT, the resultant image will not only be enhanced with respect to illumination but also will be sharper.

#### PROPOSED DWT-SVD WATERMARKING SCHEME

The proposed DWT-SVD watermarking scheme is formulated as given here.

##### A. Watermark embedding:

1) Use one-level Haar DWT to decompose the cover image  $A$  into four subbands (i.e., **LL**, **LH**, **HL**, and **HH**).

2) Apply SVD to LH and HL sub bands, i.e.,

$$A^k = X^k S^k Y^k, \quad k = 1, 2 \quad (1)$$

where  $k$  represents one of two sub bands.

3) Divide the watermark into two parts:  $W = W^1 + W^2$ , where  $W^k$  denotes half of the watermark.

4) Modify the singular values in HL and LH subbands with half of the watermark image and then apply SVD to them, respectively, i.e.,

$$S^{k+} W^k = X^k W^k S^k Y^{kT} \quad (2)$$

where  $\alpha$  denotes the scale factor. The scale factor is used to control the strength of the watermark to be inserted.

5) Obtain the two sets of modified DWT coefficients, i.e.,  $A^{*k} = X^k S^{k+} Y^{kT}$ ,  $k = 1, 2$ . (3)

6) Obtain the watermarked image  $AW$  by performing the inverse DWT using two sets of modified DWT coefficients and two sets of nonmodified DWT coefficients.

##### B. Watermark extraction:

1) Use one-level Haar DWT to decompose the watermarked (possibly distorted) image  $A^*_w$  into four subbands: LL, LH, HL, and HH.

2) Apply SVD to the LH and HL subbands, i.e.,  $A^{*k}_w = X^{*k} S^{*k} Y^{*kT}$ ,  $k = 1, 2$  (4)

where  $k$  represents one of two subbands.

3) Compute  $D^{*k} = X^{*k} S^{*k} Y^{*kT}$ ,  $k = 1, 2$ .

4) Extract half of the watermark image from each subband, i.e.,

$$W^{*k} = (D^{*k} - S^k) / \alpha, \quad k = 1, 2 \quad (5)$$

5) Combine the results of Step 4 to obtain the embedded watermark:  $W^* = W^{*1} + W^{*2}$ .

## I. EXPERIMENTAL RESULTS

Several experiments are presented to demonstrate the performance of the proposed approach. The gray-level images “Lena” of size  $256 \times 256$  and “Cameraman” of size  $128 \times 128$  are used as the cover image and the watermark, respectively. These images are shown in Fig. 1(a) and (b). Fig. 1(c) illustrates the watermarked image. It can be observed that the proposed approach preserves the high perceptual quality of the watermarked image.

As a measure of the quality of a watermarked image, the peak signal-to noise ratio (PSNR) was used. To evaluate the robustness of the proposed approach, the watermarked image was tested against five



(a)

Fig. 1. (a) Cover image.



(b)

Fig. 2. (b) Watermark image.



(c)

(c) Watermarked image

TABLE I

PEARSON'S CORRELATION COEFFICIENT VALUES OF EXTRACTED WATERMARKS FROM DIFFERENT ATTACKS

SF value	CR	RO	GN	AF
0.01	0.9452	0.9609	0.9415	0.9469
0.03	0.9633	0.9654	0.9394	0.9360
0.05	0.9803	0.9780	0.9591	0.9597
0.07	0.9824	0.9888	0.9720	0.9511
0.09	0.9843	0.9898	0.9756	0.9417
SF value	JPEG	HE	CA	GC
0.01	0.9509	0.9803	0.9862	0.9807
0.03	0.9605	0.9860	0.9929	0.9965
0.05	0.9772	0.9862	0.9936	0.9982
0.07	0.9679	0.9888	0.9951	0.9992
0.09	0.9704	0.9890	0.9958	0.9994

TABLE II

COMPARISON OF IMPERCEPTIBILITY (PSNR) FOR G&E [7], L&T [3], AND OUR ALGORITHM

SF value	0.01	0.03	0.05	0.07	0.09
G&E [7]	37.80	36.79	35.29	33.72	32.26
L&T [3]	51.50	51.26	50.05	47.84	45.56
Ours	51.14	51.14	50.89	49.52	47.49

- 1) geometrical attack: cropping (CR) and rotation(RO);
- 2) noising attack: Gaussian noise (GN);
- 3) denoising attack: average filtering (AF);
- 4) format-compression attack: JPEG compression;
- 5) image-processing attack: histogram equalization (HE), contrast adjustment (CA), and gamma correction (GC).

For comparing the similarities between the original and extracted watermarks, the Pearson's correlation coefficient was employed. In the experiments, the values of the scale factors are carried out with constant range from 0.01 to 0.09 with an interval of 0.02, and the results are illustrated in Tables I and II. It can be seen that the larger the scale factor, the stronger the robustness of the applied watermarking scheme. In contrast, the smaller the scale factor, the better the image quality.

In order to justify our approach, we also implement the DWT-SVD- based watermarking method [7] and pure SVD-based approach [3] to compare the performance. The adjustment strategy of scale factors is like our aforementioned experiment setting, and experimental results are listed in Tables II and III. After studying the experimental results, it can be seen that the proposed scheme significantly outperforms the two compared schemes. In addition to quantitative measurement, we also need the visual perceptions of the extracted watermarks. The constructed watermarks with best-quality measurement are shown in Fig. 2(a)-(x), and we can find that our scheme not only can successfully resist different kinds of attacks but can also restore watermark with high perceptual quality.

To compare the efficiency of our approach and other two methods, watermark extraction was performed on non-attacked watermarked images using the three methods. We implemented three

watermarking schemes using C# and ran them on a personal computer with Intel Core 2 Duo Processors rated at 2.13 GHz, main memory of 4 GB, and operating system of Microsoft Windows 7. Experimental results are listed in Table IV. It is clearly observed that our method can be done very efficiently in comparison with other existing watermarking schemes.

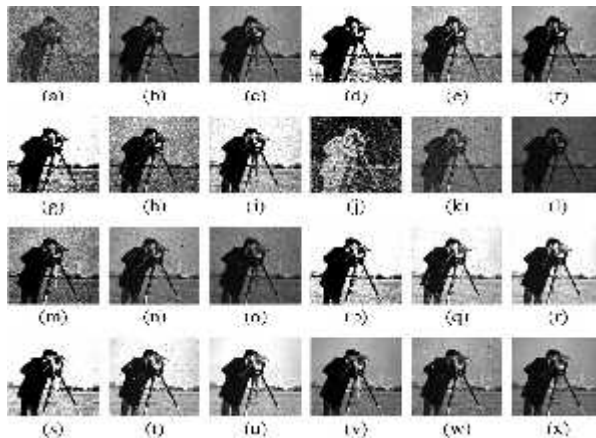


Fig. 2. Extracted watermarks obtained from G&E [7], L&T [3], and our approach in that order with different attacks: CR[(a)–(c)], RO[(d)–(f)], GN[(g)–(i)], AF[(j)–(l)], JPEG[(m)–(o)], HE[(p)–(r)], CA[(s)–(u)], and GC[(v)–(x)].

TABLE III

COMPARISON OF ROBUSTNESS FOR G&E [7], L&T [3], AND OUR ALGORITHM

Method	CR		RO		GN	
	Best	Average	Best	Average	Best	Average
G&E [7]	0.7063	0.2708	0.9091	0.8843	0.9377	0.9169
L&T [3]	0.9578	0.9412	0.9444	0.9394	0.8953	0.8857
Ours	0.9843	0.9728	0.9897	0.9772	0.9756	0.9567
Method	AF		JPEG		HE	
	Best	Average	Best	Average	Best	Average
G&E [7]	0.7047	0.8172	0.9226	0.7537	0.9700	0.9498
L&T [3]	0.8978	0.8765	0.9554	0.9356	0.9780	0.9761
Ours	0.9597	0.9500	0.9772	0.9658	0.9890	0.9862
Method	CA		GC			
	Best	Average	Best	Average		
G&E [7]	0.9759	0.9544	0.9989	0.9927		
L&T [3]	0.9848	0.9827	0.9871	0.9764		
Ours	0.9958	0.9930	0.9994	0.9957		

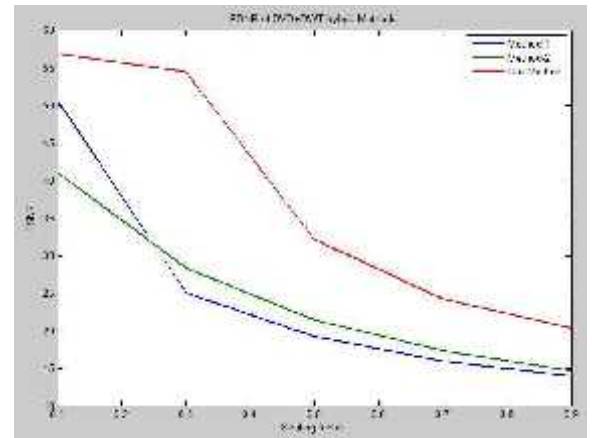


Fig: Graph of PSNR of different hybrid methods

## II. CONCLUSION

In this paper, a hybrid image-watermarking technique based on DWT and SVD has been presented, where the water mark is embedded on the singular values of the cover image’s DWT sub bands. The technique fully exploits the respective feature of these two transform domain methods: spatio-frequency localization of DWT and SVD efficiently represents intrinsic algebraic properties of an image. Experimental results of the proposed technique have shown both the significant improvement in imperceptibility and the robustness under attacks. Further work of integrating the human visual system characteristics into our approach is in progress.

## REFERENCES

- [1] J. Sang and M. S. Alam, “Fragility and robustness of binary-phase-onlyfilter- based fragile/semifragile digital image watermarking,” *IEEE Trans. Instrum. Meas.*, vol. 57, no. 3, pp. 595–606, Mar. 2008.
- [2] H.-T. Wu and Y.-M. Cheung, “Reversible watermarking by modulation and security enhancement,” *IEEE Trans. Instrum. Meas.*, vol. 59, no. 1, pp. 221–228, Jan. 2010.
- [3] R. Liu and T. Tan, “An SVD-based watermarking scheme for protecting rightful ownership,” *IEEE Trans. Multimedia*, vol. 4, no. 1, pp. 121–128, Mar. 2002.
- [4] A. Nikolaidis and I. Pitas, “Asymptotically optimal detection for additive watermarking in the DCT and DWT domains,” *IEEE Trans. Image Process.*, vol. 12, no. 5, pp. 563–571, May 2003.

- [5] V. Aslantas, L. A. Dog˘an, and S. Ozturk, "DWT-SVD based image watermarking using particle swarm optimizer," in *Proc. IEEE Int. Conf. Multimedia Expo*, Hannover, Germany, 2008, pp. 241–244.
- [6] G. Bhatnagar and B. Raman, "A new robust reference watermarking scheme based on DWT-SVD," *Comput. Standards Interfaces*, vol. 31, no. 5, pp. 1002–1013, Sep. 2009.
- [7] E. Ganic and A. M. Eskicioglu, "Robust DWT-SVD domain image watermarking: Embedding data in all frequencies," in *Proc. Workshop Multimedia Security*, Magdeburg, Germany, 2004, pp. 166–174.
- [8] Q. Li, C. Yuan, and Y.-Z. Zhong, "Adaptive DWT-SVD domain image watermarking using human visual model," in *Proc. 9th Int. Conf. Adv. Commun. Technol.*, Gangwon-Do, South Korea, 2007, pp. 1947–1951.
- [9] S. Mallat, "The theory for multiresolution signal decomposition: The wavelet representation," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 11, no. 7, pp. 654–693, Jul. 1989.
- [10] Liu Quan, Ai Qingsong, "A combination of DCT-based and SVD-based watermarking scheme", in *Proc. of 7th Int. Conf. on Signal Processing (ICSP04)*. Vol. 1, 2004, pp. 873-876.
- [11] Ming Tong ,Wei Feng, Hongbing Ji , "A robust geometrical attack resistant digital image watermarking based on fast ICA algorithm", *Congress on Image and Signal Processing*, 2008, pp.655-659.
- [12] Veysel Aslantas, "An SVD based digital image watermarking using genetic algorithm", *IEEE*, 2007.
- [13] Xiaojun Qi, Stephen Bialkowski, and Gary Brimley, "An adaptive QIM and SVD-based digital image watermarking scheme in the wavelet domain", *IEEE*, 2008, pp.421-424.
- [14] Gaurav Bhatnagar and Balasubramanian Raman, "A new robust reference watermarking scheme based on DWT-SVD", *J. of Computer Standards & Interfaces* , 2009, pp.1-11.
- [15] Ali Al-Haj and Tuqa Manasrah , "Non-invertible copyright protection of digital images using DWT and SVD", *2nd Int. Conf. on Digital Information Management*, Vol. 1, 2007, pp.448 – 453.
- [16] V. Senthil and R. Bhaskaran, "Wavelet based digital image watermarking with robustness against geometric attacks", *Int. Conf. on Computational Intelligence and Multimedia Applications*, 2007, pp 89-93.
- [17] S. G. Kejgir, Manesh Kokare, "Optimization of bit plane combination for efficient digital image watermarking", *IEEE Int. Journal of computer science and Information Security*, Aug.2009, vol.2, pp.9-18.
- [18] S. G. Kejgir, Manesh Kokare, "A wavelet-based good fidelity digital image watermarking using subband threshold computing", *IEEE Int. Conference on Signal & Image Processing*, 7-9 Dec. 2006, pp-8-12.
- [19] A.W.Calder Bank, I.Daubechies, and W.Sweldons, "Wavelet transforms that map integers to integers", *journal of applied and computational harmonic analysis*, Vol. 5, no. 3, 1998, pp.332-369.
- [20] W.H. Chang, Y.S. Lee, W.S. Peng, and C.Y. Lee, "A line-based, memory efficient and programmable architecture for 2D DWT using lifting scheme," *IEEE Int. Symposium on Circuits and Systems*, Sydney, Australia, 2001, pp. 330-333.
- [21] I. Daubechies and W. Sweldens, "Factoring wavelet transforms into lifting schemes," *The Journal of Fourier Analysis and Applications* , Vol. 4, 1998, pp. 247-269.
- [22] D. S. Taubman and M. W. Marcellin, *JPEG2000: "Image compression fundamentals, standards and practice"*, Norwell, MA: Kluwer, 2002.
- [23] W. Sweldens, "The lifting scheme: a custom-design construction of biorthogonal wavelets", *Applied and Computational Harmonic Analysis*, Vol. 3, no. 15, 1996, pp. 186 - 200.
- [24] S. Mallat, "A theory for multiresolution signal decomposition: the wavelet representation," *IEEE trans. on Pattern Analysis and Machine Intelligence*, vol. 11, no. 7, 1989, pp. 674–693.
- [25] Ingemar J. Cox, Matt L. Miller and Jeffrey A, "Bloom watermarking applications and their properties", *NEC Research Institute*.
- [26] I. J. Cox and J.-P. Linnartz, "Some general methods for tampering with watermarks", *IEEE trans. on Selected Areas of Communications*, 16(4), 1998, pp.587–593.