

# USING FUZZY TOPSIS FOR MULTI CRITERIA DECISION MAKING

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**ABSTRACT-***The objective is to create a prototype of Using Fuzzy TOPSIS for Multi Criteria Decision Making - that will assist in selecting the most attractive alternative in making a decision under uncertainty, using criteria supplied and rated according to the priorities for said criteria. The fuzzy decision support tool will be tested by solving a real decision-making problem under uncertainty. The program utilized fuzzy sets and multi-attribute decision matrices in order to select the most desirable output*

**KEYWORDS-**Decision Support, Fuzzy logic, TOPSIS, Multi criteria modeling

## I. INTRODUCTION

Decision making problem is the process of finding the best option from all of the feasible alternatives. Multi Criteria Decision Making is an effort to create a single-user program - a decision support system using fuzzy logic that will assist in selecting the most attractive alternative in making a decision under uncertainty, using criteria supplied and rated according to his priorities for said criteria. The program utilized fuzzy sets and multi-attribute decision matrices in order to select the most desirable option. It uses a suitable decision making method for solving a problem under uncertainty using multiple criteria and the attributes provided.. TOPSIS(Technique for Order Preference by Similarity to Ideal Solution) is the method used for decision making.

### A. Fuzzy Sets and Fuzzy Logic

Fuzzy sets were introduced by Zadeh in 1965 to represent/manipulate data and information possessing nonstatistical uncertainties. It was specifically designed to mathematically represent uncertainty and vagueness and to provide formalized tools for dealing with the imprecision intrinsic to many problems.

Fuzzy logic provides an inference morphology that enables approximate human

reasoning capabilities to be applied to knowledge-based systems. The theory of fuzzy logic provides a mathematical strength to capture the uncertainties associated with human cognitive processes, such as thinking and reasoning.

### B. Fuzzy inference engine

The inference engine for fuzzy is depicted below

## Operation of Fuzzy System

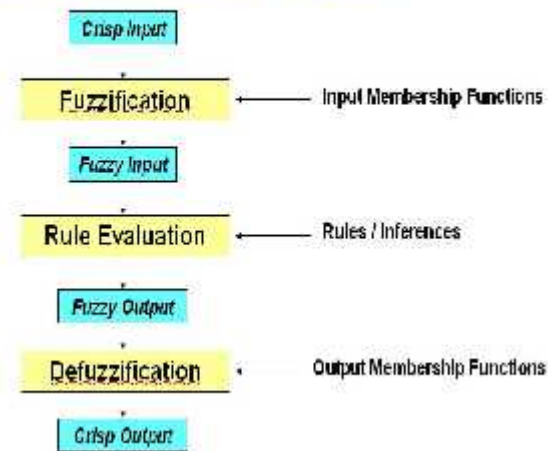


Fig 1: Inference engine fuzzy

Some of the essential characteristics of fuzzy logic relate to the following

- In fuzzy logic, exact reasoning is viewed as a limiting case of approximate reasoning.
- In fuzzy logic, everything is a matter of degree.
- In fuzzy logic, knowledge is interpreted a collection of elastic or, equivalently, fuzzy constraint on a collection of variables.
- Inference is viewed as a process of propagation of elastic constraints.
- Any logical system can be fuzzified.

## II. PROBLEM DEFINITION

To validate the method described here has done a study focusing on CRM (Customer Relationship Management) systems in a transport company. The problem consists in to establish the best customer of the company taking into account the attributes Revenue (C1), Percentage of bills late more than 30 days (C2), Regularity of payment bills(C3), Total weight carried(C4) , Amount of invoice by customer(C5) and Amount of transport invoice(C6). Except the attributes C2 and C3, all others values take into account the movement in a period of 30 days. The variables C1, C3, C5 are called positive, ie, must be maximized. The Variables C2, C4 and C6 represent cost and should be minimized. Customers are represented by  $A_n$ . The decision matrix for this problem is presented in Table

$A_n$	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$
$A_1$	200.000,00	15	79	100	5	85
$A_2$	150.000,00	0	97	70	3	89
$A_3$	37.000,00	2	77	23	7	75
$A_4$	218.000,00	23	90	89	12	100
$A_5$	90.000,00	8	34	34	9	90
$A_6$	76.000,00	10	95	30	5	54
$A_7$	55.000,00	4	34	39	2	76
$A_8$	126.000,00	0	100	6	14	120
$A_9$	142.000,00	3	43	70	12	150
$A_{10}$	69.000,00	2	7	44	8	87

Fig 2: Decision matrix

The problem is to identify the best customer of a company. The attributes accounted are

- Revenue
- Percentage of bills late more than 30 days
- Regularity of payment bills
- Total weight carried
- Amount of invoice by customer
- Amount of transport invoice

Using this attributes, the ideal solution is found by criteria supplied and weightage given. The decision engine for the above attributes can be given as

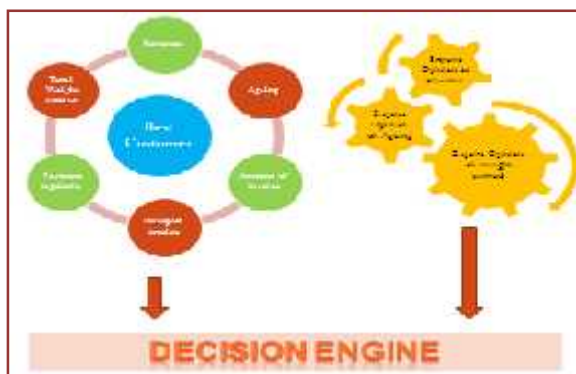


Fig 3: Decision engine

Based on this decision engine the best customer is selected using fuzzy model.

### III. IMPORTANCE OF STUDY

The aim of the project is, to analyze all the multi-criteria decision making methods, selecting the suitable one which is more efficient and accurate and to develop system software for Decision making in uncertain condition. There are decision making methods which are used to make decision under conditions where the data and values are crisp, these methods when used in uncertain condition where the linguistic variable are used these methods doesn't give a accurate solution. To get an accurate solution in uncertain condition the decision support system should be able to make accurate decision in both certain and uncertain conditions.

### IV. STATEMENT OF THE PROBLEM

There is no system present to handle uncertain environment for classic multiple criteria. Existing system is able to handle only a pair of input with a set of true/false [1/0]. With existing system user is able to measure only whether the alternative comes under true set or false set. Fuzzy logic is not implemented in existing system.

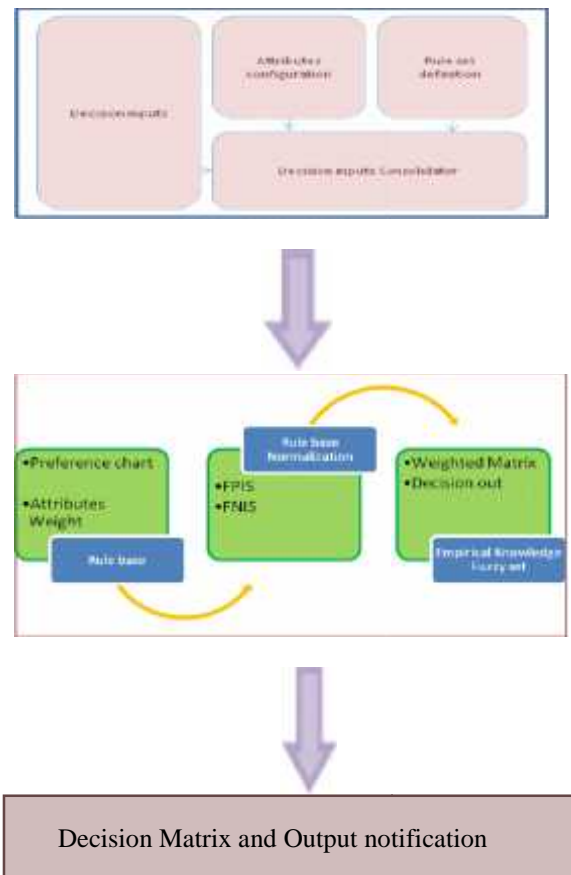
### V. OBJECTIVES

In order to achieve the objectives mentioned above the scope of the project has to be clearly defined. The Decision support system should be able make decision in uncertain conditions using the MCDM methods. The first step to achieve the objectives is to analyze and choose the appropriate methods to design the system. With the selected method the system input is designed in a way that it accept linguistic variable and process is to give accurate outputs. The resulting software will solve the problem of making decision in uncertain condition, and the same software can be used in certain and uncertain condition to make decision.

### VI. SYSTEM ARCHITECTURE

The system architecture for multi criteria decision support system is as follows

Fig 4: Multi criteria decision support system architecture



Above architecture provide an elaborated idea about the system. It contains mainly three modules they are

**Data collection :** Problem description along with number of criteria and alternative are getting on this stage. Once number of alternative and criterias are defined then details such as name, weightage, certainty are getting for the same.

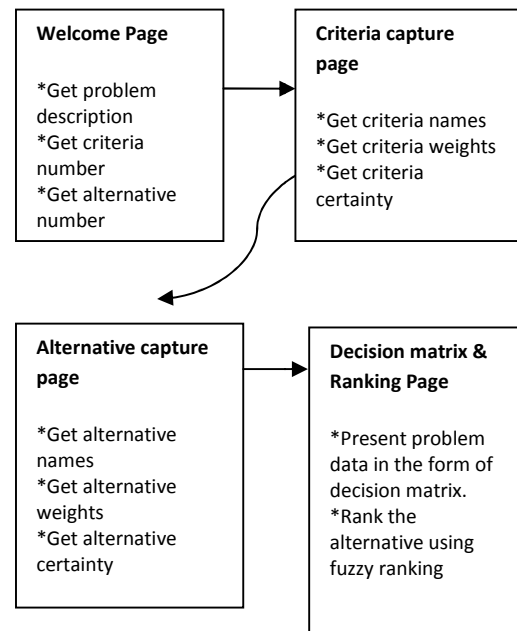
**Data process based on rules :** Weightage is calculating based on the certainty supplied by the user and the make it ready for the decision matrix.

**Formation of decision matrix and fuzzy ranking :** Based on the certainty and weightage decision matrix is create, one this part is complete alternatives are ranking using the fuzzy ranking method and the end result is providing to the user.

## VII. WEB PAGE INTERFACE

The structure of the Web pages for the fuzzy decision support prototype tool is demonstrated in Figure (2), this storyboard details the purpose of each page and shows the users progression through the system.

Fig 5: Storyboard for Web-pages



## VIII. IMPLEMENTATION

### A. Multi criteria decision making methods

This chapter introduces the decision making process and provides an overview of the varying methods for handling multi-criteria decision making (MCDM) problems. Section 8.2 defines the decision making process and its components. Section 8.3 provides an overview of the two most popular MCDM methods used in here. Section 8.4 is a more detailed explanation of the MCDM method chosen in the implementation of the DSS tool proposed in the System Request

### B. Decision making process

MCDM is the process of making a selection from a group of predetermined alternatives, based upon a set of criteria, the weights of each criterion and the measure of performance of each alternative with respects to each criterion. For the purpose of explanation the following decision making problem is posed: "Which area of Manchester should company X rent much needed additional office space?" This problem is defined by the set of alternatives choices and a set of known criteria, upon which an alternative is selected. In the given scenario the alternatives could be "Fallowfield", Deansgate" and "Hulme"; for the sake of argument the set of criteria could include cost of rent, the size of the office space and the available amenities. Each of these criteria has a value of importance (weight) to the decision maker. In making a choice a subjective

performance score is given to each alternative with respects to each criterion, taking into consideration the relative importance (weight) of the criterion. Then a number of decision-making methods could be implemented to process the decision (more on this later). This scenario is an example of a MCDM problem. The decision making process can be defined as being made up of three phases: intelligence, design and choice. The intelligence phase gathers information and defines the problem that is to be solved. The design phase constructs a model of this problem that contains evaluative criteria and alternative options. The choice phase selects an alternative that is deemed to be the best. The phases that concern this project are design and choice: assisting a user to create a model of the problem and selecting the most appropriate based on that model. A MCDM problem can be concisely expressed in a matrix format shown below:

### The Decision Matrix

	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	...	C <sub>n</sub>
	W <sub>1</sub>	W <sub>2</sub>	W <sub>3</sub>	...	W <sub>n</sub>
A <sub>1</sub>	a <sub>11</sub>	a <sub>12</sub>	a <sub>13</sub>	...	a <sub>1n</sub>
A <sub>2</sub>	a <sub>21</sub>	a <sub>22</sub>	a <sub>23</sub>	...	a <sub>2n</sub>
A <sub>3</sub>	a <sub>31</sub>	a <sub>32</sub>	a <sub>33</sub>	...	a <sub>3n</sub>

(SOURCE: FABIO J.J SANTOS AND HELOISA A.CAMARGO "FUZZY SYSTEMS FOR MULTI CRITERIA DECISION MAKING" CLEI ELECTRONIC JOURNAL VOL.13, NO.3, PP 4, DEC 2010.) WHERE A<sub>i</sub> IS THE I<sup>TH</sup> ALTERNATIVE, C<sub>j</sub> IS THE J<sup>TH</sup> CRITERION, W<sub>j</sub> IS THE WEIGHT OF THE J<sup>TH</sup> CRITERION, A<sub>ij</sub> IS THE PERFORMANCE MEASURE (SCORE) OF THE I<sup>TH</sup> ALTERNATIVE IN TERMS OF THE J<sup>TH</sup> CRITERION, AND THERE ARE M ALTERNATIVES AND N CRITERIA. THIS DECISION-MATRIX IS USED BY MANY DECISION-MAKING METHODS TO DEFINE THE MCDM PROBLEM TABLE

### C. Multi Criteria Decision making model

Following are brief descriptions of five popular decision-making models. Each method uses the decision-matrix shown in Table 1 to model MCDM problems. The following descriptions are based on those provided by Triantaphyllou and Lin

#### 1. Weighted Sum Model(WSM)

The WSM is probably the simplest and most widely used MCDM method. Suppose that there are M alternatives and N criteria in a decision-making problem. Then the priority score of the best alternative P\*WSM, is found with the following expression:

$$P^*_{WSM} = \max_{M \geq i \geq 1} \sum_{i=1}^N a_{ij} W_j \quad (1)$$

#### 2. Weighted Product Model(WPM)

The WPM is similar to the WSM, but uses multiplication rather than addition. Each alternative is compared with the others by multiplying a number of ratios, one for each criterion. Each ratio is raised to the power of the relative weight of the corresponding criterion. The following formula is used to compare two alternatives A<sub>K</sub> and A<sub>L</sub>:

$$R\left(\frac{A_K}{A_L}\right) = \prod_{j=1}^N \left(\frac{a_{Kj}}{a_{Lj}}\right)^{W_j} \quad (2)$$

#### D. Modelling Uncertainty in decision

This chapter looks at modeling uncertainty in decision-making and how uncertainty can be incorporated in the MCDM method, TOPSIS. Section 4.2.1 defines what characterizes uncertainty in decision-making. Section 8.3 explains the model chosen to model uncertainty along with its operations. Section 8.4 demonstrates the application of the uncertainty model to the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method.

#### 1. Defining Uncertainty

Taking into consideration the problem scenario presented in Section 8.2 as an example, input values are often not precise inputs like unit measurements. Linguistic variables are sometimes used to describe properties of objects that don't have quantitative values, like quality or beauty. These linguistic variables are subjective by nature; their meanings vary between decision makers, and situations. Such variables possess no real boundaries, and are open to interpretation. In some instances, even though an exact value could be given, a linguistic variable may be used instead. For example, the size of an office may be given as 'large' rather than '400 square feet'. This expression of a measurement through indistinct and potentially ambiguous terms is defined as uncertainty.

In its current state TOPSIS can only function with crisp data. A crisp set has clear boundaries: a value is either in the set or not in the set. TOPSIS must be modified so that it can solve problems that have uncertain values. There are several methods of modelling uncertainty,

including rough set theory, probability theory, and fuzzy set theory. Fuzzy set theory is the most developed in the field of MCDM. It has an intrinsic ability to handle this described ambiguity of values, and can be implemented in an MCDM system easily. Hence, TOPSIS is to be 'fuzzified' so that it can handle uncertainty, by employing fuzzy sets.

## 2. Fuzzy sets and Fuzzy Operators

Developed by Lotfi Zadeh at the University of California at Berkley, fuzzy set theory provides a simple way to arrive at a definite conclusion based upon a vague, ambiguous, imprecise, or missing data. In traditional mathematics every proposition must either be True or False, A or not A, either this or not this. An element has a degree of membership of 1 or 0.

Fuzzy sets allow degrees of membership between 0 and 1. A fuzzy subset of X is defined as a function  $f: X \rightarrow [0, 1]$ . This is, answers and degrees of set membership can be fractions. Statements may be absolutely true, absolutely false or some intermediate truth degree; one proposition can be 'more true' than another proposition. Although this does not correspond with the long-established Boolean logic used in the field of computing, it can be seen that this is a much more human way of thinking. There are four basic fuzzy membership functions: triangular, trapezoidal, Gaussian and generalized bell. MCDM systems often use an implementation of triangular fuzzy numbers (TRN). TRN are often useful in an MCDM system because they can easily implement linguistic values, such as cold, tall, etc. They are intuitive for the decision maker to use and simple to interpret. A TRN can be defined by a triplet  $(n_1, n_2, n_3)$ , with the membership function  $\mu: R[0,1]$ .

$$\tilde{m}(x) = \begin{cases} \frac{1}{m-1}x - \frac{1}{m-1}, & x \in [l, m] \\ \frac{1}{m-u}x - \frac{1}{m-u}, & x \in [m, u] \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

Where  $l \leq m \leq u$ , and  $l$  is the lower,  $m$  the modal and  $u$  the upper value of  $\mu$ .

The wider the scope of the TFN the fuzzier it is, representing a greater degree of uncertainty. Crisp numbers can be represented by TRN also, such TRNs represent absolute certainty. A TFN representation of a crisp number is called a 'singleton', this is when  $l=m=u$ .

## 3. Ranking Fuzzy Numbers

The last step of the MCDM method, TOPSIS involves ranking the alternatives. This is straightforward using crisp data, but fuzzy triangular values require a fuzzy method for ranking. There exist two categories of fuzzy ranking methods. Methods that use a function to map a TRN to a single point (thus ranking these points), and methods that use fuzzy relations to compare pairs of fuzzy numbers, providing a linguistic meaning of the relationship.

The ranking method to be implemented in this project is from the first category; the center of gravity method. The following equation finds the center of gravity C (geometric center) of a TRN  $F = (l, m, u)$

$$C = l + \frac{(u-l) + (m-l)}{3} \quad (4)$$

## IX. METHOD EXPLANATION

Fuzzy TOPSIS is an optimization technique that uses linguistic terms to evaluate the importance of attributes and their values. In order to improve their ability to deal with vagueness, in this paper we propose adding to the Fuzzy TOPSIS an attribute that represents the evaluation of a Fuzzy Rules Based System. The main purpose of this modification is to add the capabilities of fuzzy rules based systems to Fuzzy TOPSIS

### A. FUZZY TOPSIS

Fuzzy TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is the method used for decision making. In the Fuzzy TOPSIS method a MCDM (Multi-Criteria Decision-Making) problem with  $n$  alternatives and  $m$  attributes can be expressed in matrix format according to (1) and (2),

$$D = \begin{matrix} & \begin{matrix} C_1 & C_2 & C_3 & \dots & C_m \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{matrix} & \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \tilde{x}_{13} & \dots & \tilde{x}_{1m} \\ \tilde{x}_{21} & \tilde{x}_{22} & \tilde{x}_{23} & \dots & \tilde{x}_{2m} \\ \tilde{x}_{31} & \tilde{x}_{32} & \tilde{x}_{33} & \dots & \tilde{x}_{3m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \tilde{x}_{m3} & \dots & \tilde{x}_{mm} \end{bmatrix} \end{matrix} \quad (1)$$



$$\tilde{W} = [\tilde{w}_1, \tilde{w}_2, \dots, \tilde{w}_n] \quad (2)$$

where  $X_{ij}, i=1,2,\dots,m$  and  $j=1,2,\dots,n$  are numeric data of problem and  $W_i, i=1,2,\dots,n$  is the importance degree of each attribute  $c_1, c_2, \dots, c_n$ , respectively

The best way to illustrate how TOPSIS functions when fuzzified is through the use of a numerical example:

Step 1: Construct the normalized decision matrix. The following decision matrix is derived from a MCDM problem with three alternatives and four criteria:

**Table 2: Normalized Decision Matrixes**

	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>
	(0.13,0.20,0.31)	(0.08,0.15,0.25)	(0.29,0.40,0.56)	(0.17,0.25,0.38)
A <sub>1</sub>	(0.08,0.25,0.94)	(0.25,0.93,2.96)	(0.34,0.70,1.71)	(0.12,0.24,0.92)
A <sub>2</sub>	(0.23,1.00,3.10)	(0.13,0.60,2.24)	(0.03,0.05,0.09)	(0.12,0.40,1.48)
A <sub>3</sub>	(0.15,0.40,1.48)	(0.13,0.20,0.88)	(0.62,1.48,3.41)	(0.24,1.00,3.03)

(Source: Evangelos Triantaphyllou., and Chin-Tun Lin., "Development and Evaluation of Five Fuzzy Multi attribute Decision-making methods" *International Journal of Approximate Reasoning*1996.,Vol.14,pp.281-310.)

Step 2: Construct the weighted normalized decision matrix. After normalizing Table 2, the following matrix is produced:

**Table 3: Weighted Normalized Decision-Matrix**

	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>
	(0.01,0.05,0.29)	(0.02,0.14,0.74)	(0.10,0.28,0.96)	(0.02,0.06,0.35)
A <sub>1</sub>	(0.03,0.20,0.96)	(0.01,0.09,0.56)	(0.01,0.02,0.05)	(0.02,0.10,0.55)
A <sub>2</sub>	(0.02,0.08,0.46)	(0.01,0.03,0.22)	(0.18,0.59,1.91)	(0.04,0.25,1.15)

(Source: Evangelos Triantaphyllou., and Chin-Tun Lin., "Development and Evaluation of Five Fuzzy Multi attribute Decision-making methods"

*International Journal of Approximate Reasoning*1996.,Vol.14,pp.281-310.)

Step 3: Determine the ideal and negative-ideal solutions. The ideal solution A\* is:

$$A^* = \{(0.03, 0.20, 0.96), (0.02, 0.14, 0.74), (0.18, 0.59, 1.91)\}$$

The negative ideal solution A- is

$$A^- = \{(0.01, 0.05, 0.29), (0.01, 0.03, 0.22), (0.01, 0.02, 0.05), (0.02, 0.06, 0.35)\}$$

Step 4: Calculate the separation measure. The separation distance between each alternative and the ideal and negative-ideal solutions are:

$$S_1^* = (0.09, 0.39, 1.41), S_1^- = (0.09, 0.28, 1.04),$$

$$S_2^* = (0.17, 0.59, 1.95), S_2^- = (0.02, 0.16, 0.76),$$

$$S_3^* = (0.02, 0.16, 0.71), S_3^- = (0.17, 0.60, 2.03),$$

For instance,

$$S_1^* = \{[(0.01, 0.05, 0.29) - (0.03, 0.20, 0.96)]^2$$

$$* [(0.02, 0.14, 0.74) - (0.02, 0.14, 0.74)]^2$$

$$* [(0.02, 0.06, 0.35) - (0.04, 0.25, 1.15)]^2$$

$$* [(0.10, 0.28, 0.96) - (0.18, 0.59, 1.91)]^2\}^{1/2}$$

$$= (0.09, 0.39, 1.41).$$

Step 5: Calculate the relative closeness to the ideal solution. The relative closeness to the ideal solution is found using the fuzzy version of the Equation

$$C_1^* = S_1^- / (S_1^* - S_1^-)$$

$$= (0.09, 0.28, 1.04) / ((0.09, 0.39, 1.41) + (0.09, 0.28, 1.04))$$

$$= (0.04, 0.42, 5.83):$$

Similarly,

$$C_2^* = (0.01, 0.21, 3.99),$$

$$C_3^* = (0.06, 0.79, 10.42).$$

Step 6: Rank the preference order. Using the center of gravity ranking method (Equation 4), the scores produced are:

$$C_1^* = 2.1,$$

$$C_2^* = 1.4,$$

$$C_3^* = 3.75.$$

Which concludes that alternative three has the greatest amount of closeness to the ideal solution, and therefore is the best alternative ( $C_3^* > C_1^* > C_2^*$ ).

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## X CONCLUSION

Decision making becomes more precise by taking the advantage of combining theoretical and empirical knowledge. In this article, it was proposed Fuzzy TOPSIS method in order to explore the possibilities of flexibility method by analyzing the positive and negative ideal solution. Thus, the results presented in this article give clear viability of using the method in many areas of decision support. It allows the possibility of incorporating the decision making process knowledge of experts expressed in rules.

## XI. SCOPES FOR FURTHER RESEARCH

The experiments to adapt the Fuzzy F-TOPSIS method to the group decision making as well as investigating techniques for automatic generation of rule base, are among the future work planned.