

Stability Analysis By Feedback Control Based Biped Robot

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Abstract— A problem in bipedal robots is how to design a controller that generates closed-loop motions walking, running, or balancing that are periodic and stable. Due to the inherent underactuation and the changing contact conditions with the ground, this task is far from being solved through existing control methods, and makes the planning of stabilizable with dynamic motions extremely difficult.

I. INTRODUCTION

First question arise that why selecting biped? The biped robots have higher mobility than conventional wheeled robots, especially when moving on rough terrains, up and down slopes and in environments with obstacles. The geometry of the biped robot is similar to the human beings, so it is easy to adapt to the human life environment and can help the human beings to finish the complex work. With the development of the society, the needs for robots to assist human beings with activities in daily environments are growing rapidly. Therefore, a large number of researches have been done on the bipedal walking.[1]

The motivation for this paper is mainly because biped robots may open a field for new generation of machines. They may one day replace manpower in areas where hazardous tasks are to be carried out, as well as to help human being in their day to day life. Thus robotics is considered as one of the key prospective technologies of the 21st century.[2]

Design a biped walking-robots on level ground.We have used the concept of feedback control for stable walking. The principal contribution of the present work is to show that the control strategy can be designed in a way that greatly simplifies.[2]

II. STRATEGY FOR STABILITY

Stability analysis of bipedal walking is difficult, since dynamics of the bipedal robots are highly non-linear, under actuated, subject to impacts, variable external forces, and discrete changes between different modes. The common strategies, such as analysis of the eigenvalues, gain and phase margins or Lyapunov stability theory, can be applied to particular modes, such as a single or double stance, but are usually incapable to characterize stability of all modes in total. So far, stability of bipedal walking is analyzed by specific techniques, such as Zero Moment Point (ZMP) & Poincarae return maps.[3]

A. ZMP(zero moment point)

Zero Moment Point is a concept related with dynamics and control of legged locomotion, e.g.,for humanoid robots. It specifies the point with respect to which dynamic reaction force at the contact of the foot with the ground does not produce any moment, i.e. the point where total inertia force equals zero. The concept assumes the contact area is planar and has sufficiently high friction to keep the feet from sliding. The ZMP is no longer meaningful if the robot makes multiple non-planar contacts.[2]

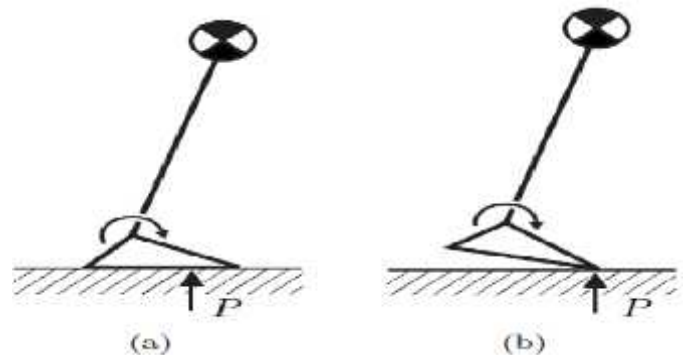


Fig.1 The ZMP (Zero Moment Point) criterion

Idealize a robot with one leg in contact with the ground as a planar inverted pendulum that is attached to a base consisting of a foot with torque applied at the ankle, and assume all other joints are independently actuated. In addition, assume adequate friction so that the foot is not sliding. In (a), the robot's nominal trajectory has been planned so that the center of pressure of the forces on the foot, P , remains strictly within the interior of the footprint. In this case, the foot will not rotate (i.e, the foot is acting as a base) and the system is therefore fully actuated.. In case (b), however, the center of pressure (CoP) has moved to the toe, allowing the foot to rotate. The system is now under actuated (two degrees of freedom and one actuator), and designing a stabilizing controller is nontrivial, especially when impact events are taken into account. The ZMP principle says to design trajectories so that case (a) holds; i.e., walk flat footed. Humans, even with prosthetic legs, use foot rotation to decrease energy loss at impact.

B. Poincare Return Map

If a walking gait exhibits a cyclic pattern, then the biped realizing such a gait will return to the same state at the end of each cycle.

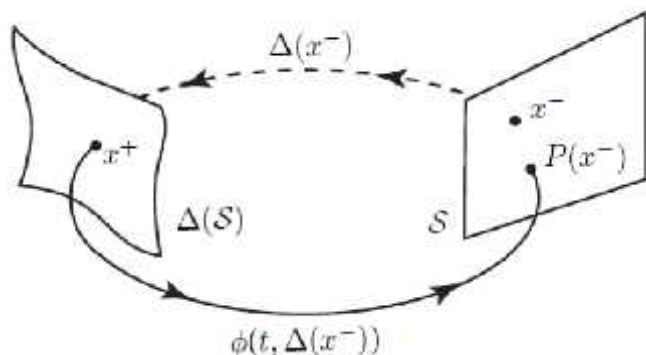


Fig.2 Cyclic pattern for Poincaré return map

The return map represents the evolution of the robot from just before an impact with the walking surface to just before the next impact, assuming that a next impact does occur. If it does not, the robot falls due to the preceding impact or fails in some other manner to complete a forward step. The notion of a Poincaré map and a periodic orbit in a system with impact effects are depicted in Fig.2

Poincaré return map is a learning method. Learn robot with thousand of trial for walking gait. Periodicity of gait is necessary for application of Poincaré return map.

III. GAIT DESIGN FOR BIPED ROBOT

The gait means pattern of biped robot walking. Our aim is to design this gait for a biped so our main focus on the biped gait must be look like human walking gait.

A walking gait can be divided into several domains, Each domain has own properties, needs own reference trajectories and control strategy.[3]

1. Push off: At $t = t_1$, the robot is in double support. The goal is to lift the swing leg, by pushing off or by lifting the swing foot from the ground.

2. Single support: This domain starts at $t = t_2$ when all contact points on the swing foot become inactive. Often (not always), the goal is to reach knee lock of the swing leg by swinging the leg forward.

3. Strike: At $t = t_3$, the swing knee gets locked. The goal is to drive the swing foot to the ground.

4. Double support: At $t = t_4$, one of the contact points on the swing foot becomes active. At the same time, the swing leg and stance leg swap their functions. The goal is to interchange the weight support from one leg to another. This domain ends when the robot is ready to push off at $t = t_5$. After this, the gait moves to domain 1 again.

These domains walking pattern are same as the human walking pattern, which made by continues and accurate observation of human gait.

A. ZMP Based Gait Design

Here, we present per domain shown in Fig. 3 how to design trajectories in each robot joint that satisfy the ZMP criterion for stable bipedal walking.

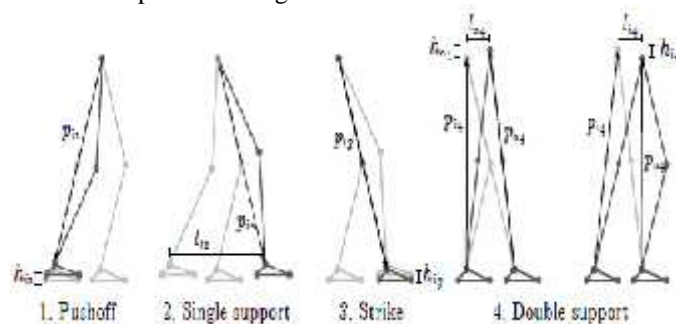


Fig.3 Domain for ZMP gait design

1. In this domain, the stance leg keeps the same configuration, $q_{a1}(t) = q_{a4}(t_1)$. This configuration is chosen such that ZMP remains above the stance foot during the complete swing phase. The swing foot needs to be lifted a distance h_{i1} from the ground.

2. For the same configuration in the stance leg as in the previous domain, which keeps ZMP above the stance foot, we have: $q_{a2}(t) = q_{a1}(t_2)$. The swing foot moves forward for a distance l_{i2} until the swing knee is locked.

3. The stance leg keeps the same configuration in this domain, $q_{a3}(t) = q_{a2}(t_3)$, so ZMP remains above the stance foot. The swing moves downwards for a distance $h_{i3} = -h_{i1}$.

4. Both feet of the robot are on the ground, which means a reduction of one dof. Here, we cannot treat the motions of two legs separately, since their motions are coupled. To resolve this situation, we divide the domain into two parts. In the first part we declare the stance leg as the master and the swing leg as the slave. This means that the stance leg determines the motion and that the swing leg has to follow. These motions are further constrained such as that ZMP remains above the support polygon, which is in this domain determined by both stance feet. In the second part of this domain, the roles swap the swing leg becomes the master and the stance leg the slave.

In above fig.3 gait is not look like human walking gait, so we require some external controller for limit cycle walking (LCW). shown in fig.3.1.

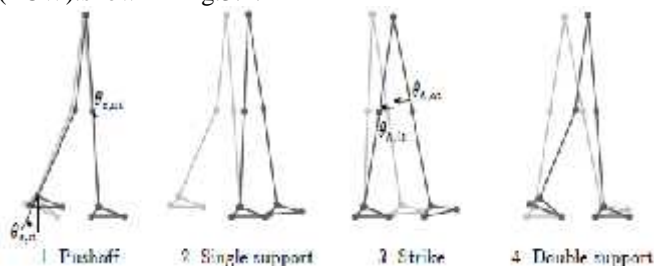


Fig.4 Design gait for LCW.

For LCW we use some extra control like knee control, foot scuffing prevention controller, step size controller etc. for achieve human like walking gait. Unfortunately, the design of LCW gaits is seemingly far more complicated than the design

of ZMP-based gaits. because its required more external controller.

B. Poincare return map gait design

If a walking gait exhibits a cyclic pattern, then the biped realizing such a gait will return to the same state at the end of each cycle. One can consider a lower dimensional subspace S of the system state-space, the Poincare return map, which is intersected by the cyclic motion. The intersection point is called a fixed point. For the stable periodic walking gait, the system state-trajectories return to approximately the same state after every step. One can make a Poincare map at, for example, every start of a step, just after heel strike. According to the terminology of bipedal locomotion, this mapping of the cyclic nonlinear dynamics is called the stride function. The stride function determines a transition between the current state and the state after one cycle:

$$q_{k+1} = S(q_k) \tag{1}$$

Only if the motion is perfectly cyclic, then the state q is a fixed point q_f:

$$q_f = S(q_f) \tag{2}$$

The stability of the cyclic motion can be analyzed by perturbing the initial fixed point and checking if it returns to the fixed point after a finite number of cycles. Linearizing the stride function around these perturbations can tell us if the state will return to the fixed point:

$$S(q_f + \delta q) \approx q_f + K \delta q \tag{3}$$

where K = ds/dq is the linear return matrix. This matrix determines if the state of the system returns to the fixed point for small perturbations. Namely, the motion is considered as stable if the eigenvalues of the matrix K fall inside the unit circle. If this is the case, then it is expected that the system state monotonously converges to the fixed point after each cycle. The smaller the absolute values of the eigenvalues, the faster the convergence to the limit cycle.

IV. MATHEMATICAL MODELING FOR BIPED ROBOT

The dynamic system of the biped robot is a nonlinear hybrid dynamic system, which consists of continuous differential equations and discrete events dynamic maps. Therefore, this system is a complex nonlinear system. Biped robot model is nonlinear characteristic. so it divide on two part swing phase model and impact model.

For swing phase mode The dynamic model is easily obtained with the method of Lagrange, which consists of first computing the kinetic energy and potential energy of each link, and then summing terms to compute the total kinetic energy, K_s, and the total potential energy, V_s.

$$L_s(q_s, \dot{q}_s) := K_s(q_s, \dot{q}_s) - V_s(q_s) \tag{4}$$

Applying the method of Lagrange the model is written in the form,

$$D_s(q_s) \ddot{q}_s + C_s(q_s, \dot{q}_s) \dot{q}_s + G_s(q_s) = B_s(q_s) u \tag{5}$$

The matrix D_s is the inertia matrix; C_s is the Coriolis matrix; G_s is the gravity vector; and B_s maps the joint torques to generalized forces.

The model is written in state space form by defining.

$$\dot{x} = f_s(x) + g_s(x)u \quad x^- \notin S \tag{6}$$

For impact model, impact means the swing leg touch to the ground.

$$x^+ = (x^-) \tag{7}$$

the hybrid model for biped is the combination of swing phase model and impact model.

$$\begin{cases} \dot{x} = f_s(x) + g_s(x)u & x^- \notin S, \\ x^+ = (x^-) & x^- \in S \end{cases} \tag{8}$$

V. FEEDBACK CONTROL DESIGN.

In feedback control design our aim is to design controller u in equation(5) to getting periodic orbit for stable walking. In feedback control we can't give any input, it totally depends on the previous condition of the biped, means the output given as a input by using feedback strategy.

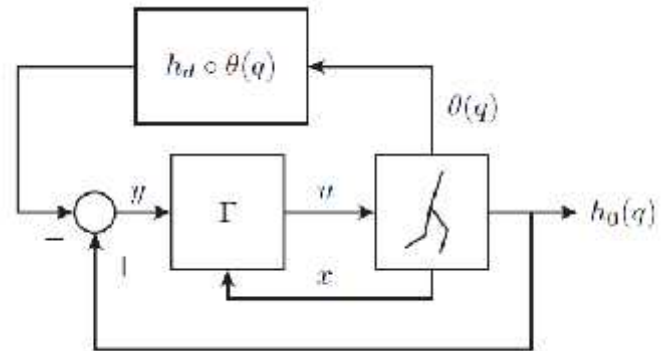


Fig.5 block diagram of feedback controller

Fig: shows block diagram of a time-invariant controller. The controller forces the signal $y = h_0(q) - h_d(q)$ to zero so that the signal $h_0(q)$ tracks the function $h_d(q)$. In this way, the control action is “clocked” to events on the robot’s path and not to an externally supplied time-based trajectory. With proper design of $h_0(q)$ and $h_d(q)$, a self-generated limit cycle through the combined actions of the controller and the environment on the robot.

A. CONTROLLER DESIGN

In order for the swing leg end to be at ground level at the end of the step, it must be the case that

$$\theta_1 = -\theta_2 \tag{9}$$

at contact. This will be taken care of in the control law design. Walking consists of two things: posture control, that is, maintaining the torso in a semi erect position, and swing leg advancement, that is, causing the swing leg to come from behind the stance leg, pass it by a certain amount, and prepare for contact with the ground. This motivates the direct control of the angles θ_2 (describing the torso) and θ_3 (describing the swing leg). On a periodic orbit corresponding to a normal walking motion, it is clear that the horizontal motion of the hips is monotonically strictly increasing. For the three-link walker, this is equivalent to $\theta_1(t)$ strictly increasing over each step of the walking cycle. It is therefore reasonable to assume that the corresponding trajectory for θ_1 has the property that $\theta_1(t)$ is strictly monotonic. It follows that θ_2 and θ_3 can each be re-parameterized in terms of $\theta_1(t)$. That is, without loss of

generality, it can be supposed that $\dot{x}_3 = h_{d,1}(x_1(t))$ and $\dot{x}_2 = h_{d,2}(x_1(t))$ for some functions $h_{d,1}$ and $h_{d,2}$.

The simplest version of posture control is to maintain the angle of the torso at some constant value, say θ^d , while the simplest version of swing leg advancement is to command the swing leg to behave as the mirror image of the stance leg, that is $\theta_2 = -\theta_1$. Thus the "behavior" of walking can be "encoded" into the dynamics of the robot by defining outputs with the control objectives being to drive the output to zero.

$$y := \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} := \begin{pmatrix} h_1(t) \\ h_2(t) \end{pmatrix} := \begin{pmatrix} \theta_3 - \theta_3^d \\ \theta_2 + \theta_1 \end{pmatrix} \quad (10)$$

Driving y to zero will force x_2 and x_3 to converge to known functions of x_1 .

As a first step the decoupling matrix is

$$L_g L_f h = \frac{1}{\det(D_2)} \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \quad (11)$$

Where

$$R_{11} = \frac{mr^3}{4} \left(\frac{5}{4}mr + M_H l + M_T r - mr(C_{12}^2) + M_T l c_{13} \right)$$

$$R_{12} = \frac{mr^3}{4} \left(\frac{5}{4}mr + M_H l + M_T r - mr(C_{12}^2) + 2M_T l c_{12} c_{13} \right)$$

$$R_{21} = \frac{-mM_T l r^2}{4} (1 + 2c_{12})(r c_{13} + l)$$

$$R_{22} = \frac{-mM_T l r^2}{4} (5ml + 4M_H l + 4M_T l + mr c_{13} + 2mr c_{12} c_{13} - 4M_T l c_{13}^2 + 2ml c_{12}^2)$$

$$\det(D_2) = \frac{mM_T l r^2}{4} \left(\frac{5}{4}m + M_H + M_T - m(c_{12})^2 - M_T (c_{13})^2 \right)$$

And,

$$c_{ij} = \cos(\theta_i - \theta_j)$$

The determinant of the decoupling matrix is zero if, and only if

$$-r(rM_H + rm + rM_T + lM_T \cos(\theta_1 - \theta_2)) = 0 \quad (12)$$

Thus, the decoupling matrix is invertible for all $x \in TQ$ as long as

$$0 < lM_T < r(m + M_T + M_H) \quad (13)$$

This imposes a very mild constraint on the position of the center of gravity of the torso of the robot in relation to the length of its legs.

Next, a controller is designed easiest way to do input-output linearize the swing phase dynamics and then impose a desired dynamic response on the outputs. So,

$$\begin{pmatrix} y_1 \\ y_2 \\ \theta_1 \end{pmatrix} = \Phi(\theta) = \begin{pmatrix} \theta_3 - \theta_3^d \\ \theta_2 + \theta_1 \\ \theta_1 \end{pmatrix} \quad (14)$$

is a diffeomorphism onto its range. With this coordinate transformation,

$$v = L_h^2 + L_g L_f h u \quad (15)$$

The swing phase dynamics can be written in the form

$$\begin{pmatrix} \dot{y} \\ \dot{\theta}_1 \end{pmatrix} = \Psi(y, \dot{y}) := \begin{pmatrix} \frac{1}{\epsilon^2} \Psi_1(y_1, \epsilon y_2) \\ \frac{1}{\epsilon^2} \Psi_2(y_2, \epsilon y_2) \end{pmatrix} \quad (16)$$

Where,

$$\Psi_\alpha(x_1, x_2) = -\text{sign}(x_2) |x_2|^\alpha - \text{sign}(\theta_2(x_1, x_2)) |\theta_2(x_1, x_2)|^{\frac{\alpha}{2}}$$

$$\theta_\alpha(x_1, x_2) := -x_1 + \frac{1}{2-\alpha} \text{sign}(x_2) |x_2|^{2-\alpha}$$

and set $\epsilon = 0.1$ and $\alpha = 0.9$. The parameter $\epsilon > 0$ allows the settling time of the controller to be adjusted. The controller is then

$$u(x) = (L_g L_f h(x))^{-1} (\Psi(h(x), L_f h(x)) - L_f^2 h(x)) \quad (18)$$

For implementing above equation in matlab programming and we can get wave form for periodic orbit with limit cycle by using feedback control.

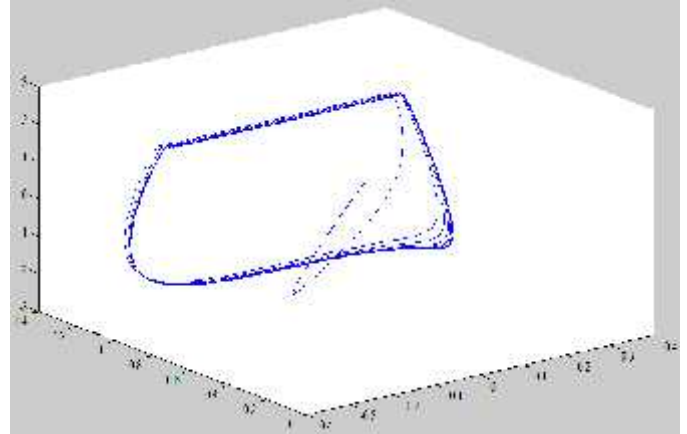


Fig.6 Projection onto $(\theta_1, \theta_2, \theta_3)$ of a trajectory asymptotically converging to an orbit

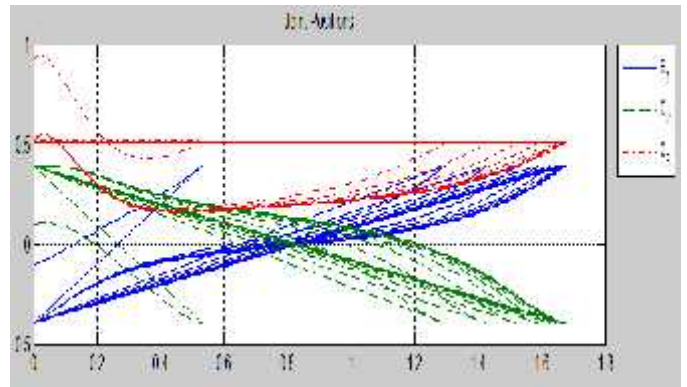


Fig.7 Joint positions are change with time.

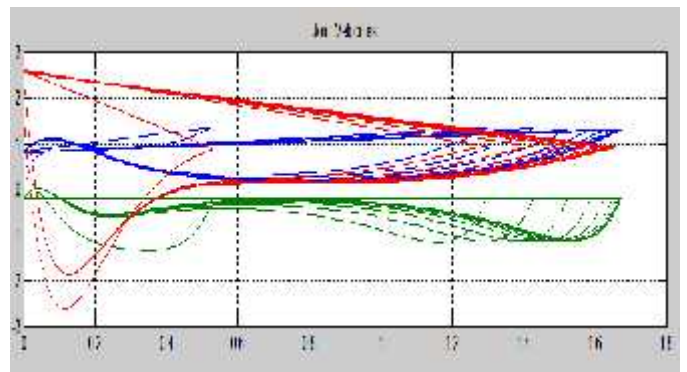


Fig.8 Joint velocities change with time

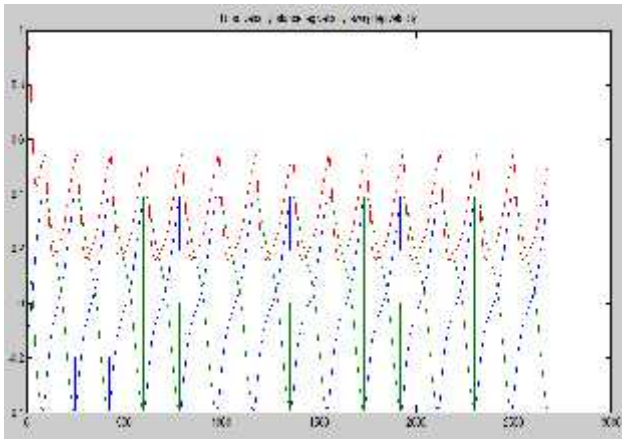


Fig.9 Torso velocity ,stance leg velocity ,swing leg velocity change with time.

VI. CONCLUSIONS

Our results illustrate that fulfillment of the ZMP criterion is sufficient for stability, but this criterion is applicable to just a portion of complete state-space of the bipedal robot. Using the Poincare return map method, we demonstrate that violation of the ZMP criterion does not necessarily mean that stable walking is not possible. This deeper physical understanding of the behavior of the robot under closed-loop controlled to experiment with simpler feedback implementations than those used in the analysis, and this led to satisfying experimental results: both because the robot walked well and understand the stability mechanism of the controllers.

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