International Journal of Advanced and Innovative Research (2278-844)/

Volume 7 Issue 9

NOTE ON TOTALLY REGULAR FUZZY DIGRAPHS

KR.BALASUBRAMANIAN[1*], N.ARUL PANDIYAN[2]

Assistant Professor[1], Research Scholar[2]

[1,2]Department of Mathematics, H. H. Rajah's College, Pudukkottai, Tamil Nadu, India.

***Abstract*— In this paper, we discussed totally regular fuzzy digraphs and properties of totally regular fuzzy digraphs. The concepts of the total degree of a vertex in fuzzy digraphs formed by the operation union is terms of the total degree of vertices in the given fuzzy digraphs for some particular cases are obtained some properties of regular fuzzy digraphs are studied and they are examined for totally regular fuzzy digraphs.**

***Keywords*— Degree of vertex, regular fuzzy graph, regular fuzzy digraph, total degree, totally regular fuzzy digraph.**

1. Introduction

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975 [9]. In 1965 Lofti A.Zadeh [3] introduced mathematical frame work to explain the concept of uncertainty in real life through the pratication of a seminal paper. This ranges from traditional mathematical subjects. Bhattacharya [1] propose the concept of some remarks on fuzzy graphs. mordeson J.N and peng.c.s [4] identified the operations on fuzzy graphs and the operations of union, join, cartesian an product and composition of two fuzzy digraphs. First we go through some basic definitions which can be found in [2,5-12]

In this paper, we review concepts involving the degree of a vertex in some fuzzy digraphs and regular property of fuzzy digraph which are obtained from two given fuzzy digraphs using the operation, join, Cartesian product and composition. Fir

2. Preliminaries

Definition: 2.1

A fuzzy graph $G = (V, \sigma, \mu)$ where v is the vertices, σ is the fuzzy subset of v and μ is the membership value on σ such that $\mu(u,v) \le \sigma(u) \wedge \sigma(v)$ for every $u, v \in v$.

Definition: 2.2

A fuzzy digraph $G_D = (\sigma_D \ \mu_D)$ is a pair of function $\sigma_D : v \longrightarrow [0,1]$ and $\mu_D : VXV \longrightarrow [0,1]$ where $\mu_D \le \sigma_D(u) \wedge \sigma_D[V]$ for every $u, v \in V$ and μ_D is a set of fuzzy directed edges called the fuzzy arcs.

Definition: 2.3

Let $G_D = (\sigma_{D,} \ \mu_D)$ be a fuzzy digraph on $G_D{}^* : (V, E)$ The degree of a vertex u is $d_{G_D}(u) = \sum_{u \neq v} \mu(u,v)$

The minimum degree of G_D is denoted by$\delta(G_D)$

$\delta(G_D) = \wedge \{d_{G_D}(v), \forall v \in V\}$

The maximum degree of G_D is denoted by $\Delta(G_D)$

$\Delta(G_D) = V\{d_{G_D}(v), \forall v \epsilon V\}$

Definition: 2.4

The order of a fuzzy digraph G_D are defined by $0(G_D) = \sum_{u \in v} \sigma_D(u)$

and The size of a fuzzy digraph G_D are defined by $s(G_D) = \sum_{u \in E} \sigma_D(u)$

Definition: 2.5

An indegree of a vertex u in a fuzzy digraph is the sum of the $\mu_D(uv)$ vertex of the edges incident. Towards the vertex $\sigma_D(u)$.

The outdegree of a vertex in a fuzzy digraph is the sum of the μ_D values of the edges incident from the vertex to all other vertices.

Here u is any vertex in v. Then the indgree and outdegree in Fuzzy digraph was denote by $d_D^+(u)$ and $d_D^-(u)$.

Definition: 2.6

Let G_D: (σ_D μ_D) be a fuzzy digraph on $G^* : (V, E)$) then a digraph is said to be fuzzy regular digraph if every vertex has the same indegree and outdegree as every other vertex.

Definition: 2.7

The union of two fuzzy digraph G_{10} and G_{20} is defined as a fuzzy digraph $G_D = G_{1D} \cup G_{2D} : (\sigma_{1D} \cup \sigma_{2D}, \mu_{1D} \cup \mu_{2D})$ on $G_D *: (V, E)$ where $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$ with.

$$(\sigma_{1D} U \sigma_{2D})(u) = \begin{cases} \sigma_{1D}(u) & if\ u \in v_{1-} v_2 \\ \sigma_{2D}(u) & if\ u \in v_{2-} v_1 \\ \sigma_{1D}(u) \vee \sigma_{2D}(u) & if\ u \in v_2 \cap v_1 \end{cases}$$

$$(\mu_{1D} U \mu_{2D})(e) = \begin{cases} \mu_{1D}(e) & if e \in E_{1-} E_2 \\ \mu_{2D}(e) & if\ e \in E_{2-} E_1 \\ \mu_{1D}(e) \vee \mu_{2D}(e) & if\ e \in E_{2\cap} \cap E_1 \end{cases}$$

Definition: 2.8

Let $G_D : (\sigma_{D,}\mu_D)$ be a fuzzy digraph. The degree of a vertex u in G_D is defined by

$$d_{G_D}(u) = \sum_{u \neq v} \mu_D(uv) = \sum_{uv \epsilon E} \mu_D(uv).$$

Definition: 2.9

Let $G_D : (\sigma_D, \mu_D)$ be a fuzzy digraph on G_D*. The total degree of a vertex $u \in v$ is defined by,

$$td_{G_D}(u) = \sum_{u \neq v} \mu_D(uv) + \sigma_D(u)$$

$$d_{G_D}(u) + \sigma_D(u)$$

If each vertex G_D has the same total degree k, then $G_D$is said to be a totally regular fuzzy digraph of total degree k or a k- totally regular fuzzy digraph.

3. Totally regular fuzzy digraph

Definition 3.1

A digraph is said to be fuzzy regular digraph if every vertex has the same indegree and outdegree as every other vertex.

Example:

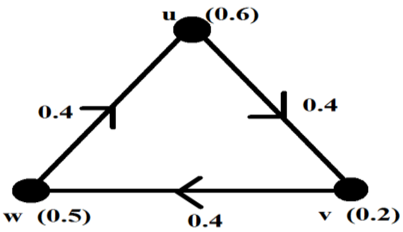

FIG 1-regular fuzzy digraph.

In figure $G_D^* : (V, E)$ where $V = \{u, v, w\}$ and $E = \{uv, vw, wu\}$ define G_D: ($\sigma_{D1}\mu_D$), σ_D(u) = 0.6 σ_D(v) = 0.2 σ_D(w) = 0.5 and $\mu_D(uv) = 0.4$ $\mu_D(vw) = 0.4$ $\mu_D(wv) = 0.4$. Then every vertex has same indegree and outdegree. So G_D is a regular fuzzy digraph.

Definition 3.2

Let G_D: (σ_D, μ_D) be a fuzzy digraph on $G_D{}^*$. The total degree of a Fuzzy digraph is defined by.

$$td_{G_D}(u) = \sum_{u \neq v} \mu_D(uv) + \sigma_D(u)$$

$$= d_{G_D}(u) + \sigma_D(u).$$

If every vertex of G_D has the same total degree $k_{1.}$ Then G_D is said to be totally regular fuzzy digraph of total degree k or a k-totally regular fuzzy digraph.

Example:

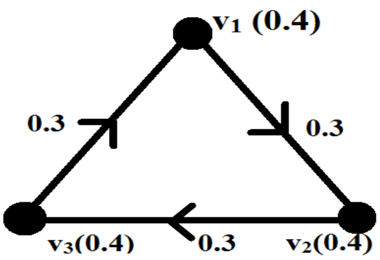

FIG 2-regular fuzzy digraph and totally regular fuzzy digraph

In figure $G^{*}:(V,E)$ where V={v_1, v_2, v_3} and E ={ v_1v_2, v_2v_3, v_3v_1} Define $G_D:(\sigma_D,\mu_D)$ by σ_D (v_1) = σ_D (v_2) = σ_D (v_3) = 0.4 and μ_D (v_1v_2) = μ_D (v_2v_3) = μ_D (v_3v_1) = 0.3. d_{GD} (v_1) = d_{GD} (v_2) = d_{GD} (v_3) = 0.3 and td_{G_D} (v_1) = td_{G_D} (v_2) = td_{G_D} (v_3).

Hence G_D is a regular fuzzy digraph and G_D is also a totally regular fuzzy digraph.

Theorem 3.3

Let $G_D:(\sigma_D,\mu_D)$ be a fuzzy digraph on $G_D{}^{*}$:(V, E). Then σ_D is a constant function if and only if the following are equivalent.

1) G_D is a regular fuzzy digraph
2) G_D is a totally regular fuzzy digraph.

Proof:

Suppose that σ_D is a constant function.

Let $\sigma_D(u) = c$, a constant, $\forall\, u \in v$.

Assume that G_D is a k – regular fuzzy digraph.

Then $d_D\ (u) = k \ \forall\, u\ \in v$

So $td_{G_D}\ (u) = d_D(u) +\ \sigma_D(u) \ \forall\, u \in v$.

$td_{G_D}\ (u) = K_1 + C \ \forall\, u \in v$.

Hence G_D is a totally regular fuzzy digraph.

Thus (1) ⇒ (2) is proved.

Now, suppose that G_D is a $K_{2.}$ totally regular fuzzy digraph

Then td_{G_D}(u) =K_2 $\forall\, u \in v$

d_D(u) + $\sigma_D(u) = K_2 \ \ \forall\, u \in v$

d_D(u) + $c = K_2 \ \forall\, u \in v$

d_D(u) $= K_2 - c \ \forall\, u \in v$

So G_D is a regular fuzzy digraph.

Thus (2) ⇒ (1) is proved.

Hence (1) and (2) are equivalent.

Conversely,

Assume that (1) & (2) are equivalent.

(i.e) G_D is regular iff G is totally regular.

Suppose σ_D is not a constant function.

Then $\sigma_D(u) \neq \sigma_D(\omega)$ for at least one pair of vertices u,$w \in v$.

Let G_D be K- regular fuzzy digraph.

Then d_D(u) = d_D(w) = K

So td_D(u) $= d_D\ (u) + \sigma_D(u)$

= K + $\sigma_D(u)$

and td_D(w) = d_D(w) = $\sigma_D(w)$

= K + $\sigma_D(w)$

Since $\sigma_D(u) \neq\ \sigma_D(w)$, we have td_D(u) $\neq\ td_D\ (w)$

So G_D is not totally regular which is a contradiction to our assumption.

Now Let G_D be a totally regular fuzzy digraph

Then td_D(u) $=\ td_D$(w)

d_D(u) + $\sigma_D(u) = d_D$(w) + $\sigma_D(w)$

d_D(u) – $\sigma_D(w) = \sigma$ (w) – $\sigma\ (u)$

d_D(u) $\neq d_D$(w).

So G_D is not regular which is a contradiction to our assumption.

Hence σ_D is a constant function.

Theorem 3.4

If a fuzzy digraph $G_D:(\sigma_D,\mu_D)$ is both regular and totally regular then σ_D is a constant function.

Proof:

Let $G_D:(\sigma_D,\mu_D)$ be a K-regular and K_2 totally regular fuzzy digraph.

So d_D(u) $= K_1 \ \forall u \in v$

and td_D(u) $= K \ \forall u \in v$

now, td_D(u) $= K_2 \ \forall u \in v$

d_D(u) + $\sigma_D(u) = K_2 \ \forall u \in v$

$K_1 + \sigma_D(u) = K_2 \ \forall u \in v$

$\sigma_D(u) = K_2 - K_1 \ \forall u \in v$

Hence σ_D is a constant function.

Example:

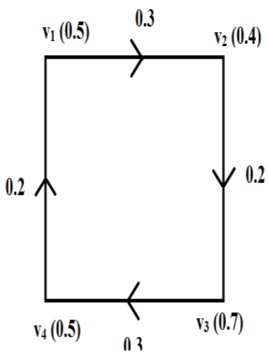

FIG – 3 not totally regular fuzzy digraph.

Consider $G_D{}^*:(V,E)$ where v=$\{v_1,v_2,v_3,v_4\}$ and

E= $\{v_1v_2,v_2v_3,v_3v_4,v_4v_1\}$.

Define $G_D:(\sigma_D,\mu_D)$ by $\sigma_D(v_1)$=0.5, $\sigma_D(v_2)$=0.4, $\sigma_D(v_3)$=0.7, $\sigma_D(v_4)$=0.5, and $\mu_D(v_1,v_2)$=0.3, $\mu_D(v_2,v_3)$=0.2, $\mu_D(v_3,v_4)$=0.3, $\mu_D(v_4,v_1)$=0.2.

Then G_D is a regular fuzzy digraph, if every vertex has same indegree and outdegree.

$d_{G_D}(v_1) = 0.3 + 0.2 = 0.5$, $d_{G_D}(v_2) = 0.3 + 0.2 = 0.5$

$d_{G_D}(v_3) = 0.3 + 0.2 = 0.5$, $d_{G_D}(v_4) = 0.3 + 0.2 = 0.5$

$\therefore$ G_D is regular fuzzy digraph.

$td_D(v_1)$ = 1.0 $td_D(v_2)$ = 1.9 $td_D(v_3)$ = 1.2 $td_D(v_4)$ = 1.0

So G_D is not totally regular fuzzy digraph.

Example:

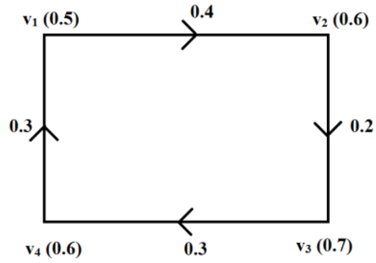

FIG – 4 total[illegible]ular fuzzy digraph.

Consider $G_D{}^*$:(V,E) where v=$\{v_1,v_2,v_3,v_4\}$ and

E= $\{v_1v_2,v_2v_3,v_3v_4,v_4v_1\}$.

Define $G_D:(\sigma_D,\mu_D)$ by $\sigma_D(v_1)$=0.5, $\sigma_D(v_2)$=0.6, $\sigma_D(v_3)$=0.7, $\sigma_D(v_4)$=0.6, and $\mu_D(v_1,v_2)$=0.4, $\mu_D(v_2,v_3)$=0.2, $\mu_D(v_3,v_4)$=0.3, $\mu_D(v_4,v_1)$=0.3.

Then, $d_{G_D}(v_1) = 0.7$ $d_{G_D}(v_2) = 0.6$ $d_{G_D}(v_3) = 0.5$

$d_{G_D}(v_4)$ =So G_D is not regular digraph.

$td_{G_D}(v_1) = 0.2$ $td_{G_D}(v_2) = 1.2$ $td_{G_D}(v_3) = 1.2$ $td_{G_D}(v_4) = 1.2$.

$\therefore$ $td_{G_D}(v_1)$ =$td_{G_D}(v_2) = td_{G_D}(v_3) = td_{G_D}(v_4)$ =1.2.

G_D is totally regular digraph.

The fuzzy digraph in example is totally regular fuzzy digraph But it is not a regular fuzzy digraph.

4. Properties of totally regular fuzzy digraphs

Theorem 4.1:

The size of a K-regular fuzzy digraph $G_D:(\sigma_D,\mu_D)$ on $G_D{}^*:(V,E)$ is $\frac{PK}{2}$ where P= $|V|$.

Proof:

The size of fuzzy digraph $G_D:(\sigma_D,\mu_D)$ is $S(G_D) = \sum_{uv\in E}\mu_D(uv)$

Since G is K-regular.

$$d_{G_D}(v) = k \;\; \forall Vu \in v$$

We have $\sum_{v\in V} d_{G_D}(v) = 2 \sum_{uv\in E}\mu_D(uv)$

$$= 2\, S(G_D).$$

$$\text{So } 2\,S(G_D) = \sum_{v\in V} d_{G_D}(v)$$

$$= \sum_{v\in V} K$$

$$= \text{P k}$$

Hence $S(G_D) = \frac{PK}{2}$

Theorem 4.2

In any fuzzy digraph $G_D:(\sigma_D,\mu_D)$ if $\sigma_D(v) > 0$ for every $v\in v$, Then $td_D(v) > 0$ for every $v \in v$.

Proof

$$\sigma_D(v) > 0 \quad \forall u \in v$$

$$td_D(v) > 0 \quad \forall u \in v.$$

Theorem 4.3

The maximum total degree of any vertex in a fuzzy digraph with P vertices is P.

Proof

For any vertex v_1

$$td_{G_D}(v) = \sum_{uv\in E}\mu_D(uv) + s_D(v)$$

$$= \sum_{uv\in E} 1 + 1$$

$$= d^*{}_{G_D}(v) + 1$$

$$= (p-1) + 1$$

$$td_D(v) = P.$$

Hence the proof

Theorem 4.4

The total degree of a vertex v is $\sigma_D(v)$ is and only is the degree of v is 0.

Proof

The total degree of a vertex v is

$$td_D(v) = S_D(v)$$

$$\sum_{uv\in E}\mu_D(uv) + S_D(v) = S_D(v)$$

$$\sum_{uv\in E}\mu_D(uv) = 0.$$

$$d_{G_D}(v) = 0.$$

Definition 4.5

For any fuzzy subset γ_D of V such that $\gamma_D \subseteq \sigma_D$. The fuzzy sub digraph of $G_D:(\sigma_D,\mu_D)$ induced by γ_D is the maximal fuzzy sub digraph of $G_D:(\sigma_D,\mu_D)$ that has fuzzy vertex set γ_D and it is the fuzzy sub digraph $H_D:(\gamma_D,\tau_D)$ where $\tau_D(u,v) = \tau_D(u) \wedge \tau_D(v) \wedge \mu_D(u,v) \forall u,v \in v$.

Theorem 4.6

Every fuzzy digraph is an induced fuzzy sub digraph of a totally regular fuzzy digraph.

Proof

Let G_D:(V,E) be any fuzzy digraph with P vertices and q edges.

If G_D is totally regular, there is nothing to prove. Suppose that G_D is not totally regular.

Let $\Delta^*{}_D$= max $\{td_D(v)/v \in V\}$ let us prove that G_D is an induced fuzzy sub digraph of a $\Delta^*{}_D$ totally regular fuzzy digraph.

Take a copy $G_D{}^1$ of G_D. Take any vertex v with total degree less than $\Delta^*{}_D$ join it to its copy V^1 in $G_D{}^1$.

Assign min $\{\sigma_D(V), \Delta^*{}_D - td_{G_P}(V)\}$ as the membership

Value of the edge VV^1

$$td_{G_D}(v) < \Delta^*{}_D$$

Let the resultant fuzzy graph be G_{1D} for any vertex V with $td_D(v) < \Delta_D{}^*$

If $\mu(VV') = \min\{\sigma_D(V), \Delta^*{}_D - td_{G_D}(V)\}$

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$= \Delta^*{}_D - td_{GD}(V)$

Then,

$= td_{G_{1D}}(V) = td_{GD}(V) + \mu_D(VV^1)$

$td_{GD} + \Delta^*{}_D - td_{GD}(V)$

$= \Delta^*{}_D$.

And all the vertices, which have total degree $\Delta^*{}_D$ in G_D and their copies in G'_D will have the same total degree $\Delta^*{}_D$ in G_{1D}.

Also, $td_{GD}(V^{'}) = td_{GD}(V^1) + M_D(VV')$

$= td_{GD}(V) + M_D(VV')$

$= \Delta^*{}_D$.

For every vertices with total degree less than $\Delta^*{}_D$ in G_D and their copies in G'_D.

If for some vertex V with $td_{GD}(V) < \Delta^*{}_D$

$= \min\{\sigma_{G_D}(V)\ \Delta^*{}_D - td_{G_D}(V)\}$

$= \sigma_{G_D}(v)$

Then $td_{G1D}(V) \quad < \Delta^*{}_D$

$G_{1D}$and $td_{G1D}(V) < \Delta^*{}_D$

The vertices have total degree Δ^* in the resultant fuzzy digraph

Let = [max $\frac{\Delta^*{}_D - td_{GD}(V)}{\sigma_D(v)}$ / $td_{GD}(V) < \Delta^*{}_D$]

Then the procedure stops after n steps with $\Delta^*{}_D$ totally regular fuzzy digraph G_{nD}.

Also G is an induced fuzzy sub digraph of G_{nD}

Here the number of vertices in

$G_{nD} = P + P + 2P + 2^2P + \cdots + 2^{n-1}P$

$= P + \left(\frac{2^{n-1}}{2-1}\right)P$

$= P + (2^{n-1})P$

$= 2^nP$.

Then the number of edges in $G_{nD} = nq + \sum_{V\in V}\left\{\frac{\Delta^*{}_D - td_{GD}(V)}{\sigma_{GD}(v)}\right\}$

Theorem: 4.7

Let $G_D : (\sigma_D, \mu_D)$ be a fuzzy digraph such that both σ_D and μ_D are constant partially regular fuzzy digraph. Then G_D is a totally regular fuzzy digraph if and only if G is a partially regular fuzzy digraph.

Proof:

Assume that G_D is a k - totally regular fuzzy digraph

Let $\mu_D(uv) = C\ \forall\ uv \in V$ and $\sigma_D(u) = C_1 \forall\ u \in V$

Where C and C_1 are constants

Then, $td_{GD}(u) = d_{GD}(u) + \sigma_D(u)$

$= \sum_{u\in E} \mu_D(uv) + \sigma_D(u)$

$K = Cd^*{}_{G_D}(u) + C_1$

$d^*{}_{G_D}(u) = \frac{k-c1}{C}\ \forall\ u \in V$

So $G^*{}_D$ is regular and hence G_D is a partially regular fuzzy digraph

Conversely assume that G is a partially regular fuzzy digraph.

Let $G^*{}_D$ be a r- regular graph then

$td_{GD}(u) = d_{GD}(u) + \sigma_D(u)$

$td_{GD}(u) = cd^*{}_{GD}(u) + c_1$

$= cr + C_1\ \forall \in V$

So G is totally regular fuzzy digraph.

Theorem 4:8

Let $G_{1D}:(\sigma_{1D}\ \mu_{1D})$ and $G_{2D}:(\sigma_{2D,}\mu_{2D})$ be two fuzzy digraph such that $\sigma_{1D} = \mu_{2D}$ Then $\sigma_{1D} = \sigma_{2D.}$

Proof:

The definition of fuzzy digraph $\mu_{2D} \leq \sigma_{2D}(u) \wedge \sigma_{2D}(V) \forall\ ur \in V_2$

We have min $\mu_{2D} \leq \sigma_{2D}$

$\sigma_{1D} \leq \mu_{2D}$

$\sigma_{1D} \leq min\ \mu_{2D}$

$\sigma_{1D} \leq min\ \mu_{2D} \leq \sigma_{2D}$

$M_{1D} \leq \sigma_{2D}$.

5. Totally regular property of union of two fuzzy digraphs

Definition: 5.1

The union of two fuzzy digraph G_{1D} and G_{2D} is defined as a fuzzy digraph $G_{1D} = G_D U G_{2D} : (\sigma_{1D}\ U\sigma_{2D},\ \mu_{1D} U\ \mu_{2D})$ on $G^*{}_D\ (V, E)$ Where $V = V_1 U V_2$ and $E = E_1\ U E_2$ with,

$$(\sigma_{1D} U \sigma_{2D})\ (u) = \begin{cases} \sigma_{1D}\ (u) & if\ u \in v_{1-} v_2 \\ \sigma_{2D}\ (u) & if\ u \in v_{2-} v_1 \\ \sigma_{1D}\ (u)\ V\ \sigma_{2D}\ (u) & if\ u \in v_2 \cap v_1 \end{cases}$$

$$(\mu_{1D} U\ \mu_{2D})\ (e) = \begin{cases} \mu_{1D}\ (e) & if e \in E_{1-} E_2 \\ \mu_{2D} (e) & if\ e \in E_{2-} E_1 \\ \mu_{1D}\ (e)\ V\ \sigma_{2D}\ (e) & if\ e \in E_2 \cap E_1 \end{cases}$$

Definition: 5.2

Let $G_{1D} : (\sigma_{1D}, \mu_{1D})$ and $G_{2D} : (\sigma_{2D}, \mu_{2D})$ be two fuzzy digraphs with underlying crisp graph (V_1, E_1) and (V_2, E_2) respectively.

1) If $u \in V_1\ U V_2$ and u is arbitary then

$$td_{G_{1D \cup G2D}}\ (u) = \begin{cases} td_{G1D}(u)\ u \in V_1 \\ td_{G2D}(u)\ u \in V_2 \end{cases}$$

Definition: 5.3

If $u \in V_1\ U V_2$ but no edge incident at u lies in $E_1\ \cap E_2$ Then any edge incident at u is either in E_1 or in E_2 but not both.

Also all these edges will be incident in $G_{1D} \cup G_{2D}$.

$$td_{G_{1D \cup G2D}}\ (u) = td_{G_{1D}}\ (u) + td_{G_{2D}}(u) - \sigma_{1D}\ (u) \wedge \sigma_{2D}\ (u)$$

Definition: 5.4

If $u \in V_1\ U V_2$ and some edges incident at a are in $E_1\ \cap E_2$. Any edge uv which is in $E_1\ \cap E_2\ \ G_{1D} U\ G_{2D}$ and for this uV.

$$td_{G1DUG2D}\ (u) = (td_{G1D}\ (u) + td_{G2D}(u) - \sigma_{1D}\ (u) \wedge \sigma_{2D}\ (u) -$$

$$\sum_{uv \in E_1 \cap E_2} \mu_{1D}\ (uv) \wedge \mu_{2D}\ (uv)).$$

Theorem 5.5

If G_{1D} and G_{2D} are two disjoint k- totally regular fuzzy digraph then $G_{1D}\ U G_{2D}$ is a k totally regular fuzzy digraph,

Proof:

Since G_{1D} and G_{2D} are disjoint fuzzy digraph

$$td_{G1DUG2D}(u) = \begin{cases} td_{G1D}(u) & if\ u \in V_1 \\ td_{G2D}\ (u) & if\ u \in V_2 \end{cases}$$

$= \text{k for every } u \in V_1 U V_2$

$G_1 U G_2$ is totally regular.

Conclusion

In this paper totally regular fuzzy digraphs, we have discussed the concept of total degree and every fuzzy digraph induced subdigraph of a totally regular fuzzy digraph. Also the total degree of a vertax in fuzzy digraphs formed by the operation union interms of the total degree of vertices in the given fuzzy digraphs for some particular cases are obtained.

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