Fuzzy Inventory Model without Shortages using Triangular Fuzzy Numbers

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***Abstract*: In this paper, we studied Economic Order Quantity (EOQ) inventory model without shortages. Our goal is to determine optimal order quantity for the proposed model. The purchasing cost, ordering cost, holding cost and screening rate are fuzzified. The function principle is used for fuzzy operation. We also provide an expression of the optimal order quantity for the case that all of the four parameters are triangular fuzzy numbers. A numerical example is provided to illustrate the proposed model and assess the effects of fuzziness of the parameters on the optimal solution.**

***Key words:* Fuzzy inventory, economic order quantity (EOQ), Function principle**.

I. INTRODUCTION

In 1915, the first inventory model was developed by Harris [2]. Later in 1965, first time the concept of fuzzy sets was introduced by Zadeh [7]. Fuzzy set theory is an extension of classical set theory where elements have degrees of membership. El Kassar et al [4]. (2012) examined the case where raw materials of imperfect quality items are used in the production process.

At the beginning of the inventory cycle, a supplier provides the raw material needed for the manufacture of the final product, and a proportion of this material thought to be ofimperfect quality. Those items are distinguished by a 100% screening process, yet all the raw material items are utilized in production. This will results in two types of finished products, one withperfect and another with imperfect quality.

The two finished products are assumed to be demanded continuously.They developed a mathematical model and attained the optimal production quantity by maximizing the profit function. The solution determined is illustrated through numerical example. Hence, several researches pay their attention to the inventory models with deficient items. Recently, Salameh and Jaber [5] assumed that the deficient items can be sold in a batch by the end of the 100% screening process. The result indicates that the economic order quantity tends to increase as the average percentage of imperfect quality items increases.

Based on the model of Salameh and Jaber [5], many new models for deficient items were extended. Wee et al. [6] extended a to the case with shortage back ordering. One of the popular ways is that the deficient items may be extended to suppliers directly. Hsu and Yu [3] have ever discussed the EOQ model with immediate return for imperfective items.

Throughout manufacturing\purchasing, two processes will be taking place: production and screening. After that, every final product is categorized as of perfect quality or imperfect quality. Let us consider p as the percentage of imperfect items generated. Then $q = 1 - p$ will refer to the percentage of perfect items produced. So, we notice that the production of items with perfect and imperfect quality takes place at a stable rate.

In this paper, the crisp case of the proposed model is first constructed in terms of annual profit. Then, the crisp model is extended in fuzzy senses. For solving the proposed model, Function principle is utilized to rank the fuzzy annual profit and determine the optimal order quantity. Duetothe fact that triangular fuzzy numbers are used extensively, we also provide an expression of the optimal order quantity for the case that all of the four parameters purchasing cost, ordering cost and screening rate, holding cost are triangular fuzzy numbers.

Chandrasiri [1] examined the case Fuzzy inventory model without shortages using triangular fuzzy numbers and signed distance method. Finally, a numerical example is given to demonstrate the applications of the proposed model and assess the effects of fuzziness of the purchasing cost, ordering cost and the screening rate, holding cost on the optimal solution.

II. NOTATIONS USED

D - demand rate (unit per year)
s - selling price per unit ($s > c$)
c - purchasing cost per unit
b - holding cost rate per unit per unit time
h - holding cost per unit per unit time, $h = bc$
a - ordering cost per order
q - perfective rate for each order
P - deficient rate for each order ($p = 1 - q$)
e - screening rate (unit per year)
w - screening cost per unit
Q - order size
$\tilde{c}$ - fuzzy purchasing cost per unit
$\tilde{a}$ - fuzzy ordering cost per order
$\tilde{e}$ - fuzzy screening rate (unit per year)

III. BASIC DEFINITIONS AND PRELIMINARIES

Definition 3.1:

Let $\tilde{A}$ be a fuzzy set on $R = (-\infty, \infty)$. it is called a fuzzy point if its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} 1; x = a \\ 0; x \neq a \end{cases}$$

Definition 3.2:

Let $[a, b : \alpha]$ be a fuzzy set on R. It is called a level α fuzzy interval, $0 \leq \alpha \leq 1, a < b$, if its membership function is

$$\mu_{[a,b;\alpha]} = \begin{cases} \alpha, if a \leq x \leq b \\ 0, if x \neq a \end{cases}$$

If $a = b$, we call [a,b;α] a level α fuzzy point at a.

Definition 3.3:

Among the various shapes of fuzzy number, triangular fuzzy number (TFN) is the most popular one. It is a fuzzy number represented with three points as follows:

$$\tilde{A} = (a, b, c)$$

This representation is interpreted as membership functions.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0; x < a \\ \dfrac{x-a}{b-a}; a \leq x \leq b \\ \dfrac{c-x}{c-b}; b \leq x \leq c \\ 0; x < c \end{cases}$$

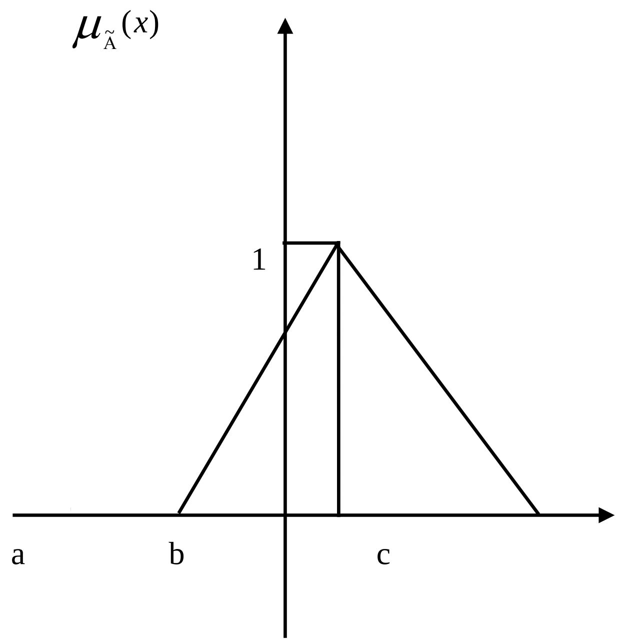

Figure 3.1: Triangular Fuzzy numbers $A = (a, b, c)$

Definition 3.4: **operations of Triangular Fuzzy numbers:**

Some important properties of operations on triangular fuzzy numbers are summarized as follows. Suppose $\tilde{A} = (a_1, b_1, c_1)$ and $\tilde{B} = (a_2, b_2, c_2)$ are two triangular fuzzy numbers, then arithmetical operations are defined as follows:

1. The addition of $\tilde{A}$ and $\tilde{B}$ is:
   $A \oplus B = (a_1, b_1, c_1) \oplus (a_2, b_2, c_2)$
   $= (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
2. The symmetric image of A is:
   $-A = -(a_1, b_1, c_1) = (c_1, b_1, a_1)$
3. The results from multiplication or division are not triangular fuzzy numbers.

Definition 3.5:

The α-level set of the triangular number $\tilde{A} = (a, b, c)$ is:

$$A(\alpha) = \{x | \mu_{\tilde{A}}(x) \geq \alpha\} = [A_L(\alpha), A_R(\alpha)]$$

where

$$A_L(\alpha) = a + (b-a)\alpha$$
$$A_R(\alpha) = c - (c-b)\alpha, \alpha \in [0,1]$$

We represent

$$\tilde{A} = (a,b,c)$$
$$= \cup [A_L(\alpha), A_R(\alpha)]; 0 \leq \alpha \leq 1.$$

Definition 3.6: The Signed Distance Method:

Defuzzification of $\tilde{A}$ can be found by signed distance method. If $\tilde{A}$ is a triangular fuzzy number than sign distance from $\tilde{A}$ to 0 is defined as:

$$d(\tilde{A},0) = \frac{1}{2}\int_0^1 ([A_L(\alpha), A_R(\alpha)], 0)\, d\alpha$$

where

$$A_\alpha = [A_L(\alpha), A_R(\alpha)]$$
$$A_\alpha = [a + (b-a)\alpha, b - (b-c)\alpha], \alpha \in [0,1]$$

is α-cut of fuzzy set $\tilde{A}$, which is a close interval.

IV. MODEL FORMULATION

(i) Finding the EOQ in crisp sense:

Consider the total cost,

$$P(Q) = sD - cD + \frac{bQ}{2e}.cD - \frac{bQ}{2}.cq - \left(\frac{a}{Q} + w\right).\frac{D}{q} - \left(\frac{bQ}{2e}\right).\frac{cD}{q}$$

$$\frac{d}{dQ}(PQ) = 0$$

$$\Rightarrow \frac{bcD}{2e} - \frac{bcq}{2} + \frac{a}{Q^2}.\frac{D}{q} - \frac{bcD}{2eq} = 0$$

$$\frac{aD}{Q^2 q} = \frac{bcD}{2eq} + \frac{bcq}{2} - \frac{bcD}{2e}$$

$$= \frac{1}{2eq}\left(bcD + bcq^2 e - bcDq\right)$$

$$\frac{aD}{Q^2 q} = \frac{bc}{2eq}\left(D + q^2 e - Dq\right)$$

$$\frac{aD}{Q^2} = \frac{bc}{2e}\left(q^2 e + D(1-q)\right)$$

$$Q^* = \sqrt{\frac{2aDe}{bc\left(eq^2 + (1-q)D\right)}}.$$

(ii)Finding the EOQ in Fuzzy sense;

$$P(Q) = sD - (c + \Delta_6 - \Delta_5) + \frac{(b + \Delta_8 - \Delta_7)Q}{2(e + \Delta_4 - \Delta_3)}$$
$$.(c + \Delta_6 - \Delta_5)D - \left(\frac{b + \Delta_8 - \Delta_7}{2}\right).Q(c + \Delta_6 - \Delta_5)q$$
$$- \frac{(a + \Delta_2 - \Delta_1)}{Q}.\frac{D}{q} - w.\frac{D}{q}$$
$$- \frac{(b + \Delta_8 - \Delta_7)Q}{2(e + \Delta_4 - \Delta_3)}.\frac{(c + \Delta_6 - \Delta_5)D}{q}$$

$$\frac{d}{dQ}.P(Q) = 0$$

$$\Rightarrow \frac{(b + \Delta_8 - \Delta_7)(c + \Delta_6 - \Delta_5)D}{2(e + \Delta_4 - \Delta_3)}$$
$$- \frac{(b + \Delta_8 - \Delta_7)}{2}(c + \Delta_6 - \Delta_5)q + \frac{(a + \Delta_2 - \Delta_1)}{Q^2}$$
$$.\frac{D}{q} - \frac{(b + \Delta_8 - \Delta_7)(c + \Delta_6 - \Delta_5)D}{2(e + \Delta_4 - \Delta_3)q} = 0$$

$$\Rightarrow \frac{(a + \Delta_2 - \Delta_1)}{Q^2}.\frac{D}{q} = \frac{(b + \Delta_8 - \Delta_7)(c + \Delta_6 - \Delta_5)D}{2(e + \Delta_4 - \Delta_3)q}$$
$$+ \frac{(b + \Delta_8 - \Delta_7)(c + \Delta_6 - \Delta_5)}{2}.q$$
$$- \frac{(b + \Delta_8 - \Delta_7)(c + \Delta_6 - \Delta_5)D}{2(e + \Delta_4 - \Delta_3)}$$

$$\Rightarrow \frac{1}{2(e+\Delta_4-\Delta_3)q}\left[\begin{array}{l}(b+\Delta_8-\Delta_7)(c+\Delta_6-\Delta_5)D \\ +(b+\Delta_8-\Delta_7)(c+\Delta_6-\Delta_5) \\ (e+\Delta_4+\Delta_3)q^2-(b+\Delta_8-\Delta_7) \\ (c+\Delta_6-\Delta_5)Dq\end{array}\right]$$

$$\frac{aD}{a^2 q} = \frac{(b+\Delta_8-\Delta_7)(c+\Delta_6-\Delta_5)}{2(e+\Delta_4-\Delta_3)q}\left[D+q^2(e+\Delta_4-\Delta_3)-Dq\right]$$

$$\tilde{Q}^* = \sqrt{\frac{2(e+\Delta_4-\Delta_3)(a+\Delta_2-\Delta_1)D}{(b+\Delta_8-\Delta_7)(c+\Delta_6-\Delta_5)\left(q^2(e+\Delta_4-\Delta_3)+D(1-q)\right)}}.$$

ALGORITHM FOR FINDING FUZZY TOTAL COST AND FUZZY OPTIMAL ORDER QUANTITY:

Step I:

Calculate total cost (TC) for the crisp model sense for the given crisp value.

Step II:

Determine fuzzy total cost $(\tilde{TC})$ using fuzzy arithmetic operations on fuzzy carrying and ordering cost, taken as fuzzy triangular numbers.

Step III:

Use signed distance method for defuzzification of $(\tilde{TC})$. Then find fuzzy optimal order quantity Q^*, using first and second derivative test.

V. NUMERICAL EXAMPLE

Apple Company needs to estimate the EOQ. However, the company have a policy with immediate return for deficient items. The company estimated that the annual demand D is likely to be 50,000 units per year. The purchasing cost c is approximately 25 per order. The perfective rate q is 0.98 for each order. The ordering cost a is projected to be around 100 per order. Holding cost b is estimated to be around 0.2 per unit. The selling price s is 175200. The screening cost e is 0.5.

Solution:

(i) Crisp model solution

$$Q^* = \sqrt{\frac{2aDe}{bc\left(eq^2 + (1-q)D\right)}}$$

$D = 50{,}000$, $c = 25$, $e = 0.5$, $a = 100$, $q = 0.98$, $b = 0.2$

$$Q^* = \sqrt{\frac{2 \times 100 \times 50{,}000 \times 0.5}{(25 \times 0.2)\left((0.98)^2 0.5 + (1-0.98)50{,}000\right)}}$$

$$Q^* = 31.61$$

(ii) Fuzzy model solution

$$\tilde{Q}^* = \sqrt{\frac{2\left(e+\Delta_4-\Delta_3\right)(a+\Delta_2-\Delta_1)D}{\left(b+\Delta_8-\Delta_7\right)\left(c+\Delta_6-\Delta_5\right)\left(q^2\left(e+\Delta_4-\Delta_3\right)+D(1-q)\right)}}.$$

$D = 50{,}000$, $c = 25$, $e = 0.5$, $a = 100$, $q = 0.98$, $b = 0.2.$

$\Delta_1 = 2$, $\Delta_2 = 4$, $\Delta_3 = 1$, $\Delta_4 = 3$, $\Delta_5 = 2$, $\Delta_6 = 5$, $\Delta_7 = 0.3$, $\Delta_8 = 1$

$$\tilde{Q}^* = \sqrt{\frac{2(0.5+2)(100+2)50{,}000}{(0.2+0.7)(2.5+3)\left((0.5+2)(0.98)^2+(1-0.98)50{,}000\right)}}$$

$$\tilde{Q}^* = 31.772$$

5.1 Sensitive Analysis:

No	Demand	Δ	$\tilde{Q}^*$
1	50,000	(2,4,1,3,2,5,0.3,1)	31.77
2	52,000	(3,5,3,5,12,6,1.5,2)	34.80
3	55,000	(5,8,3,5,2,8,2.5,3)	34.41
4	60,000	(2,6,1.5,3,4,5,2.5,3)	33.77
5	62,000	(1,4,1,3,1,2,0.25,0.75)	37.57
6	65,000	(2,5,2,4,2,3,0.5,1)	37.58

VI. CONCLUSION

In this paper, we have worked on Economic Order Quantity (EOQ) inventory model without shortages. Here, we have used signed distance method for defuzzification the total cost. We pointed out that for EOQ model, holding cost, ordering cost, purchasing cost, and screening rate can be expressed as triangular fuzzy numbers. Finally, we conclude that the Economic Order Quantity obtained by signed distance method is closer to crisp Economic Order Quantity.

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