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S S D P P Labelling of Snake Graphs

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***Abstract*—** In **SSDPP labelling, vertices of the graph are numbered with first (p-1) whole numbers and the edges (e = ab) with $|\{f(a)\}^2 + \{f(b)\}^2 - f(a)f(b)|$, where f(u) is the label of the vertex u. Greatest common divisor of the values of all edges incident on a vertex is called greatest common incidence number. The greatest common incidence number of each vertex of degree greater than or equal to 2 is one ,then the graph is called sum of the squares and product prime labelling. Here we investigate triangular snake graph, quadrilateral snake graph , pentagonal snake graph , comb triangular snake graph for the labelling .**

***Keywords*—sum of squares, incidence number, prime labelling, ssdpp graph.**

I. INTRODUCTION

Here we use the graphs obtained by replacing edges of a finite path by triangles, quadrilaterals and pentagons. Here we take basic notations and definitions from [1], [2],[3] and [4] We introduced the concept ssdpp labeling in [5]. Here we focus our study to triangular snake graph, quadrilateral snake graph , pentagonal snake graph , comb triangular snake graph.

Definition: 1.1 Let G be a graph with p vertices and q edges.The greatest common incidence number of a vertex of degree ≥ 2, is the greatest common divisor of the values of the edges incident on the vertex.

II. MAIN RESULTS

Definition 2.1 Let *G* be a graph with n vertices and m edges. Let f be a mapping from the vertex set to {0,1,2,---,n-1} defined by $f(a_j) = j-1$, $1 \le j \le n$, and $f^*_{sqsdppl}$ from set of edges of G into natural numbers by

$f^*_{sqsdppl}(ab) = |\{f(a)\}^2 + \{f(b)\}^2 - f(a)f(b)|$. $f^*_{sqsdppl}$ is said to admit ssdpp labeling, if (i) f is one-one and onto (ii) $f^*_{sqsdppl}$ is one-one and (iii) gcin of (a) = 1 for every vertex a of degree greater than one.

Definition 2.2 A graph which satisfy the above definition is called square sum difference product prime labeled graph.

Theorem 2.1 Let G be the graph obtained by replacing each edge of a path by triangles. G is a ssdpp graph if (n+1) is not a multiple of 7.

Proof: Let G be the graph and let $a_1,a_2,$----------------,a_{2n-1} are the vertices of G. G has 2n-1 vertices and 3n-3 edges. Let f be a mapping from the vertex set of G into {0,1,2,---,2n-2} defined by $f(a_j) = j-1$, $1 \le j \le 2n-1$, and $f^*_{sqsdppl}$ from set of edges of G into natural numbers by

$f^*_{sqsdppl}(a_{i+1}\, a_i) = 1-i+i^2$, i = 1,2,------,2n-2

$f^*_{sqsdppl}(a_{2i+1}\, a_{2i-1}) = 4-4i+4i^2$, i = 1,2,-------,n-1

gcin of (a_1) = 1

gcin of (a_{i+1}) = gcd of $\{f^*_{sqsdppl}(a_i\, a_{i+1})$, $f^*_{sqsdppl}(a_{i+1}\, a_{i+2})\}$

= gcd of $\{1-i+i^2, 1+i+i^2\}$

= gcd of $\{2i , 1-i+i^2\}$

= gcd of $\{i , 1-i(1-i)\}$

= 1, i = 1,2,------,2n-3

gcin of (a_{2n-1}) = gcd of $\{f^*_{sqsdppl}(a_{2n-2}\, a_{2n-1})$, $f^*_{sqsdppl}(a_{2n-3}\, a_{2n-1})\}$

= gcd of $\{4n^2-10n+7, 4n^2-12n+12\}$

= gcd of $\{2n-5, 4n^2-12n+12\}$

= gcd of $\{2n-5, 2n+2\}$

= gcd of $\{2n-5, n+1\}$

= gcd of $\{n-6, n+1\}$

= gcd of $\{n-6, 7\}$ = 1.

Here (i) f is one-one and onto (ii) $f^*_{sqsdppl}$ is one-one and (iii) gcin of (a) = 1 for every vertex a of degree greater than one.

Hence G is a ssdpp graph. ■

Theorem 2.2 Let G be the graph obtained by replacing each edge of a path by a quadrilateral. G is a ssdpp graph if (n+2) is not a multiple of 13.

Proof: Let G be the graph and let $a_1,a_2,$----------------,a_{3n-2} are the vertices of G. G has 3n-2 vertices and 4n-4 edges. Let f be a mapping from the vertex set of G into {0,1,2,---,3n-3} defined by $f(a_j) = j-1$, $1 \le j \le 2n-1$, and $f^*_{sqsdppl}$ from set of edges of G into natural numbers by

$f^*_{sqsdppl}(a_{i+1}\, a_i) = 1-i+i^2$, i = 1,2,------,3n-3

$f^*_{sqsdppl}(a_{3i-2}\, a_{3i+1}) = 9-9i+9i^2$, i = 1,2,-------,n-1

gcin of (a_1) = 1

gcin of (a_{i+1}) = 1, i = 1,2,------------,3n-4

gcin of (a_{3n-2}) = gcd of $\{f^*_{sqsdppl}(a_{3n-3}\, a_{3n-2})$, $f^*_{sqsdppl}(a_{3n-5}\, a_{3n-2})\}$

= gcd of $\{9n^2-21n+13, 9n^2-27n+27\}$

= gcd of $\{6n-14, 9n^2-27n+27\}$

= gcd of $\{3n-7, 9n^2-27n+27\}$

= gcd of $\{3n-7, 3n+6\}$

= gcd of $\{3n-7, n+2\}$

= gcd of $\{n-11, n+2\}$

= gcd of $\{n-11, 13\}$ = 1.

Here (i) f is one-one and onto (ii) $f^*_{sqsdppl}$ is one-one and (iii) gcin of (a) = 1 for every vertex a of degree greater than one. Hence G is a ssdpp graph. ■

Theorem 2.3 Let G be the graph obtained by replacing each edge of a path by a pentagon. G is a ssdpp graph if (n+3) is not a multiple of 7 and n is not a multiple of 3.

Proof: Let G be the graph and let $a_1,a_2,----,a_{4n-3}$ are the vertices of G. G has 4n-3 vertices and 5n-5 edges.Let f be a mapping from the vertex set of G into {0,1,2,---,4n-4} defined by $f(a_i) = j-1$, $1\le j \le 4n-3$, and $f^*_{sqsdppl}$ from set of edges of G into natural numbers by

$f^*_{sqsdppl}(a_{i+1}\, a_i) = 1-i+i^2$, i = 1,2,---,4n-4

$f^*_{sqsdppl}(a_{4i-3}\, a_{4i+1}) = (4i)^2+(4i-4)^2-4i(4i-4)$, i = 1,2,---,n-1

gcin of (a_1) = 1

gcin of (a_{i+1}) = 1, i = 1,2,----,4n-5

gcin of (a_{4n-3}) = gcd of $\{f^*_{sqsdppl}(a_{4n-4}\, a_{4n-3})$, $f^*_{sqsdppl}(a_{4n-3}\, a_{4n-7})\}$

= gcd of $\{(4n-4)^2+(4n-5)^2-(4n-4)(4n-5), (4n-4)^2+(4n-8)^2-(4n-4)(4n-8)\}$

= 1.

Here (i) f is one-one and onto (ii) $f^*_{sqsdppl}$ is one-one and (iii) gcin of (a) = 1 for every vertex a of degree greater than one. Hence G is a ssdpp graph. ■

Theorem 2.4 Let G be the graph obtained from a path of even number of vertices by replacing the edges by triangles alternately starting from the first vertex. G is a ssdpp graph if (n+2) is not a multiple of 14.

Proof: Let G be the graph and vertices of G are $a_1,a_2,---,a_{\frac{3n}{2}}$.

Graph has $\frac{3n}{2}$ vertices and 2n-1edges.

Let f be a mapping from the vertex set of G into {0,1,2,---, $\frac{3n-2}{2}$ } defined by $f(a_i) = j-1$, $1\le j \le \frac{3n}{2}$, and $f^*_{sqsdppl}$ from set of edges of G into natural numbers by

$f^*_{sqsdppl}(a_{i+1}\, a_i) = 1-i+i^2$, i = 1,2,---, $\frac{3n-2}{2}$

$f^*_{sqsdppl}(a_{3i-2}\, a_{3i}) = 7-12i+9i^2$, i = 1,2,---, $\frac{n}{2}$

gcin of (a_1) = 1

gcin of (a_{i+1}) = 1, i = 1,2,---, $\frac{3n-4}{2}$

gcin of $(a_{\frac{3n}{2}})$ = gcd of $\{f^*_{sqsdppl}(a_{\frac{3n}{2}}\, a_{\frac{3n-2}{2}})$, $f^*_{sqsdppl}(a_{\frac{3n}{2}}\, a_{\frac{3n-4}{2}})\}$

= gcd of $\{\frac{9n^2-18n+12}{4}, \frac{9n^2-24n+28}{4}\}$

= 1.

Here (i) f is one-one and onto (ii) $f^*_{sqsdppl}$ is one-one and (iii) gcin of (a) = 1 for every vertex a of degree greater than one. Hence G is a ssdpp graph. ■

Theorem 2.5 Let G be the graph obtained from a path of even number of vertices by replacing the edges by triangles alternately starting from the second vertex. G is a ssdpp graph.

Proof: Let G be the graph and vertices of G are $a_1,a_2,---,a_{\frac{3n-2}{2}}$.

Graph has $\frac{3n-2}{2}$ vertices and 2n-3edges.

Let f be a mapping from the vertex set of G into {0,1,2,---, $\frac{3n-4}{2}$ } defined by $f(a_i) = j-1$, $1\le j \le \frac{3n-2}{2}$, and $f^*_{sqsdppl}$ from set of edges of G into natural numbers by

$f^*_{sqsdppl}(a_{i+1}\, a_i) = 1-i+i^2$, i = 1,2,---, $\frac{3n-4}{2}$

$f^*_{sqsdppl}(a_{3i-2}\, a_{3i}) = 7-12i+9i^2$, i = 1,2,---, $\frac{n-2}{2}$

gcin of (a_1) = 1

gcin of (a_{i+1}) = 1, i = 1,2,---, $\frac{3n-6}{2}$

Here (i) f is one-one and onto (ii) $f^*_{sqsdppl}$ is one-one and (iii) gcin of (a) = 1 for every vertex a of degree greater than one. Hence G is a ssdpp graph. ■

Theorem 2.6 Let G be the graph obtained from a path of odd number of vertices by replacing the edges by triangles alternately starting from the first vertex. G is a ssdpp graph .

Proof: Let G be the graph and vertices of G are $a_1,a_2,---,a_{\frac{3n-1}{2}}$.

Graph has $\frac{3n-1}{2}$ vertices and 2n-2edges.

Let f be a mapping from the vertex set of G into {0,1,2,---, $\frac{3n-3}{2}$ } defined by $f(a_i) = j-1$, $1\le j \le \frac{3n-1}{2}$, and $f^*_{sqsdppl}$ from set of edges of G into natural numbers by

$f^*_{sqsdppl}(a_{i+1}\, a_i) = 1-i+i^2$, i = 1,2,---, $\frac{3n-3}{2}$

$f^*_{sqsdppl}(a_{3i-2}\, a_{3i}) = 7-12i+9i^2$, i = 1,2,---, $\frac{n-1}{2}$

gcin of (a_1) = 1

gcin of (a_{i+1}) = 1, i = 1,2,---, $\frac{3n-5}{2}$

Here (i) f is one-one and onto (ii) $f^*_{sqsdppl}$ is one-one and (iii) gcin of (a) = 1 for every vertex a of degree greater than one. Hence G is a ssdpp graph. ■

Theorem 2.7 Let G be the graph obtained from a path of odd number of vertices by replacing the edges by triangles alternately starting from the second vertex. G is a ssdpp graph, if (n+1) is not a multiple of 14.

Proof: Let G be the graph and vertices of G are $a_1,a_2,---,a_{\frac{3n-1}{2}}$.

Graph has $\frac{3n-1}{2}$ vertices and 2n-2edges.

Let f be a mapping from the vertex set of G into {0,1,2,---, $\frac{3n-3}{2}$ } defined by $f(a_i) = j-1$, $1\le j \le \frac{3n-1}{2}$, and $f^*_{sqsdppl}$ from set of edges of G into natural numbers by

$f^*_{sqsdppl}(a_{i+1}\, a_i) = 1-i+i^2$, i = 1,2,---, $\frac{3n-3}{2}$

$f^*_{sqsdppl}(a_{3i-2}\, a_{3i}) = 7-12i+9i^2$, i = 1,2,---, $\frac{n-1}{2}$

gcin of (a_1) = 1

gcin of (a_{i+1}) = 1, i = 1,2,---, $\frac{3n-5}{2}$

gcin of $(a_{(\frac{3n-3}{2})})$ = gcd of $\{f^*_{sqsdppl}(a_{\frac{3n-3}{2}}\, a_{\frac{3n-5}{2}})$, $f^*_{sqsdppl}(a_{\frac{3n-3}{2}}\, a_{\frac{3n-7}{2}})\}$

= gcd of $\{\frac{39-36n+9n^2}{4}, \frac{61-42n+9n^2}{4}\}$

= 1.

Here (i) f is one-one and onto (ii) $f^*_{sqsdppl}$ is one-one and (iii) gcin of (a) = 1 for every vertex a of degree greater than one. Hence G is a ssdpp graph. ■

Theorem 2.8 Let G be the graph obtained from a comb graph by replacing each path edge by triangles. G is a ssdpp graph .

Proof: Let G be the graph and let a_1,a_2,----------------,a_{3n-1} are the vertices of G. G has 3n-1 vertices and 4n-3 edges. Let f be a mapping from the vertex set of G into {0,1,2,---,3n-2} defined by $f(a_i) = j-1$, $1\leq j \leq 3n-1$, and $f^*_{sqsdppl}$ from set of edges of G into natural numbers by

$f^*_{sqsdppl}(a_{i+1}\ a_i)$ = $1-i+i^2$, i = 1,2,------,2n-1

$f^*_{sqsdppl}(a_{2i+1}\ a_{2i-1})$ = $4-4i+4i^2$, i = 1,2,-------,n-1

$f^*_{sqsdppl}(a_{2i-1}\ a_{3n-i})$ = $(3n-3i+1)^2+(3n-i-1)(2i-2)$, $1\leq i \leq n-1$

gcin of (a_1) = 1

gcin of (a_{i+1}) = 1, i = 1,2,------------,2n-2

Here (i) f is one-one and onto (ii) $f^*_{sqsdppl}$ is one-one and (iii) gcin of (a) = 1 for every vertex a of degree greater than one.

Hence G is a ssdpp graph. ■

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