Fuzzy Divisor Cordial Graph

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Abstract: **In this paper we introduce a new concept called fuzzy divisor cordial labeling. It is a conversion of crisp graph into fuzzy graph under the new condition namely fuzzy divisor cordial labeling. In divisor cordial labeling it is not possible to label all the crisp graphs due to the condition of its definition. Suppose if we consider a graph of size 5, it will be possible to label all the vertices as in the combination of vertex set {1,2,3,4,5}. So for n vertices, we need to label all the vertices as a combination of all the vertices without repetition, without neglecting any vertex among them. Here discussion about the edge labeling is trivial. So it is clear that all the crisp graphs can't be divisor cordial graphs. However in fuzzy divisor cordial graph for any vertices we can label any fuzzy membership value from [0,1]. Since the interval consists of infinite number of terms, there is infinite number of chances for labeling a vertex in fuzzy divisor cordial labeling.**

INTRODUCTION:

The existence of graphs for which a special set of integer values are assigned to its nodes or edges or both according to some given criteria has been investigated since the middle of the last century. Graph labeling have often been motivated by practical considerations such as chemical isomers, but they are also of interest in their own right due to their abstract mathematical properties arising from various structural considerations of the underlying graphs. The qualitative labelings of graph elements have inspired research in diverse field of human enquiry such as conflict resolution in social psychology, electrical circuit theory and energy crisis. Quantitative labelings of the graphs have led to quite intricate fields of applications such as coding theory problems, including the design of good radar location codes, synch-set codes, missile guidance codes and convolution codes with optimal auto correlation properties. Labeled graph often been applied in determining the ambiguities in X-ray crystallographic analysis, to design communication networks, in determining optimal circuit layouts and radio astronomy etc.

Interest in graph labeling problems began in the mid 1960's. Most graph labeling methods trace their origin to one introduced by Rosa in 1967, or one given by Graham and Sloane in 1980. The concept of cordial labeling was introduced by Cahit. The definition of fuzzy graph was first introduced by Kaufmann in the year 1973, based on L.A.Zadeh's fuzzy relations, introduced in the year 1971. Then a mathematician Azriel Rosenfeld developed the theory of fuzzy graph who considered fuzzy relations on fuzzy sets and in 1975. R.T.Yeh and S.Y.Bang have also introduced various connectedness concepts in fuzzy graph in the same year. Yeh and Bang's approach for the study of fuzzy graphs were motivated by its applicability to pattern classification and clustering analysis. They worked more with the fuzzy matrix of a fuzzy graph, introduced concepts like vertex connectivity Ω(G) , edge connectivity ʎ(G) and established the fuzzy analogue of Whiteney's theorem. They also proved that for any three real numbers a, b, c such that $0 < a \leq b \leq c$, there exists a fuzzy graph G with Ω(G) = a, ʎ(G) = b and δ(G) = c.

Fuzzy graphs have been witnessing a tremendous growth and finding applications in many branches of engineering and technology so far. Rosenfeld has obtained several concepts like bridges, paths, cycles, and trees and established some of their properties. Fuzzy end nodes and cut nodes were studied by K.R.Bhutani. Bhattacharya has established some connectivity concepts regarding fuzzy cut nodes and fuzzy bridges titled "Some remarks on fuzzy graphs".

In this paper we have introduced a new concept namely fuzzy divisor cordial graph as discussed above. It's just an application of labeling fuzzy numbers for the graphs under some new conditions. Fuzzy graph is the generalization of the crisp graph. So it is necessary to know some basic definitions and concepts of crisp graph based on divisor cordial graphs.

Def :1.1

The assignment of values subject to certain conditions to the vertices of a graph is known as graph labeling.

Def : 1.2

Let G = (V,E) be a graph. A mapping $f : V(G) \rightarrow \{0,1\}$ is called binary vertex labeling of G and f(v) is called the label of the vertex v of G under f.

For an edge e = uv, the induced edge labeling $f^* : E(G) \rightarrow \{0,1\}$ is given by $f^*(e) = |f(u)-f(v)| \leq 1$. Let $v_f(0)$ and $v_f(1)$ be the number of vertices of graph G having labels 0 and 1respectively under f and let $e_f(0)$ and $e_f(1)$ be the number of edges having labels 0 and 1 respectively under f*.

Def : 1.3

A binary vertex labeling of a graph G is called a cordial labeling if $|v_f(0)-v_f(1)| \leq 1$and $|e_f(0)-e_f(1)| \leq 1$. A graph G is cordial if it admits cordial labeling.

Def : 1.4

Let G = (V,E) be a simple graph and $f : V \rightarrow \{1,2,\ldots|V|\}$ be a bijection. For each edge uv, assign the label 1 if either f(u) divides f(v) or f(v) divides f(u) otherwise label 0. Then f is called a divisor cordial labeling of a graph G if $|e_f(0)-e_f(d)| \leq 1$.

Def : 1.5
A fuzzy graph G = (σ,μ) is a pair of functions σ : V → [0,1] and μ : V × V →[0,1], where for all u,v ∈ V, we have μ(u,v) ≤ σ(u) Λ σ(v).

Def : 1.6
A labeling of a graph is an assignment of values to the vertices and edges of a graph.

MAIN RESULTS:

Def : 1.7
A graph G = (σ,μ) is said to be a fuzzy labeling graph if σ : V → [0,1] and μ : V × V →[0,1] is bijective such that the membership value of edges and vertices are distinct and μ(u,v) ≤ σ(u) Λ σ(v) for all u, v ∈ V.

Def : 1.8
Let G = (σ,μ) be a simple graph and σ : V → [0,1] be a simple bijection. For each edge uv, assign the label d if either σ(u) | σ(v) or σ(v) | σ(v) and the label 0 if σ(u) † σ(v). σ is called a fuzzy divisor cordial labeling if $|e_\mu(0)-e_\mu(d)| \leq 1$, where d is a very small positive quantity, which is closure to 0 and d ∈ (0,1).
In other words a graph with a fuzzy divisor cordial labeling is called a fuzzy divisor cordial labeling graph.

Def : 1.9
The graph $p_{n-1}(1,2,3,\ldots n)$ is a graph obtained from a path of vertices $v_1,v_2,\ldots,v_n$ having path length n-1 by joining i pendent vertices of each of i^{th} vertex. The pendent vertices are labeled as $u_{i,1}$; $u_{i,2}$;…; $u_{i,n}$ for $1 \leq i \leq n$.

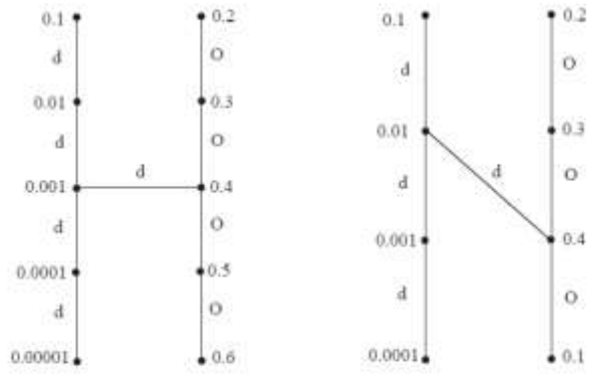
Def : 1.10
The graph $p_{n-1}(k,k,\ldots k)$ is a graph obtained from a path vertices $u_1,u_2,\ldots u_n$ having path length n-1 by joining k pendent vertex at each path vertex v_i.

Theorem : 1.11
The H-graph of path of length n is a fuzzy divisor cordial graph.

Proof :
Let G be a H-graph of path of length G. It contains 2n vertices and 2n-1 edges. Because it consists of two copies of path length n.
Let P_1 and P_2 be the first and second path respectively.
Let $v_0,v_1,v_2,\ldots v_n$ be the vertices of path P_1 and let $u_0,u_1,u_2,\ldots,u_n$ be the vertices of path P_2.
Fix v_0 = 1/10 and label $1/10^i$ for $v_1,v_2,\ldots v_n$ for all i ∈ N-{1}, where N is a set of all natural numbers.
Therefore every neighbours of vertex v_i can be divided by v_i or any vertex v_i can be divided by it's neighbours.
In path P_2, label all the vertices that no vertex can be divided one another.
Therefore $|e_\mu(0)-e_\mu(d)| \leq 1$ holds.

Note :
The path length of P_1 and P_2 are odd or even, it will not affect the result of *theorem 1*.24.

Illustration : 1.12
Consider the H-graph of size n = 8 and n = 10.



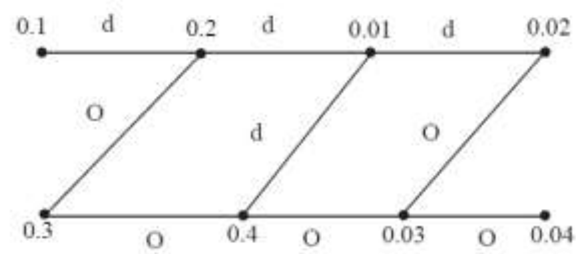
In H-graph of size 8, we have
$e_\mu(0)$ =3 , $e_\mu(d)$ = 4
$|e_\mu(0) - e_\mu(d)| = |3 - 4| = |-1| \leq 1$
Hence $|e_\mu(0) - e_\mu(d)| \leq 1$.

In H-graph of size 10, we have
$e_\mu(0)$ =4 , $e_\mu(d)$ = 5
$|e_\mu(0) - e_\mu(d)| = |4 - 5| = |-1| \leq 1$
Hence $|e_\mu(0) - e_\mu(d)| \leq 1$.

Theorem : 1.13
The graph Z-P_n is a fuzzy divisor cordial graph.

Proof :
Let G be a Z-P_n graph.
Let the vertex set of G be $\{u_i,v_i : 1 \leq i \leq n\}$.
Let the edge set of G be $\{[(u_i\, u_{i+1})$ □ $(v_i\, v_{i+1})$ □ $(v_i\, v_{i+1}) : 1 \leq i \leq n-1]\}$
Fix u_1 = 0.1 and u_2 = 0.2.
Also fix v_1 = 0.3 and v_2 = 0.4.
Label $i/10^i$ for the vertices $u_3,u_5,u_7,\ldots,u_{2k-1}$ for all i ∈ N-{1} and k ∈ N-{1}, where N is a set of all natural numbers.
Label $2/10^i$ for the vertices $u_4,u_6,u_8,\ldots u_{2k}$, where i ∈ N-{1} and k ∈ N-{1}, where N is a set of all natural numbers.
Therefore every vertex v_i cant be divided by one another.
Also every vertex u_{2k-1} divides the vertex v_{2j} when k ≥ 2 and j ≥1.
Therefore $|e_\mu(0)-e_\mu(d)| \leq 1$ holds.

Illustration : 1.14
Consider the graph Z-P_4.

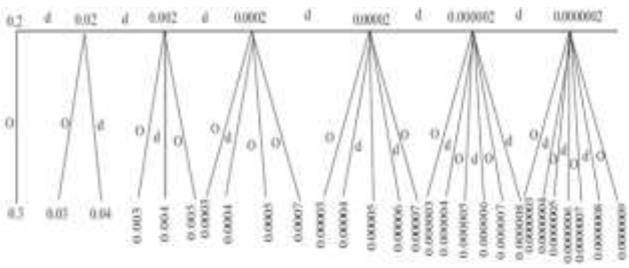


Here $e_\mu(0)$ =5 , $e_\mu(d)$ = 4
$|e_\mu(0) - e_\mu(d)| = |5 - 4| = |1| \leq 1$
Hence $|e_\mu(0) - e_\mu(d)| \leq 1$.
Therefore every graph Z-P_n is a fuzzy divisor cordial graph.

Note :
Note that the vertices u_i and v_i should have same powers of 10 in the denominator when $i \in N$

Theorem : 1.15
The graph $p_{n-1}(1,2,3\ldots,n)$ is a fuzzy divisor cordial graph for $n \geq 1$.
Proof :
Let $u_1,u_2,u_3,\ldots$un be the vertices of path p_{n-1}, where u_1 is a first vertex of the path p_{n-1}.
Therefore u_1 will have 1 pendent vertex say v_1 which is connected with u_1.
Let v_i be the pendent vertex, connected with some vertex of path
p_{n-1}.
Label all the vertices of path p_{n-1} by $2/10^i$, $i \in N$.
Each vertex u_i in path p_{n-1} has v_i pendent vertices.
For u_1 there is one pendent vertex say v_1.
Fix $\sigma(v_1) = 3/10$ and $\sigma(u_1) = 2/10$.
Note that power of 10 should be equal for both pendent vertices and path vertex, which is connected to the pendent vertices.
For u_2, fix $\sigma(u_2) = 2/10^2$, label the pendent vertices as $3/10^2$ and $4/10^2$.
For u_3, fix $\sigma(u_3) = 2/10^3$, label the pendent vertices as $3/10^3,4/10^3,5/10^3$.
But for u_4, $\sigma(u_4) = 2/10^4$, there must be labeled $\frac{n}{2}$ - 1 pendent vertices, which can be divided by $2/10^4$, others must not be divisible by $2/10^4$.
In general for u_{10+4n} for all $n \in W$, where W is a set of all whole number we have to label in the same way.
Note that u_4 and u_{10+4n} are containing even number of pendent vertices.
For the remaining path vertices and pendent vertices which is connected to the path vertices we have to label in the normal way, which we approached for u_2 and u_3.
Therefore $|e_\mu(0)-e_\mu(d)| \leq 1$ holds.

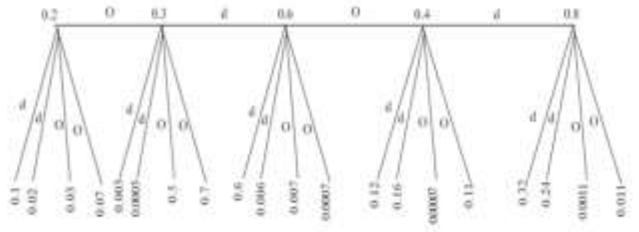
Illustration : 1.16
Consider the graph $P_7(1,2,3,4)$.



Here $e_\mu(0)$ =17 , $e_\mu(d)$ = 17
$$|e_\mu(0) - e_\mu(d)| = |17 - 17| = |0| \leq 1$$
Hence $|e_\mu(0) - e_\mu(d)| \leq 1$.
Therefore every graph $p_{n-1}(1,2,3\ldots,n)$ is a fuzzy divisor cordial graph for $n \geq 1$.

Theorem : 1.17
$P_{n-1}(m,m,m,\ldots m)$ is a fuzzy divisor cordial graph, $m,n \geq 1$.
Proof :
Let $u_1,u_2,u_3,\ldots,u_n$ be the path vertices, while labeling membership value of every path vertex u_i must divide or must be divided by at most one of its neighbours say u_{i-1} and u_{i+1}. But it is not necessary for u_1 and u_n. Because u_1 and un contain exactly one neighbour..
Let u_1 connected with m-pendent vertices. Label u_1 by any fuzzy membership value except $1/10^i$, $i \in W$.
Label m/2 pendent vertices which can be divided by u_1 if m is even.
Label $\frac{m+1}{2}$ or $\frac{m-1}{2}$ pendent vertices which can be divided by u_1 if m is odd. Remaining vertices must be labeled conversely.
Note that u_2 is a only neighbour of u_1.
Then u_2 must divide u_1 if u_2 does not divide u_3.
Also if u_2 divides u_3, then u_2 must not divide u_1.
Therefore in general u_i must divide u_{i+1}or u_{i-1}, but not both.
If the path length may odd or even, that does not affect the result but the number of pendent vertices may affect.
Suppose the number of pendent vertices is odd and if u_1 divides $\frac{m-1}{2}$ pendent vertices, then u_2 must divide $\frac{m+1}{2}$ pendent vertices, then again u_3 must divide $\frac{m-1}{2}$ pendent vertices. The remaining vertices must be labeled by any fuzzy membership function which can't be divided by the path vertex. This should be proceeded upto n number of path vertex.
Therefore $|e_\mu(0)-e_\mu(d)| \leq 1$.
Suppose the number of pendent vertices is even, then label m/2 pendent vertices which can be divided by the path vertex u_i. Then label the remaining m/2 pendent vertices must not be divided by u_i. The result holds if we proceed this upto n number of path vertex.
Therefore $|e_\mu(0)-e_\mu(d)| \leq 1$ holds.

Illustration : 1.18
Consider the graph p5(4,4,…,4).



Here $e_\sigma(0)$ =12 , $e_\sigma(d)$ = 12
$$|e_\mu(0) - e_\mu(d)| = |12 - 12| = |0| \leq 1$$
Hence $|e_\mu(0) - e_\mu(d)| \leq 1$.
Therefore every graph $P_{n-1}(m,m,m,\ldots m)$ is a fuzzy divisor cordial graph, $m,n \geq 1$

CONCLUSION:
In this paper, we have discussed some mathematical inner beauty of fuzzy graphs. If we go in to the deep of cordial labeling of crisp graph, we can realize that for every crisp graph , we can't apply cordial labeling. However it is possible in fuzzy graphs. Especially in the view of fuzzy divisor cordial graph which plays a main role in this paper. Because |V| is finite also every vertex must be labeled in crisp graph. However in fuzzy graph, there are infinite number of chances to label the vertex from 0 to 1.

So clearly we can conclude that every fuzzy graphs and crisp graphs can be represented in the form of fuzzy divisor cordial graph. This paper is the strong gateway to prove the above statement.

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