Quasi Noise Removal using Frequency Dependent Notch Filter

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Abstract: Generally, the nature of noise is random and can be removed using both static and adaptive filters. Quasi-periodic noise may leave some effect on images. This artefact is often caused by electrical interferences during image acquisition or transmission, which makes remote sensing applications especially prone to the phenomenon. Periodic noise gives a bit of sharp spikes in the spectrum of image, which could be removed by means of notch filters. The problem is to automate detection of spike, that is, design of notch filter. Some works suggest, detecting spikes in the Fourier domain as great deviations in terms of a localized median value. However, distinguishing between spikes because of a repetitive structure or a localized texture (general in man-made conditions) and spurious ones caused by periodic noise is still challenging. It has been observed in past that periodic noise is likely to be the only periodic structure present in any patch extracted from the impaired image. The quasi periodic noise changes in structure over the time, thus removal becomes more complex. In this paper, a notch filter design is presented for quasi noise removal.

Keywords: Quasi Periodic Noise, Notch Filter, Noise Removal.

1. INTRODUCTION

In the past few years, the use of internet has increased tremendously, as it now has become the major mode of communication. In internet, the transfer of data files, images and video is very common. The use of images is not limited to entertainment only but also used in steganography, watermarking, medical imaging and military applications etc [1-2]. Image transferring over the internet suffers from degradation like blurring and noise. Some noise can be easily removed using linear and non-linear filters. While for some noise adaptive filters is required. Periodic and quasi-periodic noises are such noises which require adaptive filters for the removal of noise. Periodic noises has repetitive structure thus filter design is not much complex. However, quasi-periodic noise seems to be periodic but they change structure slowly, thus a careful design of filter is needed to remove such noises [3].

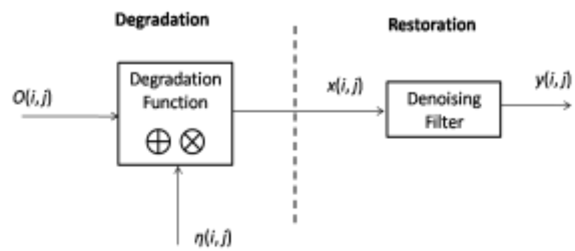


Figure 1: Image degradation and restoration process

In figure 1 image degradation and restoration process is shown. $O_{i,j}$ is an input object $\eta_{i,j}$ is degrading term (may include noise, blurring or both) so $x_{i,j}$ is

$$x_{i,j} = O_{i,j} + \eta_{i,j} \quad (1)$$

or

$$x_{i,j} = O_{i,j} \times \eta_{i,j} \quad (2)$$

$$y_{i,j} = H\left[x_{i,j}\right] \quad (3)$$

H is filter operator.

Let us consider that the degradation is represented by an operator A. Then the equation 1 can be written as $x_{i,j} = A\left[O_{i,j}\right]$, now simply using basic concept of matrix, equation can be converted into

$$A^{-1}\left[x_{i,j}\right] = A^{-1}A\left[O_{i,j}\right] = \left[\hat{O}_{i,j}\right] \quad (4)$$

where, cap denotes the expected value. In most of the image processing problems A^{-1} is singular. Therefore, estimation of $\hat{O}_{i,j}$ is not possible through inverse process. Therefore filtering based techniques are proposed to reduce noises or degradation (cannot eliminate).

Although, in the image processing, it has been proved that the linear techniques are not effective as they are not able to cope with the nonlinearities of the image formation model and don't make into note of visual system of humans. These techniques, hence, frequently create blurred images and are not sensitive to impulse noise. Image signals have the composition of flat regional parts and unexpectedly changing areas like edges, which convey essential information for visual perception. Therefore, in the course of the last 15 years, nonlinear methods have been observed to be more impressive for this task. Nonlinear methods can suppress non-Gaussian and signal dependent noise in order to preserve crucial signal elements, for example, edges and fine details and discard degradations happening at the time of signal formation or transmission through nonlinear channels [4]. Filters having decent characteristics of edge and image detail preservation are very much appropriate for image filtering and improvement. New techniques and algorithms, which can make use of the rise in computing power and can deal with more realistic suppositions, are required. Therefore, the advancement of nonlinear filtering procedures, which works equally fine under wide variety of applications, is of very high significance [5,6].

Moreover, transform domain techniques are capable of removing some part of the noises. That is why in image enhancement DFT or FFT is used.

2 RELATED WORKS

In [7] an efficient methodology to remove periodic noise from digital images is proposed. Color image needs to be transformed into gray image, and after this 2D fast Fourier transform (2DFFT) is to be implemented on the gray image. The magnitude of applying 2DFFT is to be examined for the purpose of getting the periodic filter, which is to be correlated with the magnitude matrix, and correlation result is to be applied with the angle matrix to get the de-noised gray image. However, the quality of the recovered image is not good.

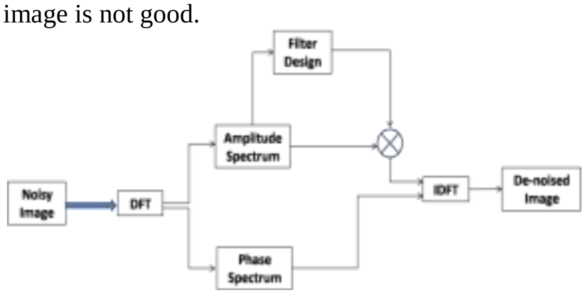


Figure 2: Methodology for periodic noise removal process as in [7]

In this paper [8] a 2-D FFT removal algorithm for reducing the periodic noise in natural and strain images is proposed. For the periodic pattern of the artifacts, we apply the 2-D FFT on the strain and natural images for peaks' extraction and removal which are corresponding with periodic noise in the frequency domain. Moreover, after this we apply the mean filter for much improved output. The performance of the proposed method is tested on both natural and strain images. In this method the PSNR of the recovered image is not very good.

In [9], a new contourlet domain image denoising framework based on SURE shrink and bilateral filter has been presented. The new algorithm combined the good properties of the non-subsampled Contourlet transform, SURE shrink and bilateral filter in the image denoising issue.

A latest algorithm is proposed for restoration of image with the help of partial differential equations applied on neighbors longer than one pixel [10]. To better preserve edges in this technique, slant edges are considered in addition to vertical and horizontal edges. A great number of experiments have been carried out to make the evaluation of performance of the proposed method and for the comparison of its performance with the current algorithms, wiener and median filters.

Quasi-periodic noise may leave some effect on images. This feature which is not desirable manifests itself by means of spurious repetitive patterns that cover the total image, properly localized in the Fourier domain [11].

Even notch filtering gives us facility to overcome this characteristic, this however needs initial detection of the resulting Fourier spikes, and, specially, to make the discrimination between spectrum patterns and noise spikes took place because of repetitive structures or spatially localized textures. In this assignment, we propose an approach, in the Fourier domain, of statistical a contrario detection of noise spikes.

3. PROBLEM FORMULATION

This section discusses the problems mathematical formulation and system modeling and how spectral density can be used in noise removal process.

A simple case quasi-periodic function obeys the equations of the form:

$f(t+T) = f(t) + C$ and $f(t+T) = cf(t)$. (5)

A useful example is the function:

$$f(t) = \sin(At) + \sin(Bt) \qquad (6)$$

If the ratio A/B is rational, this will have a true period, but if A/B is irrational there is no true period, but a succession of increasingly accurate "almost" periods.

Periodic and Quasi-periodic Signals

Certain signals repeat itself after a definite time interval, for example heart rate, AC supply frequency etc..When a signal $x(t)$ is periodic with period T, then it satisfies the relation

$$x(t) = x(t+T) \qquad (7)$$

Or

$$|x(t) - x(t+T)| = 0 \text{ for all } t. \qquad (8)$$

As we know from the fundamentals, that a periodic signal can be represented in terms of Fourier series and representation using cosine expansion would be

$$x(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}[r_n \cos(2\pi n f_0 t + \varphi_n)] \quad (9)$$

where $f_0 = \frac{1}{T}$ is the fundamental frequency and the Fourier coefficients are as follows

$$a_n = r_n \cos(\varphi_n) = \frac{2}{T}\int_0^T x(t)\cos(2\pi n f_0 t)dt$$

$$b_n = r_n \sin(\varphi_n) = -\frac{2}{T}\int_0^T x(t)\sin(2\pi n f_0 t)dt$$

The fundamental_frequency, f_0 and Fourier co-efficient a_n, b_n, r_n, or φ_n, are constants. It means that they are not functions of time. As far as the harmonic frequencies are concerned, they are exact integer multiples of the fundamental frequency.

When $x(t)$ is quasi-periodic then

$$x(t) \approx x(t + T(t)) \quad (10)$$

Or

$$\left|x(t) - x(t + T(t))\right| < \varepsilon \quad (11)$$

Where

$$0 < \varepsilon < \left\|x\right\| = \sqrt{\overline{x}^2} = \sqrt{\lim_{\tau\to\infty}\frac{1}{\tau}\int_{-\tau/2}^{\tau/2} x^2(t)dt}$$

Now the Fourier series representation would be

$$x(t) = \frac{1}{2}a_0(t) + \sum_{n=1}^{\infty}[r_n(t)\cos(2\pi n\int_0^t f_0(\tau)d\tau + \varphi_n(t))] \quad (12)$$

where $f_0(t) = \frac{1}{T(t)}$ is the possibly *time-varying* fundamental frequency and the Fourier coefficients are

$$a_n(t) = r_n(t)\cos(\varphi_n(t))$$

$$b_n(t) = r_n(t)\sin(\varphi_n(t))$$

and the instantaneous frequency is

$$f_n(t) = n f_0(t) + \frac{1}{2\pi}\frac{d}{dt}\varphi_n(t) \quad (13)$$

On the other side, in this quasiperiodic case, the fundamental frequency $f_0(t)$, the harmonic frequencies $f_n(t)$, and the Fourier coefficients $a_n(t), b_n(t)$, $r_n(t)$, or $\varphi_n(t)$, are not necessarily constant in the mandatory basis, and are time functions although *slowly varying* functions of time. Explained in a different way, these functions of time are band limited to quite less as compared to that of the fundamental frequency for $x(t)$ to be assumed to be quasi-periodic.

The partial frequencies $f_n(t)$ are quite closely harmonic but it is not in the mandatory basis. The time-derivative of $\varphi_n(t)$, that is $\frac{d}{dt}\varphi_n(t)$, has the influence of detuning the partials from their exact integer harmonic value $nf_0(t)$. A rapid pace variation $\varphi_n(t)$ implies that at the instant frequency for that partial is intensely detuned from the integer harmonic value which would indicate that $x(t)$ is not quasi-periodic.

With respect to time quasi-periodic signal vary slowly. Therefore equation can be simplified as

$$f_{i+1} = f_i + k\nabla f \quad (14)$$

System Modeling

Consider a system $G(z)$ where $u(n)$ is a deterministic input signal, $w(n)$ is a deterministic output and $v(n)$ a stochastic signal.

$$y(n) = w(n) + v(n) \quad (15)$$

If v(n) is a zero-mean stationary stochastic signal then the function,

$$R_v(k,n) = E[v(k)v(n)] \quad (16)$$

is only a function of the difference between k and n. In this case we can write this as,

$$R_v(\tau) = E[v(n)v(n-\tau)] \quad (17)$$

However, the stochastic stationarity does not hold when the signal is the sum of a zero-mean stochastic and deterministic signal. This corresponds to the typical case in an identification experiment: the output, $y(k)$, is the sum of a deterministic plant output, $G(j\omega_i)u(k)$ plus a zero-mean noise signal, $v(k)$. Therefore we define a new class of signals: quasi-stationary signals.

The covariance function of a signal $x(k)$, (defined as a function of two time indices) is,

$$R_x(k,n) = E[x(k)x(n)] \quad (18)$$

If this is bounded for all k and n and in the limit we have,

$$\lim_{n\to\infty}\frac{1}{N}\sum_{k=1}^{N} R_x(k,k-\tau) = R_x(\tau) \quad (19)$$

then the signal $x(t)$ is quasi-stationary. Note that for quasi-stationary signals we only require the covariance function to depend solely on τ in the limit.

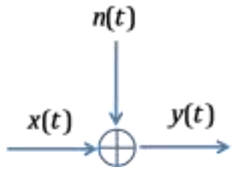


Figure 3: Additive noise process

Considering input image as $x(t)$, noise as $n(t)$ and noise corrupted image reached as the receiver is $y(t)$.

$$y(t) = x(t) + n(t) \quad (20)$$

This will passes through the filter can be represented as [29,31]

$$y_R(t) = [x(t) + n(t)] \otimes h(t) \quad (21)$$

$$y_R(t) = x(t) \otimes h(t) + n(t) \otimes h(t) \quad (22)$$

In above expression first term is filtered image and second term denotes the filtered noise. Thus both signal and noise superimpose each other.

As filters are designed in frequency domain, therefore the time delayed version of the received signal is given by

$$y(t+\tau) = x(t+\tau) + n(t+\tau) \quad (23)$$

The autocorrelation function can be written as

$$R_{YY}(\tau) = E\left[\left(x(t+\tau) + n(t+\tau)\right)\left(x(t) + n(t)\right)\right]$$

Assuming image and noise independent of each other, and noise with mean zero, then above expression can be simplified as

$$R_{YY}(\tau) = R_{XX}(\tau) + R_{nn}(\tau) \quad (24)$$

Taking Fourier transform of above expression we get, spectral density as

$$S_{YY}(\omega) = S_{XX}(\omega) + S_{nn}(\omega) \quad (25)$$

Let, $F[h(t)] = H(\omega)$

$$S_{YY}^{R}(\omega) = |H(\omega)|^2 S_{YY}(\omega)$$

$$S_{YY}^{R}(\omega) = |H(\omega)|^2 S_{XX}(\omega) + |H(\omega)|^2 S_{nn}(\omega) \quad (26)$$

As most of the images concentrated in low frequency regime, therefore in our method first we divide the image into and high frequency components by choosing suitable cut-off frequency. Then noise removal process is applied on the low frequency components and noise at high frequency components automatically removes. Thus noise reduces and image quality improves.

The modified algorithm is as under:

A. Algorithm

Input: an image '*i*'(size X×Y) and its Fourier transform '*I*', patch size '*H*'

Step 1:

First divide the image in low and high frequency components.

Step 2:

Extract non-overlapping independent patches of size H×H over the low frequency components image and independent patches of size H×H over the low frequency components image obtain *P* patches. Then evaluate F_R using step 5.

Step 3:

Obtain the power spectra of patches for any (*n*, *m*) and obtain minimum value.

Step 4:

For any (*n*, *m*), obtain NFA value.

Step 5:

Consider the spike map M_o^P on H×H and H×H spectrum such that $M_o^P(n,m) = 1$ if NFA $\left(|c_{n,m}|\right) \le 1$ and 0 otherwise

Step 6:

Interpolate the outlier map M_o^P of size H×H and H×H to X×Y providing map of M_o of the probable false spikes in the original image spectrum. Multiplying, the initial image spectrum by, $1 - M_o$ acts as notch filter and thus eliminating quasi periodic noise.

Step 6:

Retrieve $\hat{n}$, estimation of the periodic noise components as the inverse Fourier transform (IFT) of $M_o I$ and $\hat{i}$ estimation of de-noised image as, $i - \hat{n}$.

The main advantages of the modifications are:

1. The adjustment of patch size in low frequency regime reduces the error thus improves the SNR.
2. In high frequency regime, generally image components are absent, thus noise can be easily eliminated.

4. RESULTS AND DISCUSSIONS

The 2D Fourier transform of an image $i(x, y)$ is denoted by $I(u,v)$ and given by

$$I(u,v) = \sum_{x=0}^{X-1} \sum_{y=0}^{Y-1} i(x, y) e^{-2\pi j(xu/X + yv/Y)} \quad (26)$$

Where, u and v are integers values, with $(u,v) \in [-X/2, X/2-1] \times [-Y/2, Y/2-1]$, $|I(u,v)|$ is the amplitude of a periodic component of frequencies u/X and v/Y (cycles per second) along each direction. The normalized frequency given as $f=\sqrt{(u/X)^2+(v/Y)^2}$.

First, since most images have discontinuities between their left/right (respectively top/bottom) borders, their spectrum shows dominant straight lines along the horizontal axis (respectively vertical axis). To reduce these boundary effects, we multiply the patches p by a two-dimensional Hann window with the same width L as the patches.[12]

The size of the patches should be large enough to ensure both a good accuracy in the periodic noise spike detection (frequencies are distributed with $1/H$ steps in the power spectrum of a patch, and the detectability of low-frequency noise, but not too large, so as to make it possible to build enough independent patches from the noisy image of interest. Using $H \times H$ patches, it was found that a good compromise is to take a sampling step of $H/8$ in both the horizontal and vertical directions, which gives a total number of patches equal to [13]

$$[8(X-H)/H][8(Y-H)/H] \quad (27)$$

Synthetic periodic noise is added to an 8-bit noise free image i_0 of size X × Y. The periodic noise intensity is given by

$$n(x,y)=A\sin\left(\frac{2\pi p}{X}x\right)\sin\left(\frac{2\pi q}{Y}y\right) \quad (28)$$

Where p and q are the parameters defining the frequency of the periodic noise along the x and y axes, respectively. The unit of n is the gray level.

$\sqrt{(p/X)^2+(q/Y)^2}<f_2$, frequencies are considered as very low components, hence are not considered as periodic noise components and thus are not eliminated by the algorithms, in the experiment $f_2=\frac{8}{H}$.

Bilinear interpolation is used to expand outlier map. The notch filter is obtained after convolving the outlier map M_0 (after interpolation) by an isotropic Gaussian kernel of standard deviation 2 pixels, reduces ringing effects at higher frequency.

The experimental results are obtained on Baboon image and original image is shown in figure 4. The frequency domain picture is shown in Figure 5. Figure shows 2D spectrum, power and phase spectrum of the image. To view the frequency variation across the image, image is de-composed into low-mid and high frequencies zones. It is clear from the sub-figures that most part of the image concentrated in low frequency regime.

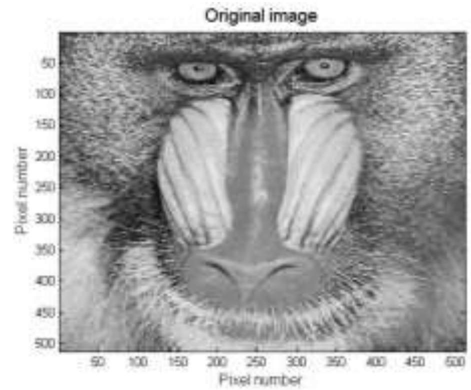


Figure 4 Image of Baboon and pixel representation

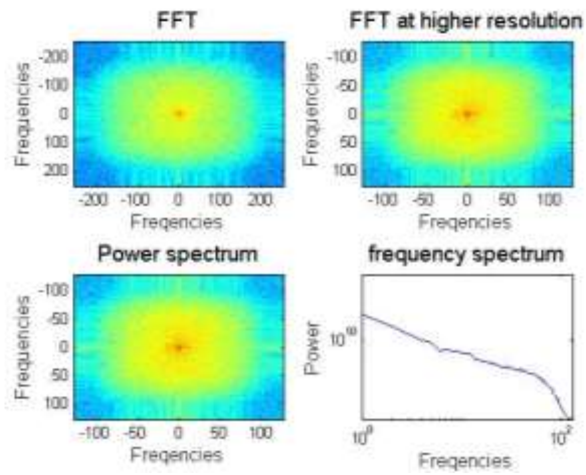


Figure 5: Baboon image FFT and associated parametres

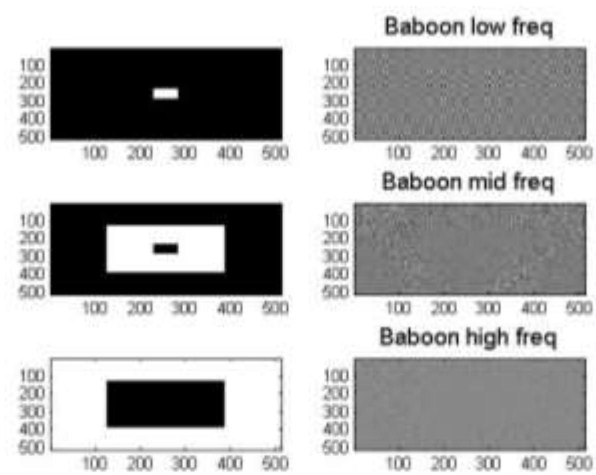
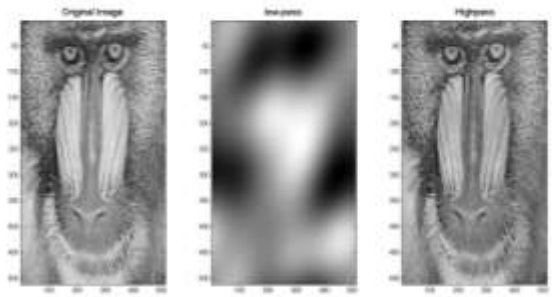


Figure 6: Frequency variation

$$f_L+f_H=\frac{\min[i(m,n)]}{2} \quad (29)$$

In the above expression, f_L is low pass frequency and f_H is high pass frequency. $[i(m,n)]$ is the size of image. Most of the practical images are low frequency image. However, the cut-off frequency varies with images.



(a) (b) (c)

Figure 7: (a) Original image (b) low pass image 0-10 Hz (c) high pass image 10-256 Hz

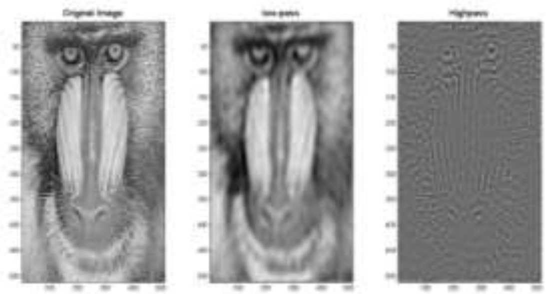


Figure 8: (a) Original image (b) low pass image 0-50 Hz (c) high pass image 50-256 Hz

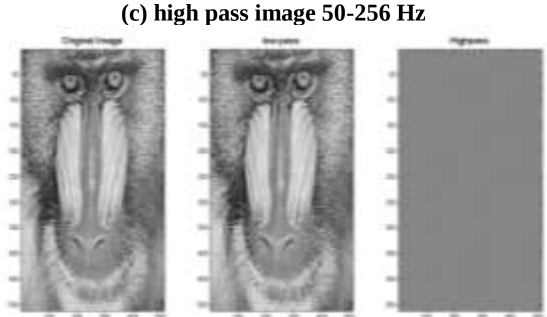


Figure 9: (a) Original image (b) low pass image 0-100 Hz (c) high pass image 100-256 Hz

In figure 7 to 9 baboon image is divided into low and high frequency components with varying cut-off frequncies. In figure 5, it is clear that the highest contained frequncies in image is arount 100 Hz. In figure 7, the cut-off frequncy is 10 Hz, therefore most of the image pixels appears in high pass image. In figure 8, the cut-off frequncy is increased to 50 Hz, thus image is appeaed in both low and high pass system. While in figure 9, the low pass frequncy is increased to 100 Hz, and thus full image appears in low pass regime.

BABOON IMAGE

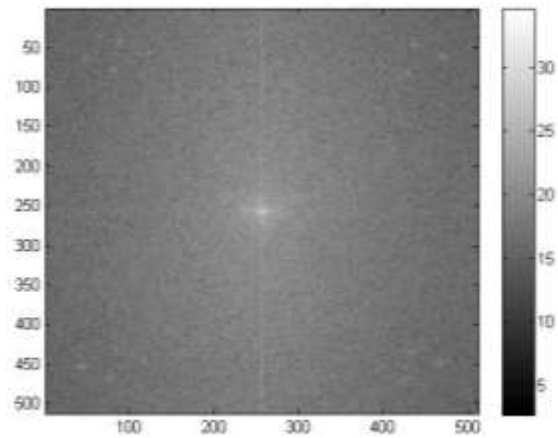


Figure 10: Power spectrum of initial image (log scale)

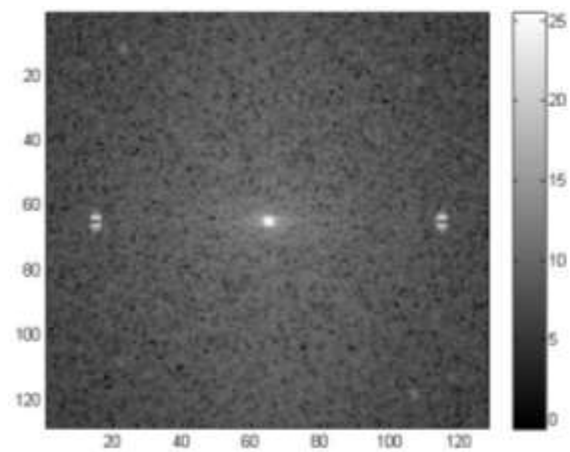


Figure 11: Minimum power spectrum on patches (log scale)

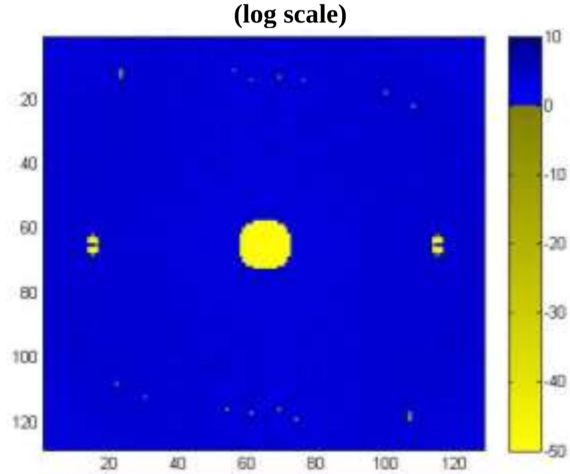


Figure 12: Logarithm of NFA (log scale)

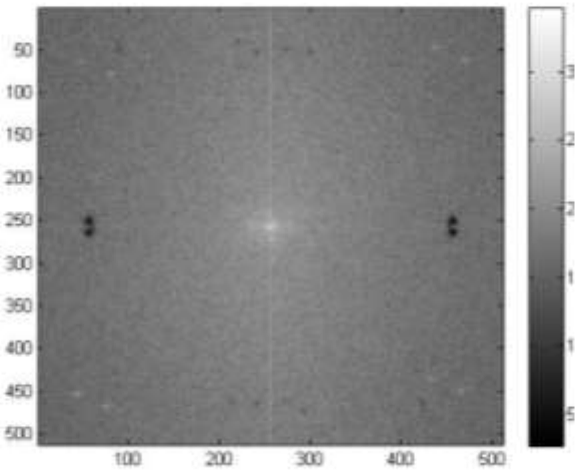


Figure 13: Corrected Power spectrum

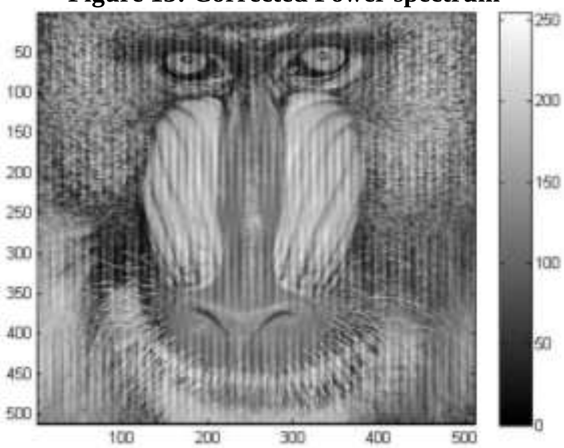


Figure 14: Origial Image

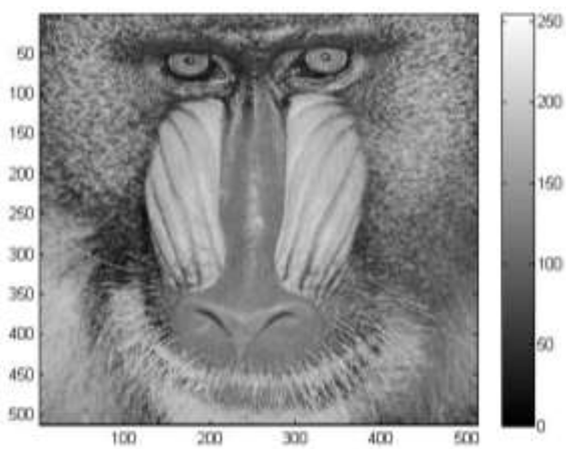


Figure 15: Denoised Image

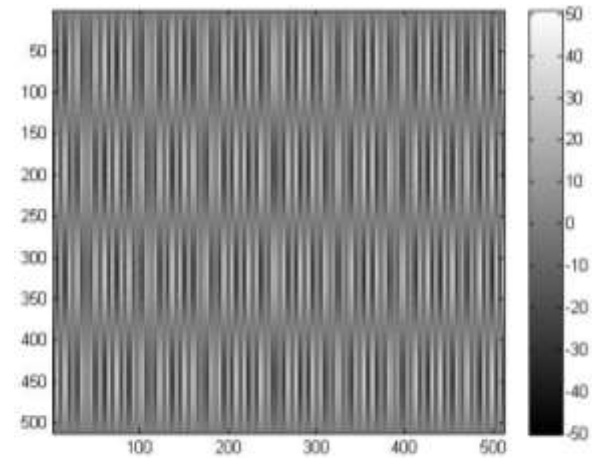


Figure 16: Noise components

Results for Baboon image is shown in Figure 10 to 16. The initial power spectrum is shown in figure 10 , which shows how power is distributed with respect to frequency.After identifying the pathces, the minimum power spectrum of the patches is shown in figure 11. Detection of number of false alarms is shown in figure 12 and therefater obtained corrected power spectrum is shown in figure 13. The original Baboon image is shown in figure 14, and after applying the algorithm, the de-noised image is shown in figure 15 and finally recovered noise components presnet in the image is shown in figure 16.

This analysis clearly reveals that, for the removal of quasi noise designed filter should be able to filter very low frequency components. However in some other images it may require to design such filter on high frequency components. Therefore the estimation of frequency components is necessary in quasi noise removal.

Peak Signal to Noise Ratio (PSNR)

Removal of noises from the images is a critical issue in the field of digital image processing. The phrase Peak Signal to Noise Ratio, often abbreviated PSNR, is an engineering term for the ratio between the maximum possible power of a signal and the power of corrupted noise that affects the fidelity of its representation. As many signals have wide dynamic. The MSE and PSNR is defined as:

$$PSNR = 20\log_{10}\left(\frac{255}{MSE}\right) \quad (30)$$

$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}\left[I(i,j) - k(i,j)\right]^2 \quad (31)$$

In table 1 results in terms of PSNR is shown, here with the proposed method the improvement in PSNR ranges from 0.2 dB to 1.3 dB. Thus the proposed method improves the PSNR, when there is scope of improvement.

Table 1: Comparison of PSNR

Image	Current [11]	Proposed
Baboon	48.2638	48.7217
Lenna	45.5598	46.8640
Barbara	42.9342	43.9953
Peppers	41.3737	41.5368

4.5 CONCLUSIONS

In this paper, results for the noise removal process are presented. Noise removal in an image is important and complicated problem due to the randomness of the noise. Most of the time noises are easily removed by measuring their pdfs. But in some cases estimation of noises is complicated which is superimposed on the signal spectral components. In such a case problem become complex as first, those frequency components has to be find out and careful removal of noise has to be done. The concept of false alarm helps in detecting those spectral components which are corrupted with noise.

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