On E_k – Cordial Graphs

J.Devaraj[#1], M.Teffilia[#2]
[1]*Associate professor(retd),Dept. of Mathematics, NMC College Marthandam,Tamil Nadu,India*
Email : devaraj.jacob@yahoo.co.in
[2]*Assistant Professor,Dept. of Mathematics,WCC College Nagercoil,Tamil Nadu,India*
Email : teffiliafranklin@gmail.com

Abstract: **A labeling f of G is said to be E_k- cordial if it is possible to label the edges with the numbers from the set {0,1,2,………,k-1} in such a way that, at each vertex v, the sum of the labels on the edges incident with v modulo k satisfies the inequalities $|v(i)-v(j)|\le 1$ and $|e(i)-e(j)|\le 1$, where v(s) and e(t) are respectively, the number of vertices labeled with s and the number of edges labelled with t. In this paper we prove that the ladder graph, $C_n \odot K_2$, attaching triangle at each vertex of the cycle, attaching $k_{1,3}$ at each vertex of the cycle , flower graph,LIC_{2n} are E_k- Cordial graphs.**

***Keywords*: Labeling, ladder graph, flower graph, LIC_{2n} graph, E_k- Cordial graph.**

I. Introduction

A labeling or valuation of a graph G is an assignment of labels to the vertices of G that induces for each edge xy a label depending on the vertex labels f(x) and f(y). For all terminology and notations in graph theory we follow Harary [1] .Yilmaz and Cahit defined a new graph technique called E_k- Cordial labeling in 1997.Let f be an edge labeling of a graph G=(V,E) such that f:E(G)$\rightarrow$ {0,1,2,…..,k-1} and the induced vertex labeling be given as $f^+(v) = \left(\sum_u f(uv)\right)$ (mod k), where u,v $\in$ V and uv$\in$E. The map f is called an E_k- Cordial labeling of G, if the following conditions are satisfied for all i, j $\in$ {0,1,2,…..,k-1}:

(1) $|e_f(i)-e_f(j)|\le 1$ and

(2) $|v_f(i)-v_f(j)|\le 1$

Where $e_f(i), e_f(j)$ denote the number of edges labeled with i and j repectively and $v_f(i), v_f(j)$ denote the number of vertices labeled with i and j repectively. The graph G is called E_k- Cordial if it admits an E_k- Cordial labeling. A graph is E- Cordial if it is E_2- Cordial.

Definition 1:

Let P_n be a path on n vertices. A ladder graph $P_n \times K_2$ is defined as the Cartesian product of P_n and K_2 and it is denoted by L_n.

Theorem 1:

Ladder graph L_n is E_k-cordial for k = n-1 and k $\not\equiv$0 (mod 3)

Proof :

The Ladder graph $L_n = P_n \times K_2$

Let $u_1,u_2,.....u_n, v_1, v_2,....... v_n$ be the vertices of L_n.

The edge set of L_n be

$E(L_n) = \{(u_iu_{i+1})/1\le i\le n-1\} \cup \{(v_iv_{i+1})/1\le i\le n-1\} \cup \{(u_iv_i)/ 1\le i\le n\}$

L_n has 2n vertices and (3n-2) edges

Define f : $E(L_n) \rightarrow\{0,1,2,.....,k-1\}$ where k = n-1 as follows.

$f(u_i u_{i+1})= i-1, 1\le i\le n-1$

$f(v_i v_{i+1})= (n-2) – (i-1) , 1\le i\le n-1$

$f(u_i v_i)= i-1, 1\le i\le n-1$

$f(u_n v_n)= n-2$

Now $e_f(0) = e_f(1) = ... = e_f(k-2) =3, e_f(k-1)=4$,

The induced vertex labels are as follows.

Now $f^+(u_1)$ = Sum of the edges incident with u_1 (mod k)

$f^+(u_1) = 0$

$f^+(u_2) =2$

$f^+(u_i) = f^+((u_{i-1})+3)$ (modk), i=3,4,....., k-1

$f^+(u_k)= k-2$

$f^+(v_i)= k-i$, i=1,2,....,k

$f^+(v_{k+1}) = k-1$

Then $v_f(0) =3, v_f(i) =2$, for i$\neq$k-2, $v_f(k-2)=3$

In both the cases,

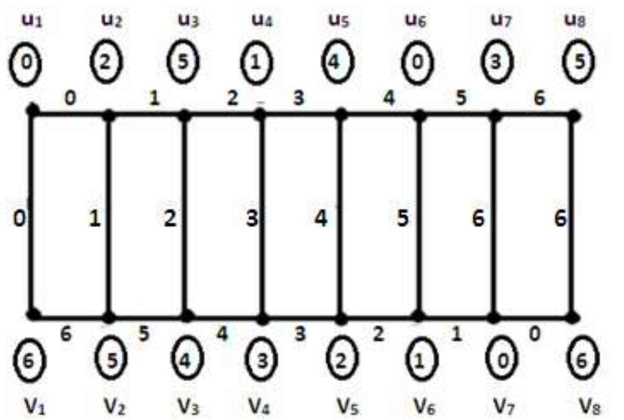
$|e_f(i) - e_f(j)| \le 1, \forall\, i,j$ and

$|v_f(i) - v_f(j)| \le 1, \forall\, i,j$

L_n is E_k – cordial for k= n-1 and k $\not\equiv$0 (mod 3)

Illustration:1

L_8 is E_7 – cordial



Theorem 2:

The graph $C_n \odot K_2$ is E_k –cordial for $n \not\equiv 0$ (mod4), $n \geq 3$ with $k = n$.

Proof :

Let C_n be the cycle. Let $v_1, v_2, \ldots v_n$ be the vertices of C_n. Let K_2 be the complete graph on two vertices. Now attach k_2 to each vertex of the cycle C_n. The newly obtained graph is denoted by $C_n \odot K_2$.

Note that $C_n \odot K_2$ is a graph with 3n vertices and 4n edges. Let a_i, b_i, i=1,2,....,n be the vertices adjacent to the rim vertices of C_n

Let the vertex set of G be

$V(G) = \{a_i / 1 \leq i \leq n\} \cup \{b_i / 1 \leq i \leq n\} \cup \{v_i \ / 1 \leq i \leq n\}$

Let the edge set of G = $C_n \odot k_2$ be

$E(G) = \{(v_i\, v_{i+1})/1 \leq i \leq n-1\} \cup \{(v_n\, v_1)\} \cup \{(a_i\, b_i)/1 \leq i \leq n\} \cup \{(v_i\, b_i) / 1 \leq i \leq n\} \cup \{(a_i\, v_i) / 1 \leq i \leq n\}$

Define $f: E(G) \rightarrow \{0,1,2,\ldots,k-1\}$ where k=n as follows.

$f(v_i\, v_{i+1}) = i-1,\ 1 \leq i \leq k$ where $v_{k+1} = v_1$

$f(v_i\, a_i) = i-1,\ 1 \leq i \leq k$

$f(a_i\, b_i) = i-1,\ 1 \leq i \leq k$

$f(v_i\, b_i) = i,\ 1 \leq i \leq k-1$

$f(v_{k+1} b_{k+1}) = 0$

Now $e_f(0) = e_f(1) = \ldots\ldots = e_f(k-1) = 4$

The induced vertex labels are as follows

$f^+(v_1) = 0$

$f^+(v_i) = [f(v_{i-1})+4)] \bmod k,\ 2 \leq i \leq k$

$f^+(a_1) = 0$

$f^+(a_i) = [f(a_{i-1})+2)] \bmod k,\ 2 \leq i \leq k$

$f^+(b_1) = 0$

$f^+(b_i) = [f(b_{i-1})+2)] \bmod k,\ 2 \leq i \leq k$

Then $v_f(0) = v_f(1) \ldots\ldots v_f(k-1) = 3$

In both the cases

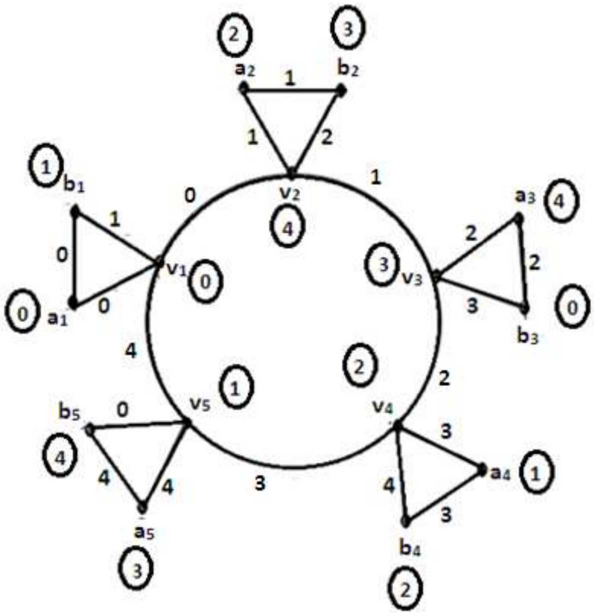
$|e_f(i) - e_f(j)| \leq 1, \forall\, i, j$ and

$|v_f(i) - v_f(j)| \leq 1, \forall\, i, j$

$\therefore C_n \odot K_2$ is E_k – cordial for $n \not\equiv 0$ (mod 4) , $n \geq 3$ with k=n

Illustration:2

$C_5 \odot K_2$ is E_5 – cordial



Theorem 3:

A graph obtained by attaching $k_{1,3}$ at each vertex of the cycle C_n is E_k – cordial, $k \not\equiv 0$ (mod 3) and k=n

Proof:

Let C_n be the cycle $u_1 u_2 \ldots u_n u_1$. Let v_i, x_i, y_i, z_i be the vertices of the i^{th} copy of $k_{1,3}$ in which v_i is the central vertex. Identify z_i with $u_i, 1 \leq i \leq n$ .Let the resultant graph be G. The edge set of G is

$E(G) = \{(u_i u_{i+1})/1 \leq i \leq n \cup \{(u_n v_1)\} \cup \{(u_i v_i, v_i x_i, v_i y_i / 1 \leq i \leq n\}$

It has 4n vertices and 4n edges.

Define $f: E(G) \rightarrow \{0,1,2,\ldots\ldots k-1\}$ by

$f(u_i u_{i+1}) = i-1,\ u_{k+1} = u_1,\ 1 \leq i \leq k$

$f(u_i v_i) = i-1,\ 1 \leq i \leq k$

$f(v_i x_i) = i-1,\ 1 \leq i \leq k$

$f(v_i y_i) = i-1,\ 1 \leq i \leq k$

Now $e_f(0) = e_f(1) = \ldots\ldots\ldots\ e_f(k-1) = k-1$

The induced vertex labels where k = n are as follows

$f^+(u_1) = k-1$

$f^+(u_i) = [f(u_{i-1}) +3] \bmod k,\ 1 \leq i \leq k$

$f^+(v_1) = 0$

$f^+(v_i) = [f(v_{i-1}) +3] \bmod k,\ 1 \leq i \leq k$

$f^+(x_i) = i-1,\ 1 \leq i \leq k$

$f^+(y_i) = i-1,\ 1 \leq i \leq k$

then $v_f(0) = v_f(1) = \ldots\ldots = v_f(k-1) = 4$

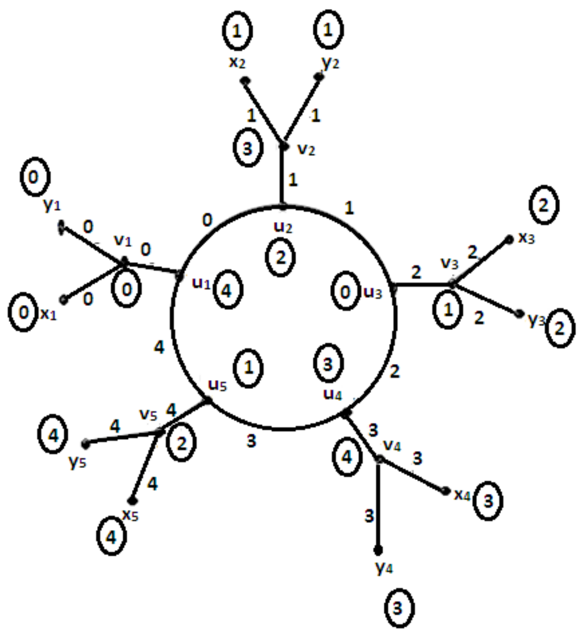
In both the cases

$|e_f(i) - e_f(j)| \leq 1, \forall\, i, j$ and $|v_f(i) - v_f(j)| \leq 1, \forall\, i, j$

$\therefore$ The graph is E_k – cordial, $k \not\equiv 0$ (mod 3) and k=n

Illustration:3

Attaching $k_{1,3}$ at each vertex of the cycle C_5 is E_5 – cordial,



II .flower graph

Definition 2:

A graph G is called a $(n \times m)$ – flower graph if it has n vertices which form an n – cycle and n sets of m-2 vertices which form m – cycles around the n – cycle on a single edge.This graph is denoted by $f_{n\times m}$.It is clear that $f_{n\times m}$ has n(m-1) vertices and mn edges.

Theorem 4:

Flower graph $f_{n\times 3}$ is E_k – cordial, where $k\not\equiv 0 \pmod 4$ and k=n.

Proof :

Let $v_1, v_2, \ldots v_n$, be the vertices of n-cycle of f_{nx3} and $\{u_i\}$ be the i^{th} sets of vertices,$1 \le i \le n$, Which form 3-cycles, around the n-cycle so that 3 cycle intersects with n-cycle on a single edge.

The edge set of the graph G is

$E(G) = \{(v_i\, v_{i+1}, v_{i+1}u_i, v_1u_n, v_nu_1/1 \le i \le n-1\} \cup \{(v_i\, u_i)/1\le i\le n\}$

Then the graph has 2n vertices and 3n edges.

Define f: $E(G) \rightarrow \{0,1,2,\ldots,k-1\}$ where k=n is as follows.

$f(v_i\, v_{i+1},)= i-1,\ 1\le i\le k,\ v_{k+1}=v_1$

$f(v_{2i-1}\, u_{2i-1})= 2i-2,\ 1\le i\le k$

$f(v_{2i}\, u_{2i})= 2i-2,\ 1\le i\le k$

$f(v_{2i}\, u_{2i-1})= 2i-1,\ 1\le i\le k$

$f(v_{2i+1}\, u_{2i})= 2i-1,\ 1\le i\le k,\ \ v_{k+1} = v_1$

Now $e_f(0)= e_f(1)= \ldots\ldots\ldots\ldots.\ e_f(k-1)= 3$

The induced vertex labels are as follows

$f^+(v_1)= k-2,$

$f^+(v_i)= [f(v_{i-1}+4)] \bmod k$

$f^+(u_1)= 1$

$f^+(u_2)= 1$

$f^+(u_{2i+1}) = [f(u_{2i-1})+4] \bmod k$

$f^+(u_{2i+2}) = [f(u_{2i})+4] \bmod k$

Then $v_f(0) = v_f(1) =\ldots\ldots\ldots\ldots.= v_f(k-1) =3$

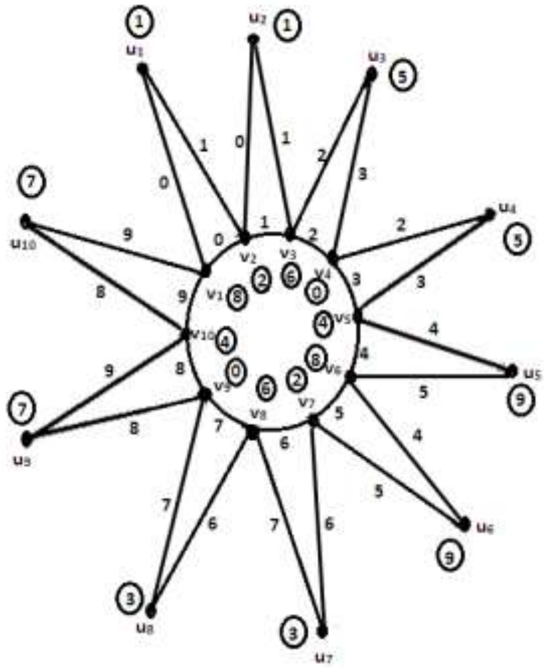
In both the cases

$|e_f(i) - e_f(j)| \le 1, \forall\, i,j$ and $|v_f(i) - v_f(j)| \le 1, \forall\, i,j$

$\therefore f_{nx3}$ is E_k – cordial, where $k\not\equiv 0 \pmod 4$ and k=n

Illustration:4

f_{nx3} is E_{10} – cordial.



III. Lotus inside a circle

Definition 3:

The graph lotus inside a circle is denoted by LIC_n , $n \ge 3$ and is defined as follows. Let S_n be the star graph with vertices $b_0,b_1,b_2\ldots,b_n$ whose centre is b_0 . Let C_n be the cycle of length n whose vertices are $a_1,a_2,a_3,\ldots,a_n$ .We join a_{i+1} with b_i and b_{i+1} for each $i \ge 1$ and join a_1 with b_1 and b_n .

Theorem 5:

The graph LIC_{2n} is E_k – cordial for $k \ge 4$, $k\not\equiv 0 \pmod 5$ and k=n

Proof :

The vertex set is $V(LIC_{2n}) = \{b_i/1\le i\le 2n\} \cup \{a_j/ 1\le j\le 2n\}$

The edge set is $E(LIC_{2n})=\{(b_0b_i) /1\le i \le 2n\} \cup \{(b_i\, a_i) /1\le i\le 2n\} \cup \{(b_i\, a_{i+1}) / 1 \le i \le 2n\ \&\ a_{2n+1} = a_1\} \cup \{(a_i\, a_{i+1})\ / 1 \le i \le 2n\ \&\ a_{2n+1} = a_1\}$

LIC_{2n} has (4n+1) vertices and 8n edges.

Define $f:E(LIC_n) \rightarrow \{0,1,2,\ldots,k-1\}$ where k=n is as follows.

$f(b_0\, b_i) = i-1,\ 1\le i \le k$

$f(b_0\, b_{i+k}) = i-1,\ 1\le i \le k$

$f(b_i\, a_i) = 2i-2\ (\bmod k),\ 1\le i \le 2k$

$f(b_i\, a_{i+1}) = 2i-1\ (\bmod k),\ 1\le i \le 2k,\ a_{2k+1} = a_1$

$f(a_{2i-2}\, a_{2i-1}) = i-1,\ 1\le i \le k-1,\ a_{2k} = a_0$

$f(a_{2i-1}\, a_{2i}) = i-1,\ 1\le i \le k-1$

Now $e_f(0) = e_f(1) = \ldots\ldots\ldots\ldots. = e_f(k-1)=8$

The induced vertex labels are as follows

$f^+(b_0) = 0$

$f^+(b_1) = 1$

$f^+(b_i) = [f(b_{i-1})+5] \bmod k$

$f^+(a_1) = k-1$

$f^+(a_i) = [f(a_{i-1})+5] \bmod k$

Then $v_f(0) = 5,\ v_f(1) = \ldots\ldots\ldots\ldots. = v_f(k-1)=4$

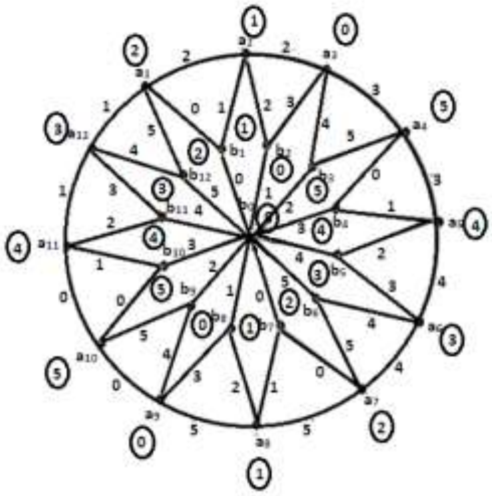
In both the cases

$|e_f(i) - e_f(j)| \leq 1, \forall\, i, j$ and $|v_f(i) - v_f(j)| \leq 1, \forall\, i, j$

$\therefore$ LIC_{2n} is E_k – cordial for $k \geq 4$, $k \not\equiv 0 \pmod 5$ where k =n

Illustration :5

LIC_{12} is E_6 – cordial



References

[1] Harary F, Graph Theory, Narosa Publishing House, New Delhi(1993)

[2] Joseph A. Gallian, Dynamic Survey of graph labeling (2015)

[3] Sridharan N. and. Umarani R., E_k – cordial labeling of graphs, Elixir Appl.Math,38(2011)4564-4567

[4] Maged Z. Youssef , On E_k – cordial labeling ,Article in Ars Combinatoria-waterloo then Winnipeg-April 2012.