Hall Effect on Flow past Porous Flat Plate

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Abstract : In a strong magnetic field, the effect of Hall Current is realized and it induces a cross flow in the transverse direction. The solution for temperature field subject to two boundary conditions that correspond to heat transfer and thermometer problems is derived. The effect of Hall and frequency parameters on temperature field caused by viscous dissipation are discussed.

Introduction

The time dependent oscillatory flow of viscous conducting fluid has been studied by Lighthill [7] by superimposing the fluctuating flow on the mean steady flow. Stuart [13] applied this assumption in studying the oscillatory flow of an incompressible viscous conducting fluid past an infinite horizontal porous plate with uniform suction. The effect of transversely applied magnetic field on Stuart's problem is presented by Rao [9, 10]. However for strong magnetic field, the effect of Hall current is realized and it induces a cross-flow in the transverse direction. The effects of Hall current on shear stress have been examined by Katagiri [6]. Datta.and Jana [5] have discussed the effect of Hall current on velocity and shear stress for small and large frequencies of oscillation. Free convection MHD flow past porous have been studied by Bezan [1], Bezan *et al.* [2], Dubewar and Soundalgeker [4], Meyer [8], Santosh et al. [11] and Singh and Chand [12].

In the present chapter, we have proposed to study the effect of oscillation and Hall current on shear stress and temperature fields. With viscous dissipation of .kinetic energy taken into accounts, the corresponding exact solution for temperature field subject to two boundary conditions that the plate is kept at constant temperature or insulated to heat is derived. From the result it is found that amplitude of the shear stress rises with frequency and for constant magnetic field it decreases with Hall parameter while phase of the shear stress lags behind the free stream oscillation. Further, the mean temperature caused by viscous dissipation is largely affected by magnetic field parameter. The effect of Hall current on amplitude and phase of the harmonic fluctuation' of temperature field is discussed for small and large frequencies.

Mathematical Analysis

Consider an unsteady incompressible boundary layer flow of an electrically conducting viscous fluid past an infinite porous flat plate placed in the x-z plane. A uniform transverse magnetic field of strength H_0 is acting in y direction. The effect of Hall current gives rise to a force in z-direction which induces a cross flow in that direction, hence the flow becomes to be three dimensional. Since the plate is infinite in extent, all physical variables (except pressure) are functions of y ant t only. The solenoidal relation ΔH =0 gives Hy = H0 = constant everywhere in the fluid, where H = (Hx, Hy, Hz).

From $\Delta\times H = j$, we obtained $J_y = 0$ at the plate, where $j = (j_x, j_y, j_z)$. By the assumption of small magnetic Reynolds number, an induced magnetic field can be neglected in comparison with the applied field H_0. In the absence of an external electric field, the effect of polarization of the ionized fluid is negligible. The generalized Ohm's law with negligible ion slip and thermoelectric effect is Cowling [3].

$$j + \frac{W_e \tau_e}{H_0}\left(j \times H\right) = \sigma\left[\tau_e V \times H + \left(1/en_e\right)\Delta p_e\right] \tag{1}$$

which gives

$$j_x - W_e \tau_e j_z = -\sigma \tau_e H_0 w \tag{2}$$

$$j_z - W_e \tau_e j_z = -\sigma \xi_e H_0 u \tag{3}$$

On solving these we get

$$j_x = \frac{\sigma \xi_e H_0}{1+m^2}\left(mu - w\right) \tag{4}$$

$$j_z = \frac{\sigma \xi_e H_0}{1+m^2}\left(u + mw\right) \tag{5}$$

where $m = w_e \tau_e$ is the Hall parameter.
where the free stream velocity U(t) is assumed as

$$U(t) = U_0\left(1 + \in e^{i\lambda t}\right) \tag{6}$$

$$\frac{\partial v}{\partial y} = 0 \tag{7}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \frac{\sigma \xi_e^2 H_0}{\rho\left(1+m^2\right)}(u + mw) \tag{8}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y}, \tag{9}$$

$$\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + v\frac{\partial^2 w}{\partial y^2} - \frac{\sigma \xi_e^2 H_0}{\rho\left(1+m^2\right)}(mu + w) \tag{10}$$

$$\rho c_p\left(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}\right)=b\frac{\partial^2 T}{\partial y^2}+\xi\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right] \quad (11)$$

The boundary conditions are

u = 0 = w at y = 0; u = U(t), w = 0 as $y \to \infty$ (12)

T = T_W at y = 0; T = T_∞ as $y \to \infty$ for isothermal wall

$\frac{\partial T}{\partial y}=0$ at y = 0; T = T_∞ as $y \to \infty$ for adiabatic wall (13)

Introducing the following non-dimensional quantities

$$v=-v_0,\ y=\frac{yv_0}{v},\quad t=\frac{tv_0^2}{v},\quad \lambda=\frac{\lambda v}{v_0^2}$$

$$u=\frac{u}{U_0},\ w=\frac{w}{U_0},\quad T=\frac{T-T_\infty}{T_W-T_\infty},\quad P=\frac{\xi C_p}{k}$$

$$E=\frac{U_0^2}{C_p\left(T_W-T_\infty\right)};\quad M=\frac{\sigma\xi_e^2 H_0^2}{\rho v_0^2} \quad (14)$$

and eliminating pressure gradient term with the help of free stream condition, the equation (8), (10) and (11) reduce to the form

$$\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=\frac{\partial U}{\partial t}+\frac{\partial^2 u}{\partial y^2}-\frac{M}{1+m^2}\left(u-U+mw\right) \quad (15)$$

$$\frac{\partial w}{\partial t}-\frac{\partial w}{\partial y}=\frac{\partial^2 w}{\partial y^2}-\frac{M}{1+m^2}\left(m\left(u-U\right)+w\right) \quad (16)$$

$$\frac{\partial T}{\partial t}-\frac{\partial T}{\partial y}=\frac{1}{P}\frac{\partial^2 T}{\partial y^2}+E\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right] \quad (17)$$

In view of equation (6), we look for the solutions of equations (15-17) as

$$u\left(y,t\right)=u_0\left(y\right)+\in e^{i\lambda t}u_1\left(y\right)$$

$$w\left(y,t\right)=w_0\left(y\right)+\in e^{i\lambda t}w_1\left(y\right) \quad (18)$$

$$T\left(y,t\right)=T_0\left(y\right)+\in e^{i\lambda t}T_1\left(y\right)$$

Substituting equation (18) into equations (15) - (17) and equating harmonic and non-harmonic terms, we get

$$\frac{d^2u_0}{dy^2}+\frac{du_0}{dy}-\frac{M}{1+m^2}\left[u_0-1+mw_0\right]=0 \quad (19)$$

$$\frac{d^2u_1}{dy^2}+\frac{du_1}{dy}-i\lambda\left(u_1-1\right)-\frac{M}{1+m^2}\left[u_1-1+mw_1\right]=0 \quad (20)$$

$$\frac{d^2w_0}{dy^2}+\frac{dw_0}{dy}+\frac{M}{1+m^2}\left[m\left(u_0-1\right)-w_0\right]=0 \quad (21)$$

$$\frac{d^2w_1}{dy^2}+\frac{dw_1}{dy}+i\lambda w_1+\frac{M}{1+m^2}\left[m\left(u_1-1\right)-w_1\right]=0 \quad (22)$$

$$\frac{d^2T_0}{dy^2}+\frac{dT_0}{dy}=-PE\left[\left(\frac{du_0}{dy}\right)^2+\left(\frac{dw_0}{dy}\right)^2\right] \quad (23)$$

$$\frac{d^2T_1}{dy^2}+\frac{dT_1}{dy}-i\lambda p=-2PE\left[\frac{du_0}{dy}\frac{du_1}{du}+\frac{dw_0}{du}\frac{dw_1}{dy}\right] \quad (24)$$

and the boundary conditions are

$u_0=0=u_1,\quad w_0=0=w_1$ at y=0

$u_0=0=u_1,\quad w_0=0=w_1,$ at $y\to\infty$ (25)

$T_0=1,\ T_1=0;$ at y=0, T_0=0=T_1 as $y\to\infty$

$\frac{dT_0}{dy}=0=\frac{dT_1}{dy}$ at y=0; T_0=0=T_1 as $y\to\infty$ (26)

The solutions of equations (19) to (22) are

$$u_0=1-e^{-A_0 y}\cos B_0 y \quad (27)$$

$$u_1 = 1 - e^{-A_1 y} \cos B_1 y \quad (28)$$

$$w_0 = -e^{-A_0 y} \cos B_0 y \quad (29)$$

$$w_1 = -e^{-A_1 y} \cos B_1 y \quad (30)$$

where

$$A_0 - iB_0 = \frac{1}{2} + \frac{1}{2}\left[\left(\frac{1+4M}{1+m^2}\right) - i\,\frac{4mM}{1+m^2}\right]^{1/2} \quad (31)$$

$$A_1 - iB_1 = \frac{1}{2} + \frac{1}{2}\left[\left(\frac{1+4M}{1+m^2}\right) - i4\left(\frac{mM}{1+m^2} - \lambda\right)\right]^{1/2} \quad (32)$$

In view of equations (27) - (30), the solution of equations (23) and (24) subject to the boundary conditions (26) are

$$T_0 = e^{-py} + \frac{PE\left(A_0^2 + B_0^2\right)}{2A_0\left(2A_0 - P\right)}\left(e^{-py} - e^{-2A_0 y}\right) \quad (33)$$

$$T_1 = 2PE\left[\frac{R\left(e^{-Ly} - e^{-Ky}\right)}{K^2 - KP - i\lambda P} + \frac{R^*\left(e^{-Ly} - e^{-k^* y}\right)}{K^{*2} - K^* P - i\lambda P}\right] \quad (34)$$

for isothermal wall conditions,

and $$T_0 = \frac{PE\left(A_0^2 + B_0^2\right)}{2A_0\left(2A_0 - P\right)}\left(\frac{2A_0}{P}e^{-py} - e^{-2A_0 y}\right) \quad (35)$$

$$T_1 = 2PE\left[\frac{R\left(\frac{K}{L}e^{-Ly} - e^{-Ky}\right)}{k^2 - kp - i\lambda p} + \frac{R^*\left(\frac{K^*}{L}e^{-Ly} - e^{-k^* y}\right)}{k^{*2} - k^* p - i\lambda p}\right] \quad (36)$$

for adiabatic wall conditions, where asterisk (*) denotes complex conjugate,

Discussions and Conclusion

For isothermal condition of the plate, mean temperature distribution in the flow field can be obtained by equation (33). When $T_w = T_\infty$ the distribution of temperature created by frictional heating depends on the magnetic field parameter and the equation (33) leads to

$$\frac{T_0 T_\infty}{\rho U_0^2 / 2c_p} = \frac{A_0^2 + B_0^2}{A_0\left(2A_0 - P\right)}\left(e^{-py} - e^{-A_0 y}\right) \quad (37)$$

and we observe that mean temperature in this case increases with Hall parameter and its highest value occur of closed distance from the plate.

Under the condition of zero heat transfer between the plate and the fluid, the mean temperature field in the flow region is solely due to frictional heating of the fluid particles and its distribution in the flow field can be obtained by equation (35). The highest mean temperature in this case occur near the adiabatic wall which is readily anticipated that under zero heat transfer condition, the heat generated due to friction is superimposed on the conducting heat of the wall. The application of magnetic field increases the mean temperature while the consideration of Hall current reduces the heat generated through viscous dissipation.

The harmonic fluctuation of temperature field can be obtained from equations (34) and (36). For isothermal boundary condition of the plate, the amplitude and phase angle of the heat transfer can be derived. It can be observed that due to application of magnetic field, the amplitude heat transfer decreases with λ. However, the effect of Hall current decreases the amplitude for fixed λ. The amplitude of harmonic fluctuation of adiabatic wall temperature decreases with λ. In the presence of magnetic field, the amplitude of fluctuating adiabatic wall temperature decays fastly as frequency approaches to a fixed value. Further, the phase of harmonic fluctuation of the adiabatic wall temperature always lags behind the free stream oscillation and the magnitude of tan δ increases with λ,. However the effect of Hall current increases the phase lag.

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