On $\psi\alpha g$-Separation axioms in Topological Spaces

[1]V. Kokilavani, [2]P.R. Kavitha
1.Assistant professor, Dept. of Mathematics, Kongunadu Arts and Science College, Coimbatore, Tamilnadu, India
2.Assistant professor, Dept. of Mathematics, Kongunadu Arts and Science College, Coimbatore, Tamilnadu, India

Abstract: **In the present paper, we introduce, study and investigate the following separation axioms: $\psi\alpha g$ - T_i spaces (for i=0,1,2). Moreover, some of special results and properties, which belong to them, are studied.**

Keywords: $\psi\alpha g$-closed; $\psi\alpha g$-T_0; $\psi\alpha g$-T_1; $\psi\alpha g$-T_2.

1. INTRODUCTION

Recently V. Kokilavani and P.R. Kavitha [12] defined the concept of $\psi\alpha g$-closed sets and studied some of their properties.

The aim of this paper is to introduce a new type of function is called $\psi\alpha g$-open and quasi $\psi\alpha g$-closed functions, quasi $\psi\alpha g$-open and quasi $\psi\alpha g$-closed functions. Also, we obtain its characterizations and its basic properties and we study a new type of weak separation axioms, namely $\psi\alpha g$-T_0, $\psi\alpha g$-T_1, $\psi\alpha g$ - T_2 and separation properties obtained by utilizing $\psi\alpha g$-closed sets.

2. PRELIMIERIES

Definition: 2.1 Let (X, τ) be a topological space. A subset A of the space X is said to be

(i) semi open set [2] if A⊆cl(int(A)).
(ii) α-open set [4] if A⊆int(cl(int(A))).

The complements of the above mentioned sets are called their respective closed sets. The ψ-closure of a subset A of a space (X, τ) is the intersection of all ψ-closed sets that contain A and is denoted by $\psi cl(A)$. The ψ-interior of a subset A of a space (X, τ) is the union of all ψ-open sets contained in A and is denoted by $\psi Int(A)$.

Definition: 2.2 A subset A of a space X is $\psi\alpha g$-closed if $\psi cl(A) \subset U$ whenever $A \subset U$ and U is αg-open in X. The family of all $\psi\alpha g$-closed subsets of the space X is denoted by $\psi\alpha gC(X)$.

Definition: 2.3 The intersection of all $\psi\alpha g$-closed sets containing a set A is called $\psi\alpha g$-closure of A and is denoted by $\psi\alpha g$- $Cl(A)$. A set A is $\psi\alpha g$-closed set if and only if $\psi\alpha g$- $Cl(A) = A$.

Definition: 2.4 The union of all $\psi\alpha g$-open sets contained in A is called $\psi\alpha g$-interior of A is denoted by $\psi\alpha g$- $Int(A)$. A set A is $\psi\alpha g$-open sets if and only if $\psi\alpha g$- $Int(A) = A$.

A function f: (X, τ)→(Y, σ) is called

(i) g-continuous[2] if $f^{-1}(V)$ is a g-closed in (X, τ) for every closed set V of (Y, σ).
(ii) α-continuous[15] if $f^{-1}(V)$ is a α-closed in (X, τ) for every closed set V of (Y, σ).
(iii) αg-continuous[6] if $f^{-1}(V)$ is a αg-closed in (X, τ) for every closed set V of (Y, σ).
(iv) $\psi\alpha g$-continuous[6] if $f^{-1}(V)$ is a $\psi\alpha g$ -closed in (X, τ) for every closed set V of (Y, σ).
(v) α-irresolute[10] if $f^{-1}(V)$ is a α-open set in (X, τ) for every α-open set V of (Y, σ).
(vi) α-quotient map[10] if f is α-continuous and $f^{-1}(V)$ is open set in (X, τ) implies V is an α-open set in (Y, σ).
(vii) Weakly continuous[9] if for each point x∈X and each open set V⊆Y containing f(x), there exists an open set U ⊆ X containing x such that f(U)⊆cl(V).

3. Quasi $\psi\alpha g$-open functions

We introduce a new definition as follows:

Definition: 3.1 A function $f: X \to Y$ is said to be quasi $\psi\alpha g$-open if the image of every $\psi\alpha g$-open set in X is open in Y.

It is evident that, the concepts quasi $\psi\alpha g$-openness and $\psi\alpha g$-continuity coincide if the function is a bijection.

Theorem:3.2 A function $f: X \to Y$ is quasi $\psi\alpha g$-open if and only if for every subset U of X,

$$f(\psi\alpha g - \text{Int(U)}) \subset Int(f(U)).$$

Proof: Let f be a quasi $\psi\alpha g$-function. Now, we have $Int(U) \subset U$ and $\psi\alpha g$-$Int(U)$ is a $\psi\alpha g$-open set. Hence, we obtain that $f(\psi\alpha g - \text{Int(U)}) \subset f(U)$. As $f(\psi\alpha g - \text{Int(U)})$ is open, $f(\psi\alpha g - \text{Int(U)}) \subset Int(f(U))$. Conversely, assume that U is a $\psi\alpha g$-open set in X. Then,

© 2016 IJAIR. All Rights Reserved

$f(U) = f(\psi\alpha g - \text{Int(U)}) \subset Int(f(U)\ )$ but Int(f(U)) ⊂ f(U) . Consequently, $f(U) = Int(f(U))$
and hence f is quasi $\psi\alpha g$-open.

Theorem:3.3 A function $f: X \to Y$ is quasi $\psi\alpha g$-open, then $\psi\alpha g$-$Int(f^{-1}(G)) \subset f^{-1}(Int(G))$ for every subset G of Y.

Proof: Let G be any arbitrary subset of Y. Then, $\psi\alpha g$-$Int(f^{-1}(G))$ is a $\psi\alpha g$-open set in X and f is quasi $\psi\alpha g$ -open, then $f(\psi\alpha g$ - $Int(f^{-1}(G))) \subset Int(f(f^{-1}(G))) \subset Int(G)$. Thus, $\psi\alpha g$-$Int(f^{-1}(G)) \subset f^{-1}(Int(G))$.

Recall that a subset S is called a $\psi\alpha g$-neighbourhood of a point x of X if there exists a $\psi\alpha g$-open set U such that $x \in U \subset S$.

Theorem:3.4 For a function $f: X \to Y$, the following are equivalent :

(i)f is quasi $\psi\alpha g$-open;

(ii)For each subset U of X, $f(\psi\alpha g -$ Int(U)) $\subset Int(f(U))$

(iii)For each $x \in X$ and each $\psi\alpha g$-neighbourhood U of x in X, there exists a neighbourhood $f(U)$ of $f(x)$ in Y such that V$\subset f(U)$.

Proof: $(1) \Rightarrow (2)$: It follows from Theorem 3.2.

$(2) \Rightarrow (3)$: Let $x \in X$ and U be an arbitrary $\psi\alpha g$-neighbourhood U of x in X. Then there exists a $\psi\alpha g$-open set V in X such that $x \in V \subset U$. Then by (ii), we have $f(V) = f(\psi\alpha g\text{-}Int(V)) \subset Int(f(V))$ and hence $f(V) = Int(f(V))$. Therefore, it follows that $f(V)$ is open in Y such that $f(x) \in f(V) \subset f(U)$.

$(3) \Rightarrow (1)$: Let U be an arbitrary $\psi\alpha g$-open set in X. Then for each $y \in f(U)$, by (iii) there exists a neighbourhood V_y of y in Y such that $V_y \subset f(U)$. As V_y is a neighbourhood of y, there exists an open set W_y in Y such that $y \in W_y \subset V_y$. Thus $f(U) = \cup\{W_y: y \in f(U)\}$ which is an open set in Y.This implies that f is quasi $\psi\alpha g$-open function.

Theorem:3.5 A function $f: X \to Y$ is quasi $\psi\alpha g$-open if and only if for any subset B of Y and for any $\psi\alpha g$-closed set F of X containing $f^{-1}(B)$, there exists a closed set G of Y containing B such that $f^{-1}(G) \subset F$.

Proof: Suppose f is $\psi\alpha g$-open. Let $B \subset Y$ and F be a $\psi\alpha g$-closed set of X containing $f^{-1}(B)$. Now put $G = Y - f(X - F)$. It is clear that $f^{-1}(G) \subset F$ implies $B \subset G$. Since f is quasi $\psi\alpha g$-open, we obtain G as a closed set of Y. Moreover, we have $f^{-1}(G) \subset F$.

Conversely, let U be a $\psi\alpha g$-open set of X and put $B = Y\backslash f(U)$. Then $X\backslash U$ is a $\psi\alpha g$-closed set in set in X containing $f^{-1}(B)$. By hypothesis, there exists a closed set F of Y such that $B \subset F$ and $f^{-1}(F) \subset X\backslash U$. Hence, we obtain $f(U) \subset Y\backslash F$. On the other hand, it follows that $B \subset F$, $Y\backslash F \subset Y\backslash B = f(U)$. Thus, we obtain $f(U) = Y\backslash F$ which is open and hence f is a quasi $\psi\alpha g$-open function.

Theorem:3.6 A function $f: X \to Y$ is quasi $\psi\alpha g$-open if and only if $f^{-1}(Cl(B)) \subset \psi\alpha g$ - $Cl(f^{-1}(B))$
for every subset B of Y.

Proof: Suppose that f is quasi $\psi\alpha g$-open. For any subset B of Y, $f^{-1}(B) \subset \psi\alpha g$ - $Cl(f^{-1}(B)\ )$. Therefore by theorem 3.5, there exists a closed set F in Y such that $B \subset F$ and $f^{-1}(F) \subset \psi\alpha g$ - $Cl(f^{-1}(B))$. Therefore, we obtain $f^{-1}(Cl(B)) \subset f^{-1}(F) \subset \psi\alpha g\text{-}Cl(f^{-1}(B))$.

Conversely, let $B \subset Y$ and f be a $\psi\alpha g$-closed of X containing $f^{-1}(B)$. Put $W = Cl_Y(B)$, then we have $B \subset W$ and W is closed and $f^{-1}(W) \subset \psi\alpha g -$ $Cl(f^{-1}(B)) \subset F$.Then by theorem 3.5, f is quasi $\psi\alpha g$-open.

Lemma: 3.7 Let $f: X \to Y$ and $g: Y \to Z$ be two functions and $g \circ f: X \to Z$ is quasi $\psi\alpha g$-open. If g is continuous injective, then f is quasi $\psi\alpha g$-open.

Proof: Let U be a $\psi\alpha g$-open set in X. Then $(g \circ f)(U)$ is open in Z since $g \circ f$ is quasi $\psi\alpha g$-open. Again g is an injective continuous function, $f(U) = f^{-1}(g \circ f)(U)$ is open in Y. This shows that f is quasi $\psi\alpha g$-open.

4. Quasi $\psi\alpha g$-closed functions

Definition:4.1 A function $f: X \to Y$ is said to be quasi $\psi\alpha g$-closed if the image of each $\psi\alpha g$-closed Set in X is closed in Y.
Clearly, every quasi $\psi\alpha g$-closed function is closed as well as $\psi\alpha g$-closed.

Remark: 4.2 Every $\psi\alpha g$-closed (resp. closed) function need not be quasi $\psi\alpha g$-closed as shown by the following example.

Example: 4.3 Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b, c\}\}$. Define a function $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = c$

© 2016 IJAIR. All Rights Reserved

and $f(c) = a$. Then clearly f is $\psi\alpha g$-closed as well as closed but not quasi $\psi\alpha g$-closed.

Theorem: 4.4 Let X and Y be topological spaces. Then the function $g: X \to Y$ is a quasi $\psi\alpha g$-closed if and only if $g(X)$ is closed in Y and $g(V)\backslash g(X\backslash V)$ is open in $g(X)$ whenever V is $\psi\alpha g$-open in X.

Proof: Necessity: Suppose $g: X \to Y$ is a quasi $\psi\alpha g$-closed function. Since X is $\psi\alpha g$-closed, $g(X)$ is closed in Y and $g(V)\backslash g(X\backslash V) = g(V) \cap g(X)\backslash g(X\backslash V)$ is open in $g(X)$ when V is $\psi\alpha g$-open in X.

Sufficiency: Suppose $g(X)$ is closed in Y, $g(V)\backslash g(X\backslash V)$ is open in $g(X)$ when V is $\psi\alpha g$-open in X, and let C be closed in X. Then $g(C) = g(X)\backslash(g(X\backslash C)\backslash g(C))$ is closed in $g(X)$ and hence, closed in Y.

Corollary: 4.5 Let X and Y be topological spaces and let $g: X \to Y$ be a $\psi\alpha g$-continuous quasi $\psi\alpha g$-closed surjective function. Then the topology on Y is $\{g(V)\backslash g(X\backslash V): V$ is $\psi\alpha g$-open in $X\}$.

Proof: Let W be open in Y. Then $g^{-1}(W)$ is $\psi\alpha g$-open in X, and $g(g^{-1}(W)\backslash g(X\backslash g^{-1}(W))) = W$. Hence, all open sets in Y are of the form $g(V)\backslash g(X\backslash V)$, V is $\psi\alpha g$-open in X. On the other hand, all sets of the form $g(V)\backslash g(X\backslash V)$, V is $\psi\alpha g$-open in X, are open in Y.

5. Separation axioms

In this section we introduce and study weak separation axioms such as $\psi\alpha g$-T_0, $\psi\alpha g$-T_1, $\psi\alpha g$-T_2 spaces and obtain some of their properties.

Definition: 5.1 A topological space X is said to be $\psi\alpha g$-T_0 space if for each pair of distinct points x and y of X, there exists a $\psi\alpha g$-open set containing one point but not the other.

Theorem: 5.2 A topological space X is a $\psi\alpha g$-T_0-space if and only if $\psi\alpha g$-closures of distinct points are distinct.

Proof: Let x and y be distinct points of X. Since X is $\psi\alpha g$-T_0- space, there exists a $\psi\alpha g$-open set G such that $x \in G$ and $y \notin G$. Consequently, $X - G$ is a $\psi\alpha g$-closed set containing y but not x. But $\psi\alpha gCl(A)\{y\}$ is the intersection of all $\psi\alpha g$-closed sets containing y. Hence $y \in \psi\alpha gCl\{y\}$
But $x \notin \psi\alpha gCl(A)\{y\}$ as $x \notin X - G$. Therefore, $\psi\alpha gCl(A)\{x\} \neq \psi\alpha gCl(A)\{y\}$.

Conversely, let $\psi\alpha gCl(A)\{x\} \neq \psi\alpha gCl(A)\{y\}$ for $x \neq y$. Then there exists at least one point $z \in X$ such that $z \in \psi\alpha gCl(A)\{x\}$ but $z \notin \psi\alpha gCl(A)\{y\}$. We claim $x \notin \psi\alpha gCl(A)\{y\}$, because if $x \in \psi\alpha gCl(A)\{y\}$ then $\{x\} \subset \psi\alpha gCl(A)\{y\}$ implies $\psi\alpha gCl(A)\{x\} \subset \psi\alpha gCl(A)\{y\}$. So $z \in \psi\alpha gCl(A)\{y\}$, which is a contradiction. Hence $x \notin \psi\alpha gCl(A)\{y\}$, which implies $x \in X - \psi\alpha gCl(A)\{y\}$, which is a $\psi\alpha g$-open set containing x but not y. Hence X is $\psi\alpha g$-T_0- space.

Theorem: 5.3 If $f: X \to Y$ is a bijection strongly $\psi\alpha g$-open and X is $\psi\alpha g$-T_0- space, then Y is also $\psi\alpha g$-T_0- space.

Proof: Let y_1 and y_2 be two distinct points of Y. Since f is bijective there exist distinct points x_1 and x_2 of X such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Since X is $\psi\alpha g$-T_0- space there exists a $\psi\alpha g$-open set G such that $x_1 \in G$ and $x_2 \notin G$. Therefore $y_1 = f(x_1) \in f(G)$ and $y_2 = f(x_2) \notin f(G)$. Since f being strongly $\psi\alpha g$-open function, $f(G)$ is $\psi\alpha g$-open in Y. Thus, there exists a $\psi\alpha g$-open set $f(G)$ in Y such that $y_1 \in f(G)$ and $y_2 \notin f(G)$. Therefore Y is $\psi\alpha g$-T_0- space.

Definition: 5.4 A topological space X is said to be $\psi\alpha g$-T_1-space if for any pair of distinct points x and y, there exists a $\psi\alpha g$-open sets G and H such that $x \in G$, $y \notin G$ and $x \notin H$, $y \in H$.

Theorem: 5.5 A topological space X is $\psi\alpha g$-T_1-space if and only if singletons are $\psi\alpha g$-closed sets.

Proof: Let X be a $\psi\alpha g$-T_1-space and $x \in X$. Let $y \in X - \{x\}$. Then for $x \neq y$, thre exists $\psi\alpha g$-open set U_y such that $y \in U_y$ and $x \notin U_y$. Consequently, $y \in U_y \subset X - \{x\}$. That is $X - \{x\} = \cup \{U_y: y \in X - \{x\}\}$, which is $\psi\alpha g$-open set. Hence $\{x\}$ is $\psi\alpha g$-closed set.

Conversely, suppose $\{x\}$ is $\psi\alpha g$closed set for every $x \in X$. Let x and $y \in X$ with $x \neq y$. Now $x \neq y$ implies $y \in X - \{x\}$. Hence $X - \{x\}$ is $\psi\alpha g$-open set containing y but not x. Similarly, $X - \{y\}$ is $\psi\alpha g$-open set containing x but not y. Therefore X is $\psi\alpha g$-T_1 space.

Theorem: 5.6 The property being $\psi\alpha g$-T_1 space is preserved under bijection and strongly $\psi\alpha g$-open function.

Proof: Let $f: X \to Y$ be bijection and strongly $\psi\alpha g$-open function. Let X be a $\psi\alpha g$-T_1-space and y_1, y_2 be any two distinct points of Y. Since f is bijective there exist distinct points x_1, x_2 of X such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Now X being a $\psi\alpha g$-T_1-space, there exist $\psi\alpha g$-open sets G and H such that $x_1 \in G$, $x_2 \notin G$ and $x_1 \notin H$, $x_2 \in H$. Therefore $y_1 = f(x_1) \in f(G)$ but $y_2 = f(x_2) \notin f(G)$ and $y_2 = f(x_2) \in f(H)$ and $y_1 = f(x_1) \notin f(H)$. Now f being strongly

$\psi\alpha g$-open, $f(G)$ and $f(H)$ are $\psi\alpha g$-open subsets of Y such that $y_1 \in f(G)$ but $y_2 \notin f(G)$ and $y_2 \in f(H)$ and $y_1 \notin f(H)$. Hence Y is $\psi\alpha g$-T_1-space.

Theorem: 5.7 Let $f: X \to Y$ be bijective and $\psi\alpha g$-open function. If X is $\psi\alpha g$-T_1 and $T_{\psi\alpha g}$ -space, then Y is $\psi\alpha g$-T_1-space.

Proof: Let $y_{1,}\ y_2$ be any two distinct points of Y. Since f is bijective there exist distinct points $x_{1,}$ x_2 of X such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Now X being a $\psi\alpha g$-T_1-space, there exist $\psi\alpha g$-open sets G and H such that $x_1 \in G$, $x_2 \notin G$ and $x_1 \notin H$, $x_2 \in H$. Therefore $y_1 = f(x_1) \in f(G)$ but $y_2 = f(x_2) \notin f(G)$ and $y_2 = f(x_2) \in f(H)$ and $y_1 = f(x_1) \notin f(H)$. Now X is $T_{\psi\alpha g}$ -space which implies G and H are open sets in X and f is $\psi\alpha g$-open function, $f(G)$ and $f(H)$ are $\psi\alpha g$-open subsets of Y. Thus there exist $\psi\alpha g$-open sets such that $y_1 \in f(G)$ but $y_2 \notin f(G)$ and $y_2 \in f(H)$ and $y_1 \notin f(H)$. Hence Y is $\psi\alpha g$-T_1-space.

Theorem: 5.8 If $f: X \to Y$ is $\psi\alpha g$ -continuous injection and Y is T_1 then X is $\psi\alpha g$-T_1-space.

Proof: Let $f: X \to Y$ be $\psi\alpha g$-continuous injection and Y is T_1. For any two distinct points $x_{1,}$ x_2 of X there exist distinct points $y_{1,}\ y_2$ of Y such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since Y is T_1-space, there exists open sets U and V in Y such that $y_1 \in U$, $y_2 \notin U$ and $y_1 \notin V$, $y_2 \in V$. That is $x_1 \in f^{-1}(U)$, $x_1 \notin f^{-1}(V)$ and $x_2 \in f^{-1}(V)$, $x_2 \notin f^{-1}(U)$. Since f is $\psi\alpha g$ -continuous $f^{-1}(U), f^{-1}(V)$ are $\psi\alpha g$-open sets in X. Thus, for two distinct points $x_{1,}\ x_2$ of X there exist $\psi\alpha g$-open sets $f^{-1}(U)$ and $f^{-1}(V)$ such that $x_1 \in f^{-1}(U)$, $x_1 \notin f^{-1}(V)$ and $x_2 \in f^{-1}(V)$, $x_2 \notin f^{-1}(U)$. Therefore X is $\psi\alpha g$-T_1space.

Theorem: 5.9 If $f: X \to Y$ is $\psi\alpha g$ -irresolute injective function and Y is $\psi\alpha g$-T_1-space then X is $\psi\alpha g$-T_1 space.

Proof: Let $x_{1,}\ x_2$ be pair of distinct points in X. Since f is injective, there exist distinct points $y_{1,}\ y_2$ of Y such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since Y is $\psi\alpha g$-T_1-space, there exists $\psi\alpha g$-open sets U and V in Y such that $y_1 \in U$, $y_2 \notin U$ and $y_1 \notin V$, $y_2 \in V$. That is, $x_1 \in f^{-1}(U)$, $x_1 \notin f^{-1}(V)$ and $x_2 \in f^{-1}(V)$, $x_2 \notin f^{-1}(U)$. Since f is $\psi\alpha g$- irresolute $f^{-1}(U), f^{-1}(V)$ are $\psi\alpha g$-open sets in X. Thus, for two distinct points $x_{1,}\ x_2$ of X there exist $\psi\alpha g$-open sets $f^{-1}(U)$ and $f^{-1}(V)$ such that $x_1 \in f^{-1}(U)$, $x_1 \notin f^{-1}(V)$ and $x_2 \in f^{-1}(V)$, $x_2 \notin f^{-1}(U)$. Therefore X is $\psi\alpha g$-T_1-space.

Definition: 5.10 A topological space X is said to be $\psi\alpha g$-T_2 space if for any pair of distinct points x and y, there exists disjoint $\psi\alpha g$-open sets G and H such that $x \in G$, and $y \in H$.

Theorem: 5.11 If $f: X \to Y$ is $\psi\alpha g$ -continuous injection and Y is T_2 then X is $\psi\alpha g$-T_2 space.

Proof: Let $f: X \to Y$ be $\psi\alpha g$-continuous injection and Y is T_2. For any two distinct points $x_{1,}\ x_2$ of X there exist distinct points $y_{1,}\ y_2$ of Y such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since Y is T_2-space, there exists distinct open sets U and V in Y such that $y_1 \in U$, $y_2 \in V$. That is $x_{1,} \in f^{-1}(U)$, and $x_2 \in f^{-1}(V)$. Since f is $\psi\alpha g$ -continuous $f^{-1}(U), f^{-1}(V)$ are $\psi\alpha g$-open sets in X. Further f is injective, $f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V) = f^{-1}(\phi) = \phi$. Thus, for two distinct points $x_{1,}$ x_2 of X there exist disjoint $\psi\alpha g$-open sets $f^{-1}(U)$ and $f^{-1}(V)$ such that $x_1 \in f^{-1}(U)$ and $x_2 \in f^{-1}(V)$. Therefore X is $\psi\alpha g$-T_2 -space.

Theorem: 5.12 If $f: X \to Y$ is $\psi\alpha g$ -irresolute injective function and Y is and $\psi\alpha g$-T_2-space then X is $\psi\alpha g$-T_2-space.

Proof: Let $x_{1,}\ x_2$ be pair of distinct points in X. Since f is injective there exist distinct points $y_{1,}\ y_2$ of Y such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since Y is $\psi\alpha g$-T_2-space there exists $\psi\alpha g$-open sets U and V in Y such that $y_1 \in U$, $y_2 \in V$. That is $x_{1,} \in f^{-1}(U)$ and $x_2 \in f^{-1}(V)$. Since f is $\psi\alpha g$ -irresolute injective $f^{-1}(U), f^{-1}(V)$ are $\psi\alpha g$-open sets in X.Thus, for two distinct points $x_{1,}\ x_2$ of X there exist $\psi\alpha g$-open sets $f^{-1}(U)$ and $f^{-1}(V)$ such that $x_1 \in f^{-1}(U)$ and $x_2 \in f^{-1}(V)$. Therefore X is $\psi\alpha g$-T_2-space.

Theorem: 5.13 In any topological space the following are equivalent:

(1) X is $\psi\alpha g$-T_2-space,

(2) For each $x \neq y$, there exists a $\psi\alpha g$-open set U such that $x \in U$ and $y \notin \psi\alpha gCl(U)$,

(3) For each $x \in X$, $\{x\} = \cap \{\psi\alpha gCl(U): U$ is a $\psi\alpha g$-open set in X and $x \in U\}$.

Proof: (1)⇒(2): Assume (1) holds. Let $x \in X$ and $x \neq y$, then there exist disjoint $\psi\alpha g$-open sets U and V such that $x \in U$ and $y \in V$. Clearly, $X - V$ is $\psi\alpha g$-closed set. Since $U \cap V = \phi$, $U \subset X - V$. Therefore $\psi\alpha gCl(U) \subset \psi\alpha gCl(X - V) = X - V$. Now $y \notin X - V$ implies $y \notin \psi\alpha gCl(U)$.

(2)⇒(3): For each $x \neq y$, there exists a $\psi\alpha g$-open set U such that $x \in U$ and $y \notin \psi\alpha gCl(U)$. So $y \notin \cap \{\psi\alpha gCl(U): U$ is a $\psi\alpha g$-open set in X and $x \in U\} = \{x\}$.

© 2016 IJAIR. All Rights Reserved

(3)⇒(1): Let $x, y \in X$ and $x \neq y$. By hypothesis there exists a $\psi\alpha g$-open set U such that $x \in U$ and $\psi\alpha gCl(U$. This implies there exists a $\psi\alpha g$-closed set V such that $y \notin V$. Therefore $y \in X - V$ and $X - V$ is $\psi\alpha g$-open set. Thus, there exist two disjoint $\psi\alpha g$-open sets U and $X - V$ such that $x \in U$ and $y \in X - V$. Therefore X is $\psi\alpha g$-T_2-space.

References

[1] C. Dorsett. Semi-normal spaces. Kyungpook Math. J., 25:173:180, 1985.
[2] N. Levine. Semi-open sets and semi-continuity in topological spaces. Amer. Math. Montly, 70:36:41, 1963.
[3] N. V. Velico. H-closed topological spaces. Amer. Math. Soc. Trans., 78:103:118, 1968.
[4] O. Njasted. On some classes of nearly open sets. Pacific J. Math., 15:961:970, 1986.
[5] O. Ravi, G. Ramkumar and S. Chandrasekar. On Quasi sg-open and Quasi sg-closed functions. Int. J. Diff. Equ. ISSN 0973-6069 Vol 6, Num 1(2011), pp. 11-16.
[6] S. N. Maheshwari and R. Prasad. On s-regular spaces. Glasnik Math., 10(30):347:350, 1975.
[7] S. N. Maheshwari and R. Prasad. On s-normal spaces. Bull. Math. Soc. Sci. R.S. Roumanie, 22(70):27:29, 1978.
[8] S.P. Arya and T. Nour. Characterizations of s-normal spaces. Indian J. Pure. Appl. Math., 8(21):717:719, 1990.
[9] S.S. Benchalli and Umadevi I Neeli. Generalized δ-semi closed sets in topological spaces. Int. J. Appl. Math., 24:21:38, 2011.
[10] S.S. Benchalli and Umadevi I Neeli. On $g\delta s$-Seperation Axioms in topological spaces. J. Adv. Stud. In Topo, Vol. 3, No. 4, 2012, 93-101.
[11] T. Noiri. A generalization of closed mappings. Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur., 54(8):412:415, 1973.
[12] V. Kokilavani and P.R. Kavitha. On $\psi\alpha g$-closed sets in topological spaces.

© 2016 IJAIR. All Rights Reserved