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On the Construction of Balanced Bipartite Block Designs

Bhavika L. Patel
Aroma College of Commerce, Usmanpura, Ahmedabad 380013, INDIA.
Email: blpatel08@gmail.com

Abstract: This paper provides some methods of construction of balanced bipartite block (BBPB) designs which are based on incidence matrices of the known balanced incomplete block (BIB) designs. The designs are useful for comparing a set of test treatments to a set of control treatments. Examples are given for application of the results.

Key Words: BBPB designs, Balanced bipartite block designs with unequal block sizes (BBPBUB), BIB designs.

I. Introduction

This paper deals with the situation where two sets of treatments are compared. One set of v_1 treatments are called test treatments (denoted by $1, 2, \ldots, v_1$) and the other set of $v_2\,(\geq 2)$ treatments are called control treatments (denoted by $v_1 + 1, \ldots, v_1 + v_2 (= v)$). Balanced treatment incomplete block (BTIB) designs have been defined by Bechhofer and Tamhane (1981) for test treatments-control comparison. Angelis and Moyssiadis (1991) have defined balanced treatment incomplete block designs with unequal block sizes (BTIUB) for test treatments-control comparison as a natural extension of BTIB designs. The need for blocks of unequal sizes in biological experiments has been noted by Pearce (1964). Angelis and Moyssiadis (1991) and Angelis, Moyssiadis and Kageyama (1993) have obtained some methods of construction of A-efficient BTIUB designs. Jacroux (1992) has derived some methods of construction of A- and MV-optimal balanced treatment unequal block designs. For comparing test treatments with a control treatment Parsad and Gupta (1994) have obtained the structure of optimal BTIUB designs. For comparing a set of test treatments with a set of control treatments balanced bipartite block (BBPB) designs have been introduced by Kageyama and Sinha (1988) as an extension of BTIB designs.

Definition 1.1: An incomplete block binary design with a set of v_1 treatments occurring r_1 times and another set of v_2 treatments occurring r_2 times ($r_1 \neq r_2$) arranged into b blocks of constant block size k is said to be a BBPB design if
(i) any two distinct treatments in the i^{th} set occur together in λ_{ii} blocks, $i = 1,2$;
(ii) any two treatments from different sets occur together in $\lambda_{12} = \lambda_{21} (> 0)$ blocks.

Majumdar (1986) has given certain sufficient conditions for a block design to be A-optimal for test treatments-control treatments comparison. Kageyama and Sinha (1988) and Sinha and Kageyama (1990) have given some methods of construction of BBPB designs. Parsad, Gupta and Singh (1996) have studied the optimal designs for comparing two sets of treatments. Balanced bipartite block designs with unequal block sizes (BBPBUB) for both binary and non-binary block designs have been defined by Jaggi, Parsad and Gupta (1999) using the definition of the BBPB designs given by Kageyama and Sinha (1988) and the BTIUB designs by Angelis and Moyssiadis (1991). Several researchers have studied more results for comparing a set of test treatments with a set of control treatments (see e.g. Majumdar (1996), Gupta and Parsad (2001) and Jacroux (2000, 2002)).

International Journal of Advanced and Innovative Research (2278-7844) /

Volume 7 Issue 6

We give some methods of constructing BBPBUB designs for comparing test treatments-control treatments comparison by using BIB designs in following section. The definition of BIB design can be seen in Raghavrao (1971).

In what follows, we denote by $\otimes$ the kronecker product of matrices, $\mathbf{1}_p'$ the $1 \times p$ row vector of ones, $\mathbf{1}_p' \otimes N$ the p replications of N, I_p the identity matrix of order p, $J_{p\times q}$ the matrix of ones of order $p \times q$, $O_{p\times q}$ the null matrix of order $p \times q$ and by p_1, p_2, p_3, p_4, p_5 the positive integers.

II. Methods of Construction of BBPBUB Designs

In this section, we describe some methods of construction of BBPBUB designs making use of the incidence matrices of BIB designs, etc.

Theorem 2.1: Let N_L $(L = 1,2,3,4,5)$ be the $v_L \times b_L$ incidence matrix of a BIB design with parameters v_L, b_L, r_L, k_L, λ_L such that $v_2 = v_4$, $v_3 = v_5$ and $v_1 = v_2 + v_3$, then

$$N = \left[\mathbf{1}_{p_1}' \otimes N_1 : \begin{matrix} \mathbf{1}_{p_2}' \otimes N_2 & O_{v_2\times p_3 b_3} & \mathbf{1}_{p_4}' \otimes N_4 \\ O_{v_3\times p_2 b_2} & \mathbf{1}_{p_3}' \otimes N_3 & J_{v_3\times p_4 b_4} \end{matrix} \right.$$

$$\left. \begin{matrix} I_{v_2} \otimes \mathbf{1}_{b_5}' & I_{v_2} & O_{v_2\times v_3} \\ \mathbf{1}_{v_2}' \otimes N_5 & O_{v_3\times v_2} & I_{v_3} \end{matrix} \right] \qquad (2.1)$$

is the incidence matrix of a BBPB design D with unequal block sizes with parameters $v_1^* = v_2$, $v_2^* = v_3$, $b = p_1 b_1 + p_2 b_2 + p_3 b_3 + p_4 b_4 + v_2 b_5 + v_2 + v_3$, $\boldsymbol{r}' = \{(p_1 r_1 + p_2 r_2 + p_4 r_4 + b_5 + 1)\mathbf{1}_{v_2}', (p_1 r_1 + p_3 r_3 + p_4 b_4 + v_2 r_5 + 1)\mathbf{1}_{v_3}'\}$, $\boldsymbol{k}' = \{k_1 \mathbf{1}_{p_1 b_1}', k_2 \mathbf{1}_{p_2 b_2}', k_3 \mathbf{1}_{p_3 b_3}', (k_4 + v_3)\mathbf{1}_{p_4 b_4}', (k_5 + 1)\mathbf{1}_{v_2 b_5}', 1\mathbf{1}_{v_2}', 1\mathbf{1}_{v_3}'\}$ if and only if the positive integers p_1, p_2, p_3, p_4 and p_5 satisfy

$$\frac{p_1 r_1 (k_1 - 1)}{k_1} + \frac{p_2 r_2 (k_2 - 1)}{k_2} + \frac{p_4 r_4 (k_4 + v_3 - 1)}{(k_4 + v_3)} + \frac{b_5 k_5}{(k_5 + 1)}$$

$$- (v_2 - 1)\left\{\frac{p_1 \lambda_1}{k_1} + \frac{p_2 \lambda_2}{k_2} + \frac{p_4 \lambda_4}{(k_4 + v_3)}\right\}$$

$$-v_3 \left\{\frac{p_1 \lambda_1}{k_1} + \frac{p_4 r_4}{(k_4 + v_3)} + \frac{r_5}{(k_5 + 1)}\right\} = 0$$

and

$$\frac{p_1 r_1 (k_1 - 1)}{k_1} + \frac{p_3 r_3 (k_3 - 1)}{k_3} + \frac{p_4 b_4 (k_4 + v_3 - 1)}{(k_4 + v_3)} + \frac{v_2 r_5 k_5}{(k_5 + 1)}$$

$$- (v_3 - 1)\left\{\frac{p_1 \lambda_1}{k_1} + \frac{p_3 \lambda_3}{k_3} + \frac{p_4 b_4}{(k_4 + v_3)} + \frac{v_2 \lambda_5}{(k_5 + 1)}\right\}$$

$$-v_2 \left\{\frac{p_1 \lambda_1}{k_1} + \frac{p_4 r_4}{(k_4 + v_3)} + \frac{r_5}{(k_5 + 1)}\right\} = 0.$$

Proof: For the block design with incidence matrix N given in (2.1) we have

$$C = \begin{bmatrix} (a_1 + s_1) I_{v_2} - s_1 \mathbf{1}_{v_2} \mathbf{1}_{v_2}' & -s_0 \mathbf{1}_{v_2} \mathbf{1}_{v_3}' \\ -s_0 \mathbf{1}_{v_3} \mathbf{1}_{v_2}' & (a_2 + s_2) I_{v_3} - s_2 \mathbf{1}_{v_3} \mathbf{1}_{v_3}' \end{bmatrix}$$

where the off-diagonal elements of $C(= c_{ij})$ matrix are:

$$c_{ij} = \frac{p_1 \lambda_1}{k_1} + \frac{p_2 \lambda_2}{k_2} + \frac{p_4 \lambda_4}{(k_4 + v_3)} = s_1(\text{say}) \qquad ; i, j \le v_2 \ \& \ i \ne j$$

$$c_{ij} = \frac{p_1 \lambda_1}{k_1} + \frac{p_4 r_4}{(k_4 + v_3)} + \frac{r_5}{(k_5 + 1)} = s_0(\text{say}) \qquad ; i \le v_2, j \ge (v_2 + 1)$$

$$c_{ij} = \frac{p_1 \lambda_1}{k_1} + \frac{p_3 \lambda_3}{k_3} + \frac{p_4 b_4}{(k_4 + v_3)} + \frac{v_2 \lambda_5}{(k_5 + 1)} = s_2(\text{say}) \qquad ; i, j \ge (v_2 + 1) \ \& \ i \ne j$$

International Journal of Advanced and Innovative Research (2278-7844) /

Volume 7 Issue 6

and the diagonal elements of C matrix are:

$$\frac{p_1r_1(k_1-1)}{k_1}+\frac{p_2r_2(k_2-1)}{k_2}+\frac{p_4r_4(k_4+v_3-1)}{(k_4+v_3)}$$

$$+\frac{b_5k_5}{(k_5+1)}=a_1(\text{say})$$

and

$$\frac{p_1r_1(k_1-1)}{k_1}+\frac{p_3r_3(k_3-1)}{k_3}+\frac{p_4b_4(k_4+v_3-1)}{(k_4+v_3)}$$

$$+\frac{v_2r_5k_5}{(k_5+1)}=a_2(\text{say})$$

Then by Jaggi, Parsad and Gupta (1999), $a_1-(v_2-1)s_1-v_3s_0=0$ and $a_2-(v_3-1)s_2-v_2s_0=0$ i.e.

$$\frac{p_1r_1(k_1-1)}{k_1}+\frac{p_2r_2(k_2-1)}{k_2}$$

$$+\frac{p_4r_4(k_4+v_3-1)}{(k_4+v_3)}+\frac{b_5k_5}{(k_5+1)}$$

$$-(v_2-1)\left\{\frac{p_1\lambda_1}{k_1}+\frac{p_2\lambda_2}{k_2}+\frac{p_4\lambda_4}{(k_4+v_3)}\right\}$$

$$-v_3\left\{\frac{p_1\lambda_1}{k_1}+\frac{p_4r_4}{(k_4+v_3)}+\frac{r_5}{(k_5+1)}\right\}=0$$

and

$$\frac{p_1r_1(k_1-1)}{k_1}+\frac{p_3r_3(k_3-1)}{k_3}$$

$$+\frac{p_4b_4(k_4+v_3-1)}{(k_4+v_3)}+\frac{v_2r_5k_5}{(k_5+1)}$$

$$-(v_3-1)\left\{\frac{p_1\lambda_1}{k_1}+\frac{p_3\lambda_3}{k_3}+\frac{p_4b_4}{(k_4+v_3)}+\frac{v_2\lambda_5}{(k_5+1)}\right\}$$

$$-v_2\left\{\frac{p_1\lambda_1}{k_1}+\frac{p_4r_4}{(k_4+v_3)}+\frac{r_5}{(k_5+1)}\right\}=0.$$

Hence the proof.

Example 2.1: Consider five BIB designs with parameters (11,11,5,5,2), (7,7,3,3,1), (4,4,3,3,2), (7,7,4,4,2) and (4,6,3,2,1) respectively. Then taking $p_1=p_4=1$, $p_2=2$ and $p_3=3$, the design D with incidence matrix N as in (2.1) is a non-proper non-equireplicate BBPB design with parameters $v_1^*=7$, $v_2^*=4$, $b=97$, $\boldsymbol{r}'=\{22\mathbf{1}_7', 43\mathbf{1}_4'\}$, $\boldsymbol{k}'=\{5\mathbf{1}_{11}', 3\mathbf{1}_{14}', 3\mathbf{1}_{12}', 8\mathbf{1}_7', 3\mathbf{1}_{42}', 1\mathbf{1}_7', 1\mathbf{1}_4'\}$.

Corollary 2.1: In theorem 2.1, if we remove last v_2 and v_3 blocks, then we get a BBPB design D with unequal block sizes with parameters $v_1^*=v_2$, $v_2^*=v_3$, $b=p_1b_1+p_2b_2+p_3b_3+p_4b_4+v_2b_5$, $\boldsymbol{r}'=\{(p_1r_1+p_2r_2+p_4r_4+b_5)\mathbf{1}_{v_2}', (p_1r_1+p_3r_3+p_4b_4+v_2r_5)\mathbf{1}_{v_3}'\}$, $\boldsymbol{k}'=\{k_1\mathbf{1}_{p_1b_1}', k_2\mathbf{1}_{p_2b_2}', k_3\mathbf{1}_{p_3b_3}', (k_4+v_3)\mathbf{1}_{p_4b_4}', (k_5+1)\mathbf{1}_{v_2b_5}'\}$.

Example 2.2: In example 2.1, if we remove last v_2 and v_3 blocks, then we get a non-proper non-equireplicate BBPB design D with $p_1=p_4=1$, $p_2=2$ and $p_3=3$. The parameters of the design are $v_1^*=7$, $v_2^*=4$, $b=86$, $\boldsymbol{r}'=\{21\mathbf{1}_7', 42\mathbf{1}_4'\}$, $\boldsymbol{k}'=\{5\mathbf{1}_{11}', 3\mathbf{1}_{14}', 3\mathbf{1}_{12}', 8\mathbf{1}_7', 3\mathbf{1}_{42}'\}$.

Remark: Following theorems can be proved on the similar lines of theorem 2.1. So we avoided proofs of the theorems.

Theorem 2.2: Let N_L (L = 1,2,3,4,5) be the $v_L\times b_L$ incidence matrix of a BIB design with parameters v_L, b_L, r_L, k_L, λ_L such that $v_2=v_4$, $v_3=v_5$ and $v_1=v_2+v_3$, then

$$N=\left[\mathbf{1}_{p_1}'\otimes N_1:\begin{matrix}\mathbf{1}_{p_2}'\otimes N_2 & O_{v_2\times p_3b_3} & \mathbf{1}_{p_4}'\otimes N_4 & I_{v_2}\otimes\mathbf{1}_{b_5}' & I_{v_2} & O_{v_2\times v_3}\\ O_{v_3\times p_2b_2} & \mathbf{1}_{p_3}'\otimes N_3 & O_{v_3\times p_4b_4} & \mathbf{1}_{v_2}'\otimes N_5 & O_{v_3\times v_2} & I_{v_3}\end{matrix}\right] \qquad (2.2)$$

is the incidence matrix of a BBPB design D with unequal block sizes with parameters $v_1^*=v_2$, $v_2^*=v_3$, $b=p_1b_1+p_2b_2+p_3b_3+p_4b_4+v_2b_5+v_2+v_3$, $\boldsymbol{r}'=\{(p_1r_1+p_2r_2+p_4r_4+b_5+1)\mathbf{1}_{v_2}', (p_1r_1+p_3r_3+v_2r_5+1)\mathbf{1}_{v_3}'\}$, $\boldsymbol{k}'=\{k_1\mathbf{1}_{p_1b_1}',$

$k_2\mathbf{1}'_{p_2b_2}, k_3\mathbf{1}'_{p_3b_3}, k_4\mathbf{1}'_{p_4b_4}, (k_5+1)\mathbf{1}'_{v_2b_5}, 1\mathbf{1}'_{v_2}, 1\mathbf{1}'_{v_3}\}$ having off-diagonal elements of its C matrix as

$$s_1 = \frac{p_1\lambda_1}{k_1} + \frac{p_2\lambda_2}{k_2} + \frac{p_4\lambda_4}{k_4},$$

$$s_0 = \frac{p_1\lambda_1}{k_1} + \frac{r_5}{(k_5+1)},$$

$$s_2 = \frac{p_1\lambda_1}{k_1} + \frac{p_3\lambda_3}{k_3} + \frac{v_2\lambda_5}{(k_5+1)}$$

and diagonal elements of C matrix as

$$a_1 = \frac{p_1r_1(k_1-1)}{k_1} + \frac{p_2r_2(k_2-1)}{k_2} + \frac{p_4r_4(k_4-1)}{k_4} + \frac{b_5k_5}{(k_5+1)},$$

$$a_2 = \frac{p_1r_1(k_1-1)}{k_1} + \frac{p_3r_3(k_3-1)}{k_3} + \frac{v_2r_5k_5}{(k_5+1)}.$$

Example 2.3: Consider five BIB designs with parameters (11,11,5,5,2), (6,6,5,5,4), (5,10,4,2,1), (6,15,5,2,1) and (5,5,4,4,3) respectively. Then taking $p_1 = p_2 = p_3 = p_4 = 1$, the design D with incidence matrix N as in (2.2) is a non-proper non-equireplicate BBPB design with parameters $v_1^* = 6$, $v_2^* = 5$, $b = 83$, $\boldsymbol{r}' = \{21\mathbf{1}_6', 34\mathbf{1}_5'\}$, $\boldsymbol{k}' = \{5\mathbf{1}_{11}', 5\mathbf{1}_6', 2\mathbf{1}_{10}', 2\mathbf{1}_{15}', 5\mathbf{1}_{30}', 1\mathbf{1}_6', 1\mathbf{1}_5'\}$.

Corollary 2.2: In theorem 2.2, if we remove last v_2 and v_3 blocks, then we get a BBPB design D with unequal block sizes with parameters $v_1^* = v_2$, $v_2^* = v_3$, $b = p_1b_1 + p_2b_2 + p_3b_3 + p_4b_4 + v_2b_5$, $\boldsymbol{r}' = \{(p_1r_1 + p_2r_2 + p_4r_4 + b_5)\mathbf{1}_{v_2}', (p_1r_1 + p_3r_3 + v_2r_5)\mathbf{1}_{v_3}'\}$, $\boldsymbol{k}' = \{k_1\mathbf{1}'_{p_1b_1}, k_2\mathbf{1}'_{p_2b_2}, k_3\mathbf{1}'_{p_3b_3}, k_4\mathbf{1}'_{p_4b_4}, (k_5+1)\mathbf{1}'_{v_2b_5}\}$.

Example 2.4: In example 2.3, if we remove last v_2 and v_3 blocks, then we get a non-proper non-equireplicate BBPB design D with $p_1 = p_2 = p_3 = p_4 = 1$. The parameters of the design are $v_1^* = 6$, $v_2^* = 5$, $b = 72$, $\boldsymbol{r}' = \{20\mathbf{1}_6', 33\mathbf{1}_5'\}$, $\boldsymbol{k}' = \{5\mathbf{1}_{11}', 5\mathbf{1}_6', 2\mathbf{1}_{10}', 2\mathbf{1}_{15}', 5\mathbf{1}_{30}'\}$.

Theorem 2.3: Let N_L (L = 1,2,3,4,5) be the $v_L \times b_L$ incidence matrix of a BIB design with parameters v_L, b_L, r_L, k_L, λ_L such that $v_2 = v_4$, $v_3 = v_5$ and $v_1 = v_2 + v_3$, then

$$N = \begin{bmatrix} \mathbf{1}_{p_1}' \otimes N_1 : & \begin{matrix} \mathbf{1}_{p_2}' \otimes N_2 & O_{v_2\times p_3b_3} & \mathbf{1}_{p_4}' \otimes N_4 \\ O_{v_3\times p_2b_2} & \mathbf{1}_{p_3}' \otimes N_3 & O_{v_3\times p_4b_4} \end{matrix} \\ & \begin{matrix} J_{v_2\times p_5b_5} & I_{v_2} & O_{v_2\times v_3} \\ \mathbf{1}_{p_5}' \otimes N_5 & O_{v_3\times v_2} & I_{v_3} \end{matrix} \end{bmatrix} \quad (2.3)$$

is the incidence matrix of a BBPB design D with unequal block sizes with parameters $v_1^* = v_2$, $v_2^* = v_3$, $b = p_1b_1 + p_2b_2 + p_3b_3 + p_4b_4 + p_5b_5 + v_2 + v_3$, $\boldsymbol{r}' = \{(p_1r_1 + p_2r_2 + p_4r_4 + p_5b_5 + 1)\mathbf{1}_{v_2}', (p_1r_1 + p_3r_3 + p_5r_5 + 1)\mathbf{1}_{v_3}'\}$, $\boldsymbol{k}' = \{k_1\mathbf{1}'_{p_1b_1}, k_2\mathbf{1}'_{p_2b_2}, k_3\mathbf{1}'_{p_3b_3}, k_4\mathbf{1}'_{p_4b_4}, (k_5+v_2)\mathbf{1}'_{p_5b_5}, 1\mathbf{1}'_{v_2}, 1\mathbf{1}'_{v_3}\}$ having off-diagonal elements of its C matrix as

$$s_1 = \frac{p_1\lambda_1}{k_1} + \frac{p_2\lambda_2}{k_2} + \frac{p_4\lambda_4}{k_4} + \frac{p_5b_5}{(k_5+v_2)},$$

$$s_0 = \frac{p_1\lambda_1}{k_1} + \frac{p_5r_5}{(k_5+v_2)},$$

$$s_2 = \frac{p_1\lambda_1}{k_1} + \frac{p_3\lambda_3}{k_3} + \frac{p_5\lambda_5}{(k_5+v_2)}$$

and diagonal elements of C matrix as

$$a_1 = \frac{p_1r_1(k_1-1)}{k_1} + \frac{p_2r_2(k_2-1)}{k_2} + \frac{p_4r_4(k_4-1)}{k_4} + \frac{p_5b_5(k_5+v_2-1)}{(k_5+v_2)},$$

$$a_2 = \frac{p_1r_1(k_1-1)}{k_1} + \frac{p_3r_3(k_3-1)}{k_3} + \frac{p_5r_5(k_5+v_2-1)}{(k_5+v_2)}.$$

Example 2.5: Consider five BIB designs with parameters (9,12,4,3,1), (5,10,4,2,1), (4,4,3,3,2), (5,5,4,4,3) and (4,6,3,2,1) respectively. Then taking $p_1 = 1$, $p_2 = 2$, $p_3 = p_4 = 3$ and $p_5 = 4$, the design D with incidence matrix N as in (2.3) is a non-proper non-equireplicate BBPB design with parameters $v_1^* = 5$, $v_2^* = 4$, $b = 92$, $\boldsymbol{r}' = \{49\mathbf{1}_5', 26\mathbf{1}_4'\}$, $\boldsymbol{k}' = \{3\mathbf{1}_{12}', 2\mathbf{1}_{20}', 3\mathbf{1}_{12}', 4\mathbf{1}_{15}', 7\mathbf{1}_{24}', 1\mathbf{1}_5', 1\mathbf{1}_4'\}$.

Corollary 2.3: In theorem 2.3, if we remove last v_2 and v_3 blocks, then we get a BBPB design D with unequal block sizes with parameters $v_1^* = v_2$, $v_2^* = v_3$, $b = p_1b_1 + p_2b_2 + p_3b_3 + p_4b_4 + p_5b_5$, $\boldsymbol{r}' = \{(p_1r_1 + p_2r_2 + p_4r_4 + p_5b_5)\mathbf{1}_{v_2}', (p_1r_1 + p_3r_3 + p_5r_5)\mathbf{1}_{v_3}'\}$, $\boldsymbol{k}' = \{k_1\mathbf{1}_{p_1b_1}', k_2\mathbf{1}_{p_2b_2}', k_3\mathbf{1}_{p_3b_3}', k_4\mathbf{1}_{p_4b_4}', (k_5 + v_2)\mathbf{1}_{p_5b_5}'\}$.

Example 2.6: In example 2.5, if we remove last v_2 and v_3 blocks, then we get a non-proper non-equireplicate BBPB design D with $p_1 = 1$, $p_2 = 2$, $p_3 = p_4 = 3$ and $p_5 = 4$. The parameters of the design are $v_1^* = 5$, $v_2^* = 4$, $b = 83$, $\boldsymbol{r}' = \{48\mathbf{1}_5', 25\mathbf{1}_4'\}$, $\boldsymbol{k}' = \{3\mathbf{1}_{12}', 2\mathbf{1}_{20}', 3\mathbf{1}_{12}', 4\mathbf{1}_{15}', 7\mathbf{1}_{24}'\}$.

Theorem 2.4: Let N_L (L = 1,2,3,4,5) be the $v_L \times b_L$ incidence matrix of a BIB design with parameters v_L, b_L, r_L, k_L, λ_L such that $v_2 = v_4$, $v_3 = v_5$ and $v_1 = v_2 + v_3$, then

$$N = \left[\mathbf{1}_{p_1}' \otimes N_1 : \begin{matrix} \mathbf{1}_{p_2}' \otimes N_2 & O_{v_2 \times p_3b_3} & \mathbf{1}_{p_4}' \otimes N_4 \\ O_{v_3 \times p_2b_2} & \mathbf{1}_{p_3}' \otimes N_3 & O_{v_3 \times p_4b_4} \end{matrix} \right.$$
$$\left. \begin{matrix} O_{v_2 \times p_5b_5} & I_{v_2} & O_{v_2 \times v_3} \\ \mathbf{1}_{p_5}' \otimes N_5 & O_{v_3 \times v_2} & I_{v_3} \end{matrix} \right] \quad (2.4)$$

is the incidence matrix of a BBPB design D with unequal block sizes with parameters $v_1^* = v_2$, $v_2^* = v_3$, $b = p_1b_1 + p_2b_2 + p_3b_3 + p_4b_4 + p_5b_5 + v_2 + v_3$, $\boldsymbol{r}' = \{(p_1r_1 + p_2r_2 + p_4r_4 + 1)\mathbf{1}_{v_2}', (p_1r_1 + p_3r_3 + p_5r_5 + 1)\mathbf{1}_{v_3}'\}$, $\boldsymbol{k}' = \{k_1\mathbf{1}_{p_1b_1}', k_2\mathbf{1}_{p_2b_2}', k_3\mathbf{1}_{p_3b_3}', k_4\mathbf{1}_{p_4b_4}', k_5\mathbf{1}_{p_5b_5}', 1\mathbf{1}_{v_2}', 1\mathbf{1}_{v_3}'\}$ having off-diagonal elements of its C matrix as

$$s_1 = \frac{p_1\lambda_1}{k_1} + \frac{p_2\lambda_2}{k_2} + \frac{p_4\lambda_4}{k_4},$$

$$s_0 = \frac{p_1\lambda_1}{k_1},$$

$$s_2 = \frac{p_1\lambda_1}{k_1} + \frac{p_3\lambda_3}{k_3} + \frac{p_5\lambda_5}{k_5}$$

and diagonal elements of C matrix as

$$a_1 = \frac{p_1r_1(k_1-1)}{k_1} + \frac{p_2r_2(k_2-1)}{k_2} + \frac{p_4r_4(k_4-1)}{k_4},$$

$$a_2 = \frac{p_1r_1(k_1-1)}{k_1} + \frac{p_3r_3(k_3-1)}{k_3} + \frac{p_5r_5(k_5-1)}{k_5}.$$

Example 2.7: Let $N_L (L = 1, 2, 3,4,5)$ be the incidence matrix of five BIB designs with parameters (11,11,5,5,2), (6,15,5,2,1), (5,5,4,4,3), (6,6,5,5,4) and (5,10,4,2,1) respectively. Then taking $p_1 = p_2 = p_3 = p_4 = p_5 = 1$, the design D with incidence matrix N as in (2.4) is a non-proper non-equireplicate BBPB design with parameters $v_1^* = 6$, $v_2^* = 5$, $b = 58$, $\boldsymbol{r}' = \{16\mathbf{1}_6', 14\mathbf{1}_5'\}$, $\boldsymbol{k}' = \{5\mathbf{1}_{11}', 2\mathbf{1}_{15}', 4\mathbf{1}_5', 5\mathbf{1}_6', 2\mathbf{1}_{10}', 1\mathbf{1}_6', 1\mathbf{1}_5'\}$.

Corollary 2.4: In theorem 2.4, if we remove last v_2 and v_3 blocks, then we get a BBPB design D with unequal block sizes with parameters $v_1^* = v_2$, $v_2^* = v_3$, $b = p_1b_1 + p_2b_2 + p_3b_3 + p_4b_4 + p_5b_5$, $\boldsymbol{r}' = \{(p_1r_1 + p_2r_2 + p_4r_4)\mathbf{1}_{v_2}', (p_1r_1 + p_3r_3 + p_5r_5)\mathbf{1}_{v_3}'\}$, $\boldsymbol{k}' = \{k_1\mathbf{1}_{p_1b_1}', k_2\mathbf{1}_{p_2b_2}', k_3\mathbf{1}_{p_3b_3}', k_4\mathbf{1}_{p_4b_4}', k_5\mathbf{1}_{p_5b_5}'\}$.

Example 2.8: In example 2.7, if we remove last v_2 and v_3 blocks, then we get a non-proper non-equireplicate BBPB design D with $p_1 = p_2 = p_3 = p_4 = p_5 = 1$. The parameters of the design are $v_1^* = 6$, $v_2^* = 5$, $b = 47$, $\boldsymbol{r}' = \{15\mathbf{1}_6', 13\mathbf{1}_5'\}$, $\boldsymbol{k}' = \{5\mathbf{1}_{11}', 2\mathbf{1}_{15}', 4\mathbf{1}_5', 5\mathbf{1}_6', 2\mathbf{1}_{10}'\}$.

III. Conclusion

For comparing test-control treatments a number of balanced bipartite block designs with unequal block sizes obtained by the new methods of construction given here. Such methods are flexible enough to incorporate number of incidence matrices of BIB designs. The obtained designs are found to have applications in agricultural, pharmaceutical and industrial experiments.

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