Modified Gaussian Noise De-Noising using UDWT / DWT

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***Abstract*—In this work we have proposed an image De-noising method of noisy image using Undecimated Discrete Wavelet Transform (UDWT). The proposed method based on the concept of wavelet thresholding by using Undecimated wavelet transform. The performance of image de-noising of noisy image is shown in terms of Peak Signal to Noise Ratio (PSNR), Mean Square Error (MSE)& is compared with Discrete Wavelet Transform (DWT). The performance of calculated result shows improvement in terms of Mean Square Error and Peak Signal to Noise Ratio. Experimental results on several test images like 'Lena' by using proposed method shows that this method yields significantly superior image quality and better Peak Signal to Noise Ratio (PSNR). Here, to prove the efficiency of this method in image denoising, we have compared this with various denoising methods like wiener filter, Average filter, VisuShrink and BayesShrink, HMT etc.**

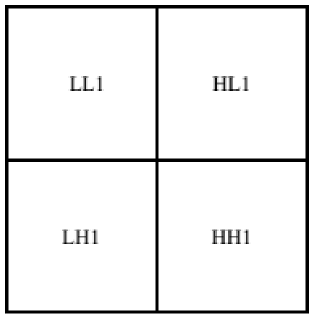
***Keywords*— Undecimated Discrete Wavelet Transform (UDWT), Discrete Wavelet Transform (DWT), Gaussian Noise, Image Denoising, Filter Banks and Thresholding.**

I. INTRODUCTION

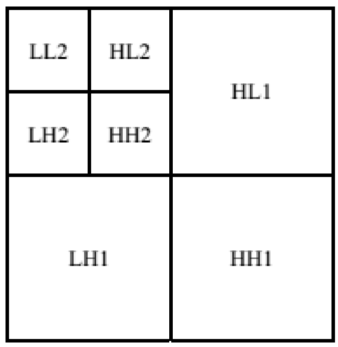
An image is often corrupted by noise in its acquisition and transmission. For example during the image acquisition, the performance of imaging sensors is affected by a variety of factors, such as environmental conditions and by the quality of the sensing elements themselves. For instance, in acquiring images with a CCD camera, light levels and sensor temperature are major factors affecting the amount of noise in the resulting image. Images are also corrupted during transmission, due to interference in the channel used for transmission. Image denoising techniques are necessary to remove such random additive noises while retaining as much as possible the important signal features. The main objective of these types of random noise removal is to suppress the noise while preserving the original image details. Statistical filters like Average filter [1] [2], Wiener filter [3] can be used for removing such noises but the wavelet based denoising techniques proved better results than these filters. In general, image de-noising imposes a compromise between noise reduction and preserving significant image details. To achieve a good performance in this respect, a denoising algorithm has to adapt to image discontinuities. The wavelet representation naturally facilitates the construction of such spatially adaptive algorithms. It compresses essential information in a signal into relatively few, large coefficients, which represent image details at different resolution scales. In recent years there has been a fair amount of research on wavelet thresholding and threshold selection for signal and image denoising [4] [5] [6] [7] [8] [9], because wavelet provides an appropriate basis for separating noisy signal from image signal. Many wavelet based thresholding techniques like VisuShrink [10], BayesShrink [11] have proved better efficiency in image denoising. We describe here an efficient thresholding technique for denoising by analyzing the statistical parameters of the wavelet coefficients.

II. DISCRETE WAVELET TRANSFORM

The DWT is identical to a hierarchical subband system where the subbands are logarithmically spaced in frequency and represent octave-band decomposition. Due to the decomposition of an image using the DWT [12] the original image is transformed into four pieces which is normally labeled as LL, LH, HL and HH as in the schematic depicted in Fig.1(a). The LL subband can be further decomposed into four subbands labeled as LL2, LH2, HL2 and HH2 as shown in
Fig.1(b).



(a) One-Level



(b) Two-Level
Fig. 1 Image decomposition by using DWT

The LL piece comes from low pass filtering in both directions and it is the most like original picture and so is called the approximation. The remaining pieces are called detailed components. The HL comes from low pass filtering in the vertical direction and high pass filtering in the horizontal direction and so has the label HL. The visible detail in the sub-image, such as edges, have an overall vertical orientation since their alignment is perpendicular to the direction, of the high pass filtering and they are called vertical details. The remaining components have analogous explanations. The filters LD and HD shown in Fig. 2 are one-dimensional Low Pass Filter (LPF) and High Pass Filter (HPF) respectively for image decomposition. To obtain the next level of decomposition, sub band LL1 alone is further decomposed. This process continues until some final scale is reached. The decomposed image can be reconstructed using a reconstruction filter as shown in Fig. 3. Here, the filters LR and HR represent low pass and high pass reconstruction filters respectively. Here, since the image size is not changed after decomposition this DWT is called critically sampled transform without having any redundancy.

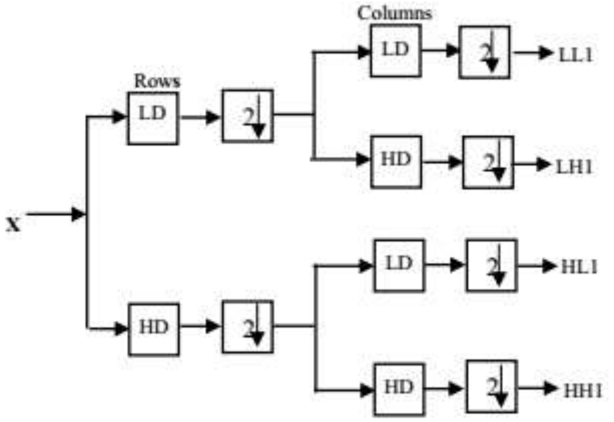


Fig. 2 Wavelet Filter bank for one-level image decomposition

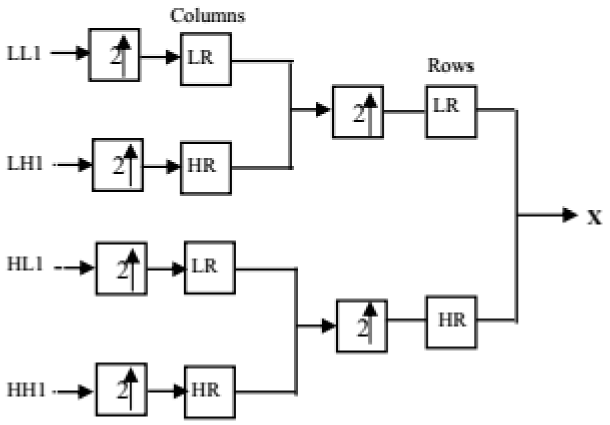


Fig. 3 Wavelet Filter bank for one-level image Reconstruction

An image is often corrupted by noise during its acquisition or transmission. The de-noising process is to remove the noise while retaining and not distorting the quality of the processed image .The traditional way of image de-noising is filtering. Recently, a lot of research about non-linear methods of signal de-noising has been developed. These methods are mainly based on thresholding the Discrete Wavelet Transform (DWT) coefficients, which have been affected by additive white Gaussian noise. Simple denoising algorithms that use DWT consist of three steps.

- Discrete wavelet transform is adopted to decompose the noisy image and get the wavelet coefficients.
- These wavelet coefficients are denoised with wavelet threshold.
- Inverse transform is applied to the modified coefficients and get denoised image.

The second step, known as thresholding, is a simple nonlinear technique, which operates on one wavelet coefficient at a time. In its most basic form, each coefficient is thresholded by comparing threshold, if the coefficient is smaller than threshold, set to zero; otherwise it kept as it is or it is modified. Replacing the small noisycoefficient by zero and inverse wavelet transform on the resulted coefficient may lead to reconstruction with the essential signal characteristics and with less noise.

During the last decade, a lot of new methods based on wavelet transforms have emerged for removing Gaussian random noise from images. The denoisingv process is known as wavelet shrinkage or thresholding. Both VisuShrink and SureShrink are the best known methods of wavelet shrinkage proposed by Donoho and Johnstone.

For VisuShrink, the wavelet coefficients w of the noisy signal are obtained first. Then with the universal threshold T (is the noise level and N is the length of the noisy signal), the coefficients are shrinked according to the softshrinkage rule is used to estimate the noiseless coefficients. Finally, the estimated noiseless signal is reconstructed from the estimated coefficients. VisuShrink is very simple, but its disadvantage is to yield overly smoothed images because the universal threshold T is too large.

Just like VisuShrink, SureShrink also applies the soft shrinkage rule, but it uses independently chosen thresholds for each subband through the minimization of the Stein's unbiased risk estimate (SURE) (Stein, 1981). VisuShrink performs better than SureShrink, producing more detailed images.

III. UN-DECIMATED WAVELET TRANSFORM (UDWT)

UDWT is based on the idea of no decimation. It applies the wavelet transform & omits both down-sampling in the forward & up-sampling in the inverse transform. More precisely, at each point of the image the transform is applied & the detail coefficients are saved & uses the low-frequency coefficients for the next level. The coefficients array size does not diminish from level to level. By using all coefficients at each level, we get very well allocated high-frequency information. From level to level there is very small step in the width of the scaling filter - instead of 8 pixels at the third level of DWT; here its width is 5 pixels. Generally, the step is not a power of 2 but a sum with 2. This property is good for noise removal because the noise is usually spread over small number of neighboring pixels. With this transform the number of pixels involved in computing a given coefficient grows slower & so the relation between the frequency & spatial information is more precise. In the ideal case, this means removal of the noise only at the places that it really exists, without affecting the neighboring pixels. It gives the best outputs in terms of visual quality (less blurring for larger noise removal).

UDWT VS DWT

The discrete wavelet transform is very efficient from the computational point of view. Its only drawback is that it is not translation invariant. Translations of the original signal lead to different wavelet coefficients. In order to overcome this & to get more complete characteristic of the analyzed signal the Undecimated Wavelet Transform (UDWT) was proposed. The general idea behind it is that it doesn't decimate the signal. Thus it produces more precise information for the frequency localization. From the computational point of view the Undecimated Wavelet Transform (UDWT) has larger storage space requirements & involves more computations.

IV. WAVELET THRESHOLDING

The first step in the denoising process is to obtain the wavelet transform of the signal x (n) using a suitable basis function. Then, a threshold is obtained using one of the above thresholding techniques [5]. Figure 4 shows the nature of thresholding.

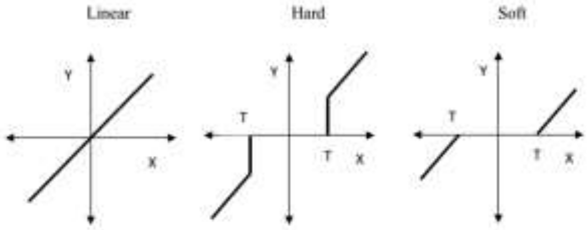


Figure 4: Hard and Soft Thresholding functions

The hard thresholding zeroes out, or shrinks the coefficients that have magnitudes below the threshold, and leaves the rest of the coefficients unchanged Soft thresholding extends hard thresholding by shrinking the magnitude of the remaining coefficients by , producing a smooth rather than abrupt transition to zero. The smooth transition to zero results in noticeably fewer artifacts upon reconstruction, especially when dealing with image denoising. Hence, soft thresholding is generally better for denoising due to its inherent smoothing, whereas hard thresholding is better suited for data compression. In either case, perfect reconstruction is not possible since some of the signal components are thrown away with the undesired noise. Furthermore, any thresholding technique other than the universal threshold will preserve some of the noise-only coefficients. Some significant research has been done using wavelet based de-noising.

The hard-thresholding T_H can be defined as:

$$T_H = \begin{cases} x, & |x| \geq t \\ 0, & inother \end{cases}$$

Here t is the threshold value. A plot of T_H is shown in figure 5;

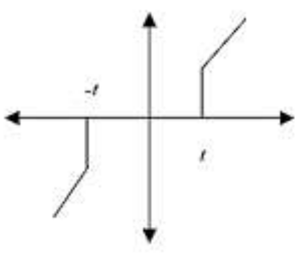


Figure 5: Hard Thresholding

Thus, all coefficients whose magnitude is greater than the selected threshold value *t* remain as they are and the others with magnitudes smaller than *t* are set to zero. It creates a region around zero where the coefficients are considered negligible. Soft thresholding is where the coefficients with greater than the threshold are shrunk towards zero after comparing them to a threshold value. It is defined as follows.

$$T_s = \begin{cases} sign\,(x)(|x| - t), & |x| > t \\ 0, & otherwise \end{cases}$$

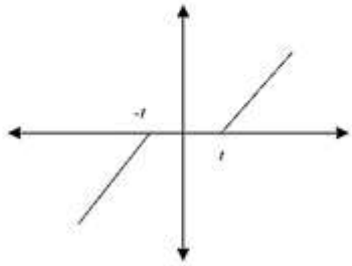


Figure 6: Soft Thresholding

In general, it is observed that the hard thresholding technique is much better than soft thresholding and yields more visually pleasant images. This is because the soft thresholding technique is discontinuous and yields abrupt artifacts in the recovered images. Also, the hard thresholding technique yields a smaller minimum mean squared error compared to hard form of thresholding.

V. IMAGE DENOISING ALGORITHM

Modified Gaussian Noise De-Noising using UDWT / DWT: This section contains the stepwise, detailed methodology that is followed while denoising images using un-decimated wavelet transforms& discrete wavelet transforms. For better and easy understanding, a complete flowchart of the discussed methodology has been shown at the end of this chapter. The proposed algorithm steps are as follows:

Step 1:

Read the test image (original).

Step 2:

Resize the test image and convert it into Gray scale image. The images taken for rectification have a lot of variation in their sizes and hence cannot be compared on the same basis. For large sized images, such as 1024×1024 the computation time for denoising is found to be more difficult and if the image size is taken smaller.

Step 3:

Noise is added to the standard test images. In this work AWGN is added for generation of noisy image. The gaussian noise adds normal distributed noise to the original image. Main feature of this noise, it is independent of the image on which it is going to be applied. The pixel value altered by the additive Gaussian noise can be shown as:

$$J(k, l) = x(k, l) + n$$

Where n is the noise, $n \sim N(0, v)$, being distributed normally with variance v.

Step 4:

Make the noisy image to undergo un-decimated wavelet transform, UDWT& DWT.

- In general, linear approximation systems are often sub-optimal, due mainly to the functional complexity involved in any cases. Thus, instead of following the rule of selecting N approximating terms, it is preferable to adhere to adaptive criteria and nonlinear schemes like wavelet transform.
- A well-known orthogonal basis expansion is obtained by discrete wavelet transformWT^d, by which a map $f \to w$ is

implemented via a bank of quadrature mirror filters by $w = W^d f$ and co-efficient at high/low scale (with high and low frequency content, respectively) are obtained. If an orthogonal wavelet basis is used, such as daublets, symlets or coiflets, then

$$y = f + \xi$$

Become transformed in:

$$W^d y = W^d f + W^d \xi \equiv g + \eta$$

This transformation preserves Gaussianity (as from the noise ξ) and produces decorrelation for auto-correlated systems.

- An extension of the above is non-orthogonal non-decimated wavelet transform WT^u , i.e. a conservative transform for which the expansion coefficients are not eliminated while obtaining them resolution-wise, unlike with transforms where the decimation occurs when changing scale. It is characterized by a matrix W^u of size $\bar{N} \times N$, for $\bar{N} \geq N$ and a redundant system is found, together with a pseudo-inverse transform W^{u-}, such that $W^{u-}W = I$
  Now for $y = f + z.WT^u$ decomposed as:
  $$W^U y = W^U f + W^u z \equiv h + \epsilon$$
  While the Gaussian property is still preserved.
- There are various wavelet families that can be used to approximate many types of functions that when transformed assume a sparser or simplified structure. Wavelets refer to a set of functions generated by dilation and translation of a compactly supported scaling function (or father wavelet) and a mother wavelet, $\phi and \psi$, respectively associated with a multi-resolution analysis of $L_2(R)$. Multi-resolution technique represent both adaptive and time-frequency localized solutions, deal with non-linear complex dynamics and non-stationary systems, and have strong computational and theoretical motivation. With WT^d a sequence of smoothed signals and of details, giving information at finer resolution levels, is found and may be used to represent a signal expansion:
  $$f(x) = \sum_k c_{j_0 k} \phi_{j_0 k}(x) + \sum_{j > j_0} \sum_k d_{jk} \psi_{jk}(x)$$
  Where $\phi_{j_0 k}$ is associated with the corresponding coarse resolution coefficients $c_{j_0 k}$&d_{jk} are the details coefficient given as:
  $$c_{jk} = \int f(x) \phi_{jk}(x) dx$$
  $$d_{jk} = \int f(x) \psi_{jk}(x) dx$$

Step 5:

After the noisy image is decomposed into approximation and detail coefficients using wavelet transform, it is made to undergo the following thresholding rules having various threshold values. In addition, two cases have been considered- one where the low pass components are not thresholded and the other being the one where the low pass components have been thresholded. The thresholding techniques applied are as follows,

- **Soft thresholding-** Refers to the procedure where firstly the input elements with absolute value lower than the defined threshold value, are set to zero and are then scaled to the non-zero coefficients toward zero. It eliminates discontinuity and gives more visually pleasant images.
  $x = abs(y)$
  $$x = sign(y).*(x \geq thld).*(x - thld)$$
  Where y is the input, thld is the threshold value and x is the thresholded output.

- **Hard thresholding-** refers to the procedure where the input elements with absolute value lower than the defined threshold value, are set to zero. It is discontinuous at the point where $|x| = thld$ and yields abrupt artifacts in the recovered images especially when the noise energy is significant.
  $x = (abs(y) > thld).*y$

Step 6:

After the decomposed image coefficients are thresholded using the above mentioned three threshold values with each of the thresholding technique, the denoised image is reconstructed using inverse un-decimated wavelet transform IUDWT & IDWT.

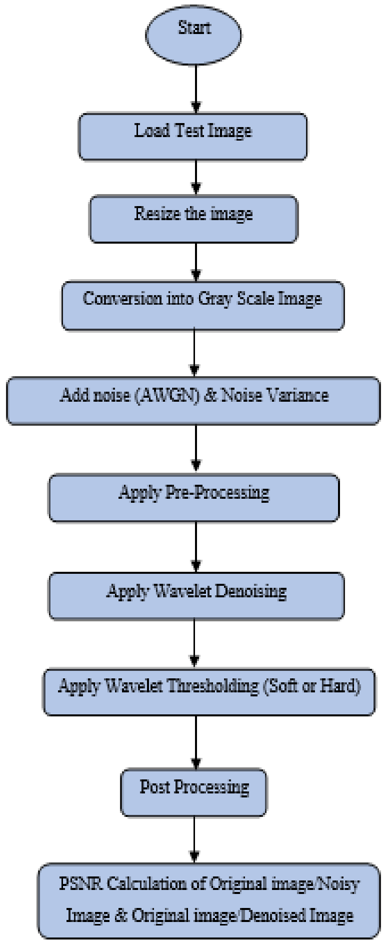


Figure 7: Flowchart of Proposed UDWT/DWT Image Denoising

Step 7:

PSNR is calculated for all the standard images with their noisy and denoised image counterparts, respectively. Hence I get good amount of comparison between the noisy and denoised image keeping the set standard image intact.

- PSNR- PSNR stands for the peak signal to noise ratio. It is a term used to calculate the ratio of the maximum power of a test signal and the power of noise corrupted version of the test signal. Since most of the signals have a large dynamic range, PSNR is generally represented in terms of the logarithmic decibel (dB) scale. It is most commonly used as a measure of quality of reconstruction in image compression etc. It is calculated as the following:

$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}\|I(i,j) - K(i,j)\|^2$$

$$PSNR = 10.* \log_{10}\left(\frac{MSE_I^2}{MSE}\right)$$

Where I and K are the original and noisy / denoised image, respectively. MAX_I is the maximum pixel value of the image under test. For an image having 8 bits per sample, pixels representation, this is equivalent to $2^8 = 255$.

At one time, we calculate PSNR for original with noisy image and refer it as PSNR (O/N). After the image is denoised, it is calculated for original with denoised image and is then referred as PSNR (O/D). Hence, it shows the improvement in the noisy image after denoising, if any.

VI. EXPERIMENTAL RESULTS AND DISCUSSION

The above algorithm has been applied on natural gray scale test images like Lena, of size 512 x 512, at different Gaussian noise of levels: (Standard Deviation) σ = 15, 20, 25, 30, 35. Here, we used Haar & Symlet wavelets, the least asymmetric compactly supported wavelet at 5 levels of decomposition. In this work the hard thresholding technique is used to compose the noisy data into an orthogonal & non-orthogonal wavelet basis, to suppress the wavelet coefficients smaller than the given amplitude and also to transform the data back into the original domain. The original image is corrupted with the Gaussian noise with a variance value in order to get noisy data. The Undecimated Wavelet Transform (UDWT) has also been used for decomposing the signal to provide visually better solution. UDWT is shift invariant transform, hence it avoids visual artifacts such as pseudo-Gibbs phenomenon. Though the improvement in results is much higher, use of UDWT adds a large overhead of computations thus making it less feasible. For the simulation the proposed algorithm in MATLAB is implemented.

(a) Original image

(b) Noisy Image corrupted with AWGN

(c) Image Denoised By Discrete Wavelet Transform (DWT)

(d) Image Denoising Using Un-Decimated Wavelet Transform (UDWT)

Figure 8: Simulation results for test image 'Lena' using proposed method

Figure 8 shows the simulation results of test image Lena taken for simulation of our proposed algorithms by using DWT & UDWT. Simulation results values of PSNR & MSE for different wavelets are tabulated in table 1.

Table 1: Test Image Lena Simulation Resultsusing proposed method

Noise Variance	Denoising using Proposed DWT		Denoising using Proposed DWT		Denoising using Proposed UDWT		Denoising using Proposed UDWT	
	Using Haar Wavelet		Using Symlet Wavelet		Using Haar Wavelet		Using Symlet Wavelet	
σ	PSNR (dB)	MSE	PSNR (dB)	MSE	PSNR (dB)	MSE	PSNR (dB)	MSE
5	34.15	24.97	34.00	25.88	37.15	12.52	35.33	19.03
10	28.13	99.83	30.98	51.86	34.40	23.60	32.30	38.24
15	24.58	226.4	29.07	80.48	32.65	35.29	30.36	59.82
20	22.11	399.3	27.74	109.21	31.51	47.20	28.91	83.53
25	20.17	624.9	26.63	141.16	30.41	59.12	27.72	109.84
30	18.60	896.1	25.72	174.06	29.66	71.83	26.70	138.85

In above table 1, it has been shown that proposed UDWT method gives better results as compared to DWT for different values of noise variance in dB. Also we can see from the table that with Haar Wavelet both DWT and UDWT performs better than Symlet wavelets. The proposed UDWT method has minimum Mean Square and highest PSNR with Haar wavelets.

VII. RESULTS COMPARISON

Table 2 shows that comparison of proposed UDWT method with Haar Wavelet. It is clearly shown that proposed method gives better results as compared to other existing methods mentioned in literature for different values of noise variance for 10 dB, 20 dB & 30 dB. We have taken Lena image for comparison. The proposed UDWT method has minimum Mean Square and highest PSNR with Haar wavelets. The proposed UDWT has 0.45 dB improvement for σ =10 dB, 0.20 dB improvement for σ =20 dB & 0.10 dB improvement for σ =30 dB as compared to Contourlet Domain Image Denoising based on the Bessel k-form Distribution (CD-B-k D) [1].

Noise Variance σ → / ↓ Algorithm	σ =10	σ =20	σ =30
Proposed UDWT (Haar)	**34.40**	**31.51**	**29.66**
CD-B-k D [1]	33.95	31.49	29.65
HMT [2]	33.81	30.36	28.45
NIG-NSCT [4]	33.74	31.18	29.09
NIG-WT [4]	31.97	28.42	26.27
Bayes-Shrink [2]	33.29	30.14	28.26
NIG-CT [6]	33.32	31.06	29.33
AS-CT [9]	33.77	31.48	29.64
Visu-shrink [11]	30.65	27.76	26.33

Table 2: Test Image Lena Simulation Results Comparison

VIII. CONCLUSION

In this paper, study of several well-known algorithms for image denoising is carried out & their performance with their methodologies are comparatively assessed. A new algorithm based on the Haar & Symlet wavelet using Un-decimated Wavelet Transform is developed. This time invariant UDWT shows better performance in comparison with the performance of other algorithms. In addition, it has been shown to enjoy the advantage of implementation simplicity. There are different types of noises that may corrupt an image in real life such as, salt-pepper noise, Sparkle noise shot noise, amplification noise, quantization noise etc. However, AWGN or Gaussian noise was considered in this work. A major part of the thesis was devoted to the review, implementation and performance assessment of published image denoising algorithms based on various techniques including the UDWT & Wavelet transform.

REFERENCES

[1] Liqiang Shi, "An Improved Image Denoising Algorithm", Seventh IEEE International Conference on Measuring Technology and Mechatronics Automation, 2015.

[2] H. Sadreazami et-al, "Contourlet Domain Image Denoising based on the Bessel k-form Distribution", IEEE 28th Canadian Conference on Electrical and Computer Engineering Halifax, Canada, May 3-6, 2015.

[3] Cuong Cao Pham et-al, "Efficient image sharpening and denoising using adaptive guided image filtering", Published in IEEE, IET Image Processing Magazine, Pp. 71 – 79, 2015.

[4] WangmengZuo et-al, "Gradient Histogram Estimation and Preservation for Texture Enhanced Image Denoising", IEEE Transactions on Image Processing, Volume 23, Nn. 6, JUNE 2014.

[5] Vikas Gupta et-al, "Image Denoising using Wavelet Transform method", Tenth IEEE International Conference on Wireless and Optical Communications Networks (WOCN), Pp 1-4, 2013.

[6] FuqingJia et-al, "Image Denoising Using Hyper-Laplacian Priors and Gradient Histogram Preservation Model", 12th IEEE International Conference on Signal Processing (ICSP), 2014.

[7] Ajay Boyat et-al, "Image Denoising using Wavelet Transform and Median Filtering", Nirma University IEEE International Conference on Engineering (NUiCONE), 2013.

[8] Paras Jain & VipinTyagi, "Spatial and frequency domain filters for restoration of noisy images", IETE Journal of Education, 54(2), 108-116, 2013.

[9] Maggioni, M., Katkovnik, V., Egiazarian, K., Foi, "A.: Nonlocal transform-domain filter for volumetric data denoising and reconstruction", IEEE Transaction on Image Processing, 22(1), 119–133, 2013.

[10] Silva, R.D., Minetto, R., Schwartz, W.R., Pedrini, H.: Adaptive edge-preserving image denoising using wavelet transforms. Pattern Analysis and Applications. Springer, Berlin doi:10.1007/s10044-012-0266-x, 2012.

[11] Zhang, Y., Li, C., Jia, J, "Image denoising using an improved bivariate threshold function in tetrolet domain", 2013.

[12] Dai, L., Zhang, Y., Li, Y.: Image denoising using BM3D combining tetrolet prefiltering. Inf. Technol. J. 12(10), 1995–2001, 2013.

[13] He, K., Sun, J., Tang, X.: Guided image filtering. In: Proceedings European Conference on Computer Vision, pp. 1–14, 2010.

[14] Porikli, F, "Constant time O(1) bilateral filtering", In Proceeding IEEE Conference on Computer Vision and Pattern Recognition, Anchorage, pp. 1–8, 2008.

[15] Yang, Q., Tan, K.H., Ahuja, N, "Real-time O(1) bilateral filtering", In Proceedings IEEE Conference on Computer Vision and Pattern Recognition, Miami, pp. 557–564, 2009.

[16] Farbman, Z., Fattal, R., Lischinski, D., Szeliski, "R.: Edge-preserving decompositions for multi-scale tone and detail manipulation", ACM Transactions on Graphics 27(3), 1–10, 2008.

[17] Paris, S., Durand, F, "A fast approximation of the bilateral filter using signal processing approach", In the Proceeding of European Conference on Computer Vision, pp. 568–580, 2006.

[18] Gonzalez, R.C., Woods, R.E.: Digital image processing, 3rd edn. Prentice-Hall, Upper Saddle River, 2008.

[19] Blu, T., Luisier, F, "The SURE-LET approach to image denoising", IEEE Transaction Image Processing, 16(11), 2778–2786, 2007.

[20] Paris, S., Durand, F.: A fast approximation of the bilateral filter using signal processing approach. In: Proceeding European Conference on Computer Vision, pp. 568–580, 2006.

[21] Dabov, K., Foi, A., Katkovnik, V., Egiazarian, K.: Image denoising with block-matching and 3D filtering. In: SPIE electronic imaging: algorithms and systems, vol. 6064, pp. 606414-1–606414-12, 2006.

[22] Yuan, X., Buckles, B.: Subband noise estimation for adaptive wavelet shrinkage. In: Proceeding 17th International Conference on Pattern Recognition, vol. 4, pp 885–888, 2004.

[23] Elad, M.: On the origin of the bilateral filter and ways to improve it. IEEE Transaction Image Processing, 11(10), 1141–1151, 2002.

[24] Sendur, L., Selesnick, I.W.: Bivariate shrinkage functions for wavelet-based denoising exploiting interscale dependency. IEEE Trans. Signal Process. 50(11), 2744–2756, 2002.

[25] Chang, S., Yu, B., Vetterli, M.: Spatially adaptive wavelet thresholding based on context modeling for image denoising. IEEE Trans. Image Process. 9(9), 1522–1531, 2000.

[26] D. L. Donoho De-noising by soft-thresholding IEEE Transactions on Information Theory (Volume:41 , Issue: 3), 2002.