$S\alpha RPS$-Closed Maps and $S\alpha RPS$-Open Maps in Topological Spaces

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***Abstract*- In this paper, the authors introduce $s\alpha rps$-closed maps and $s\alpha rps$-open maps in topological spaces and study some of their basic properties.**

***Keywords*: $s\alpha rps$-closed maps, $s\alpha rps$-open maps.**

I. INTRODUCTION

Different types of generalized closed maps and generalized open maps were studied by various researchers. Recently the authors introduced $s\alpha rps$-closed sets and $s\alpha rps$-open sets in topological spaces. Using these two sets, the authors introduce $s\alpha rps$-closed maps and $s\alpha rps$-open maps in topological spaces and continue the study of their relationship with various generalized closed maps and open maps.

II. PRELIMINARIES

Throughout this paper X and Y represent the topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a topological space X, *cl*A and *int*A denote the closure of A and the interior of A respectively. X \ A denotes the complement of A in X. We recall the following definitions.

Definition 2.1:
A subset A of a space X is called
(i) semi-open [9] if A $\subseteq$ *cl int*A and semi-closed if *int cl*A $\subseteq$ A.
(ii) α-open [17] if A $\subseteq$ *int cl int*A and α-closed if *cl int cl*A $\subseteq$ A.
(iii) regular open [21] if A = *int cl*A and regular closed if *cl int*A = A.

Definition 2.2:
A subset A of a space X is called
(i) generalized closed [10] (briefly g-closed) if *clA* $\subseteq$ U whenever A $\subseteq$ U and U is open.
(ii) regular generalized closed [18] (briefly rg-closed) if *cl*A $\subseteq$ U whenever A $\subseteq$ U and U is regular open.
(iii) α-generalized closed [12] (briefly αg-closed) if $\alpha clA \subseteq$ U whenever A $\subseteq$ U and U is open.
(iv) generalized semi-closed [3] (briefly gs-closed) if *scl*A $\subseteq$ U whenever A $\subseteq$ U and U is open.
(v) generalized pre-regular closed [7] (briefly gpr-closed) if *pcl*A $\subseteq$ U whenever A $\subseteq$ U and U is regular open.
(vi) generalized semi-pre-closed [6] (briefly gsp-closed) if *spcl*A $\subseteq$ U whenever A $\subseteq$ U and U is open.
(vii) weakly generalized closed [16] (briefly wg-closed) if *cl int*A $\subseteq$ U whenever A $\subseteq$ U and U is open.
(viii) regular weakly generalized closed [23] (briefly rwg-closed) if *cl int*A $\subseteq$ U whenever A $\subseteq$ U and U is regular open.
(ix) generalized b-closed [1] (briefly gb-closed) if *bcl*A $\subseteq$ U whenever A $\subseteq$ U and U is open.
(x) regular generalized b-closed [14] (briefly rgb-closed) if *bcl*A $\subseteq$ U whenever A $\subseteq$ U and U is regular open.

Definition 2.3 [22]
A subset A of a space X is called semi α-regular pre-semi closed (briefly $s\alpha rps$-closed) if *scl*A $\subseteq$ U whenever A $\subseteq$ U and U is αrps-open.

The complements of the above mentioned closed sets are their respective open sets. For example, a subset B of a space X is generalized open (briefly g-open) if X \ B is g-closed.

Definition 2.4
A map f: $(X,\tau)\rightarrow(Y,\sigma)$ is called semi-closed [9] (resp. regular closed [11], resp. α-closed [15], resp. gs-closed [24], resp. gsp-closed [6], resp. gb-closed [24], resp. rgb-closed [8], resp. g-closed [13], resp. αg-closed [5], resp. wg-closed [16], resp. rwg-closed [16], resp. rg-closed [2], resp. gpr-closed [7]) if for every closed subset F of (X,τ), the set f(F) is semi-closed (resp. regular closed, resp. α-closed, resp. gs-closed, resp. gsp-closed, resp. gb-closed, resp. rgb-closed, resp. g-closed, resp. αg-closed, resp. wg-closed, resp. rwg-closed, resp. rg-closed, resp. gpr-closed) in (Y,σ).

Definition 2.5
A map f: $(X,\tau)\rightarrow(Y,\sigma)$ is called semi-open [9] (resp. regular open [11], resp. α-open [15], resp. gs-open [25], resp. gsp-open [6], resp. gb-open [24], resp. rgb-open [8], resp. g-open [13], resp. αg-open [5], resp. wg-open [16], resp. rwg-open [16], resp. rg-open [2], resp. gpr-open [7]) if for every open subset U of (X,τ), the set f(U) is semi-open (resp. regular open, resp. α-open, resp. gs-open, resp. gsp-open, resp. gb-open, resp. rgb-open, resp. g-open, resp. αg-open, resp. wg-open, resp. rwg-open, resp. rg-open, resp. gpr-open) in (Y,σ).

Definition 2.6
For a subset A of a space X, $s\alpha rps$-closure is defined as, $s\alpha rps$-*cl*A = $\cap$ {F: A $\subseteq$ F and F is $s\alpha rps$-closed in X}.

Lemma 2.7
(i) If A $\subseteq$ B, then $s\alpha rps$-*cl*A $\subseteq$ $s\alpha rps$-*cl*B.

(ii) If A $\subseteq$ B, then $s\alpha rps$-intA $\subseteq$ $s\alpha rps$-*int*B.

(iii) A $\subseteq$ $s\alpha rps$-*cl*B.

(iv) $s\alpha rps$-intA $\subseteq$ A.

(v) A is $s\alpha rps$-closed if and only if $s\alpha rps$-*cl*A = A.

(vi) A is $s\alpha rps$-open if and only if $s\alpha rps$-*int*A = A.

Lemma 2.8 [19]

If a subset N of space X is $s\alpha rps$-open, then N is a $s\alpha rps$-nbhd of each of its points.

Definition 2.9

(i) A function f: (X, τ) $\rightarrow$ (Y, σ) is called g-continuous [4] if f^{-1}(V) is g-closed in (X, τ) for every closed subset V of (Y, σ).

(ii) A function f: (X, τ) $\rightarrow$ (Y, σ) is called $s\alpha rps$-continuous [20] if f^{-1}(V) is $s\alpha rps$-closed in (X, τ) for every closed subset V of (Y, σ).

(iii) A function f: (X, τ) $\rightarrow$ (Y, σ) is called $s\alpha rps$-irresolute [20] if f^{-1}(V) is $s\alpha rps$-closed in (X, τ) for every $s\alpha rps$-closed subset V of (Y, σ).

(iv) A function f: (X, τ) $\rightarrow$ (Y, σ) is called strongly $s\alpha rps$-continuous if f^{-1}(V) is closed in (X, τ) for every $s\alpha rps$-closed subset V of (Y, σ).

Definition 2.10 [10]

A space X is called a $T_{1/2}$ space if every g-closed set is closed.

III. $S\alpha RPS$ -CLOSED MAPS IN TOPOLOGICAL SPACES

In this section, we introduce $s\alpha rps$-closed maps in topological spaces.

Definition 3.1

A map f: (X, τ) $\rightarrow$ (Y, σ) is called $s\alpha rps$-closed if for every closed subset F of (X, τ), f(F) is a $s\alpha rps$-closed set in (Y, σ).

Proposition 3.2

Every closed map is a $s\alpha rps$-closed map.

Proof

Let f: (X, τ) $\rightarrow$ (Y, σ) be a closed map. Let F be a closed subset of (X, τ). Since f is a closed map, f(F) is a closed set in (Y, σ). Since every closed set is $s\alpha rps$-closed, f(F) is $s\alpha rps$-closed in (Y, σ). Therefore f is a $s\alpha rps$-closed map.

Converse of the above Proposition need not be true as shown in Example 3.4.

Proposition 3.3

(i) Every semi-closed map is a $s\alpha rps$-closed map.

(ii) Every regular closed map is a $s\alpha rps$-closed map.

(iii) Every α -closed map is a $s\alpha rps$-closed map .

Proof

Let f: (X, τ) $\rightarrow$ (Y, σ) be a semi-closed (resp. regular closed, resp. α -closed) map. Let F be a closed subset of (X, τ). Since f is a semi-closed (resp. regular closed, resp. α -closed) map, f(F) is a semi-closed (resp. regular closed, resp. α -closed) set in (Y, σ). By Proposition 3.2 of [22], f(F) is $s\alpha rps$-closed in (Y, σ). Therefore f is a $s\alpha rps$-closed map.

Converse of the above Proposition need not be true as shown in Example 3.4.

Example 3.4

Let X = Y = {a,b,c} with topologies τ ={ ϕ ,{a},X} and σ ={ ϕ ,{a},{b,c},Y} on X and Y respectively. Let the map f: (X, τ) $\rightarrow$ (Y, σ) be defined as f(a) = b, f(b) = c, f(c) = a. Here the closed sets in (X, τ) are ϕ, {b,c} and X. Now f({b,c}) = {a,c},which is $s\alpha rps$-closed in (Y, σ). Therefore for every closed set F in (X, τ), f(F) is $s\alpha rps$-closed in (Y, σ). Hence f is $s\alpha rps$-closed. But {a,c} is not closed, not semi-closed, not regular closed, not α -closed in (Y, σ). Hence f is not closed, not semi-closed, not regular closed, not α -closed.

Proposition 3.5

(i) Every $s\alpha rps$-closed map is a gs-closed map.

(ii) Every $s\alpha rps$-closed map is a gsp-closed map.

(iii) Every $s\alpha rps$-closed map is a gb-closed map.

(iv) Every $s\alpha rps$-closed map is a rgb-closed map.

Proof

Let f: (X, τ) $\rightarrow$ (Y, σ) be a $s\alpha rps$-closed map. Let F be a closed subset of (X, τ). Since f is a $s\alpha rps$-closed map, f(F) is a $s\alpha rps$-closed set in (Y, σ). By Proposition 3.4 of [22], f(F) is gs-closed (resp. gsp-closed, resp. gb-closed, resp. rgb-closed) in (Y, σ). Therefore f is a gs-closed (resp. gsp-closed, resp. gb-closed, resp. rgb-closed) map.

Converse of the above Proposition need not be true as shown in Example 3.6.

Example 3.6

Let X = Y = {a,b,c,d} with topologies τ ={ ϕ ,{b,d},X} and σ ={ ϕ ,{a},{b},{a,b},{b,c},{a,b,c},Y} on X and Y respectively. Let the map f: (X, τ) $\rightarrow$ (Y, σ) be defined as f(a) = b, f(b) = c, f(c) = d, f(d) = a. Here the closed sets in (X, τ) are ϕ ,{a,c} and X. Now f({a,c}) = {b,d} is gs-closed (resp. gb-closed, resp. gsp-closed, resp. rgb-closed) in (Y, σ). Therefore f is gs-closed (resp. gb-closed, resp. gsp-closed, resp. rgb-closed). But {b,d} is not $s\alpha rps$-closed in (Y, σ). Hence f is not $s\alpha rps$-closed.

The concept $s\alpha rps$-closed map is independent from the concepts αg -closed map, g-closed map, rg-closed map, gpr-closed map, wg-closed map, rwg-closed map as shown in the following examples.

Example 3.7

Let X = Y={a,b,c,d} with topologies τ ={ ϕ ,{a,c},X} and σ ={ ϕ ,{a},{b},{a,b},{b,c},{a,b,c},Y} on X and Y respectively. Let the map f: (X, τ) $\rightarrow$ (Y, σ) be defined as f(a) = b, f(b) = c, f(c) = d, f(d) = a. Here the closed sets in (X, τ) are ϕ ,{b,d} and X. Now f({b,d}) = {a,c} is

$s\alpha rps$-closed, but not αg-closed in (Y, σ). Hence f is $s\alpha rps$-closed, but not αg-closed.

Example 3.8

Let X = Y = {a,b,c,d} with topologies τ ={ϕ,{b,d},X} and σ ={ϕ,{a},{b},{a,b},{b,c},{a,b,c},Y} on X and Y respectively. Let the map f: (X, τ) $\rightarrow$ (Y, σ) be defined as f(a) = b, f(b) = c, f(c) = d, f(d) = a. Here the closed sets in (X, τ) are ϕ, {a,c} and X. Now f({a,c}) = {b,d} is αg-closed, but not $s\alpha rps$-closed in (Y, σ). Hence f is αg-closed, but not $s\alpha rps$-closed.

Example 3.9

Let X = Y = {a,b,c,d} with topologies τ ={ϕ,{a,b},X} and σ ={ϕ,{a},{b},{a,b},{a,b,c},{a,b,d},Y} on X and Y respectively. Let the map f: (X, τ) $\rightarrow$ (Y, σ) be defined as f(a) = b, f(b) = c, f(c) = f(d) = a. Here the closed sets in (X, τ) are ϕ,{c,d} and X. Now f({c,d}) = {a} is $s\alpha rps$-closed, but not rg-closed in (Y, σ). Hence f is $s\alpha rps$-closed, but not rg-closed.

Example 3.10

Let X = Y = {a,b,c,d} with topologies τ ={ϕ,{d},X} and σ ={ϕ,{a},{b},{a,b},{a,b,c},{a,b,d},Y} on X and Y respectively. Let the map f: (X, τ) $\rightarrow$ (Y, σ) be defined as f(a) = a, f(b) = b, f(c) = c, f(d) = d. Here the closed sets in (X, τ) are ϕ,{a,b,c} and X. Now f({a,b,c}) = {a,b,c} is rg-closed, but not $s\alpha rps$-closed in (Y, σ). Hence f is rg-closed , but not $s\alpha rps$-closed.

Example 3.11

Let X = Y = {a,b,c,d} with topologies τ ={ϕ,{b,c},X} and σ ={ϕ,{a},{a,b},Y} on X and Y respectively. Let the map f: (X, τ) $\rightarrow$ (Y, σ) be defined as f(a) = b, f(b) = c, f(c) = d, f(d) = b. Here the closed sets in (X, τ) are ϕ,{a,d} and X. Now f({a,d}) = {b} is $s\alpha rps$-closed, but not g-closed in (Y, σ). Hence f is $s\alpha rps$-closed, but not g-closed.

Example 3.12

Let X = Y={a,b,c,d} with topologies τ ={ϕ,{a,c},X} and σ ={ϕ,{a},{a,b},Y} on X and Y respectively. Let the map f: (X, τ) $\rightarrow$ (Y, σ) be defined as f(a) = b, f(b) = c, f(c) = d, f(d) = a. Here the closed sets in (X, τ) are ϕ,{b,d} and X. Now f({b,d}) = {a,c} is g-closed, but not $s\alpha rps$-closed in (Y, σ). Hence f is g-closed, but not $s\alpha rps$-closed.

Example 3.13

Let X = Y = {a,b,c} with topologies τ ={ϕ,{a},X} and σ ={ϕ,{a},{b},{a,b},Y} on X and Y respectively. Let the map f: (X, τ) $\rightarrow$ (Y, σ) be defined as f(a) = b, f(b) = f(c) = a. Here the closed sets in (X, τ) are ϕ,{b,c} and X. Now f({b,c}) = {a} is $s\alpha rps$-closed, but not gpr-closed in (Y, σ). Hence f is $s\alpha rps$-closed, but not gpr-closed.

Example 3.14

Let X = Y={a,b,c} with topologies τ ={ϕ,{a},X} and σ ={ϕ,{a},{b},{a,b},Y} on X and Y respectively. Let the map f: (X, τ) $\rightarrow$ (Y, σ) be defined as f(a) = c, f(b) = b, f(c) = a. Here the closed sets in (X, τ) are ϕ,{b,c} and X. Now f({b,c}) = {a,b} is gpr-closed, but not $s\alpha rps$-closed in (Y, σ). Hence f is gpr-closed, but not $s\alpha rps$-closed.

Example 3.15

Let X = Y={a,b,c,d} with topologies τ ={ϕ,{a},X} and σ ={ϕ,{a},{b},{a,b},{a,b,c},Y} on X and Y respectively. Let the map f: (X, τ) $\rightarrow$ (Y, σ) be defined as f(a) = b, f(b) = f(c) = f(d) = a. Here the closed sets in (X, τ) are ϕ,{b,c,d} and X. Now f({b,c,d}) = {a} is $s\alpha rps$-closed, but not wg-closed and not rwg-closed in (Y, σ). Hence f is $s\alpha rps$-closed, but not wg-closed and not rwg-closed.

Example 3.16

Let X = Y={a,b,c,d} with topologies τ ={ϕ,{a},X} and σ ={ϕ,{a},{b},{a,b},{a,b,c},Y} on X and Y respectively. Let the map f: (X, τ) $\rightarrow$ (Y, σ) be defined as f(a) = c, f(b) =a, f(c) = d, f(d) = b. Here the closed sets in (X, τ) are ϕ, {b,c,d} and X. Now f({b,c,d}) = {a,b,d} is wg-closed and rwg-closed, but not $s\alpha rps$-closed in (Y, σ). Hence f is wg-closed and rwg-closed, but not $s\alpha rps$-closed.

Theorem 3.17

A map f: (X, τ) $\rightarrow$ (Y, σ) is $s\alpha rps$-closed if and only if for each subset S of (Y, σ) and for each open set U containing f^{-1}(S), there exists a $s\alpha rps$-open set V of (Y, σ) such that S $\subseteq$ V and f^{-1}(V) $\subseteq$ U.

Proof

Assume that f is $s\alpha rps$-closed. Let S be a subset of (Y, σ) and U be an open set containing f^{-1}(S). Then X \ U is a closed set in (X, τ) not containing f^{-1}(S). Since f is $s\alpha rps$-closed, f(X \ U) is $s\alpha rps$-closed in (Y, σ) not containing S. Therefore Y \ f(X \ U) is a $s\alpha rps$-open set in (Y, σ) containing S. Let V = Y \ f(X \ U). Hence V is a $s\alpha rps$-open set in (Y, σ) such that S $\subseteq$ V and f^{-1}(V) $\subseteq$ U.

Conversely, let F be a closed set in (X, τ). Then X \ F is a open set in (X, τ) and f(F) is a subset of (Y, σ) and so Y \ f(F) is a subset of (Y, σ). Therefore f^{-1}(Y \ f(F)) is a subset of (X, τ). Take S = Y \ f(F) and U = X \ F. Therefore f^{-1}(S) $\subseteq$ U. By hypothesis, there exists a $s\alpha rps$-open set V of (Y, σ) such that S $\subseteq$ V and f^{-1}(V) $\subseteq$ U. That is, Y \ f(F) $\subseteq$ V and f^{-1}(V) $\subseteq$ X \ F. Then we have F $\subseteq$ X \ f^{-1}(V) and Y \ V $\subseteq$ f(F) $\subseteq$ f(X \ f^{-1}(V)) $\subseteq$ Y \ V which implies Y \ V = f(F). Since V is $s\alpha rps$-open in (Y, σ), Y \ V is $s\alpha rps$-closed in (Y, σ). Therefore f(F) is $s\alpha rps$-closed in (Y, σ). Hence f is $s\alpha rps$-closed.

Theorem 3.18

If f: $(X,\tau)\rightarrow(Y,\sigma)$ is a continuous, $s\alpha rps$-closed surjection and if (X,τ) is a normal space, then (Y,σ) is normal.

Proof

Let A and B be two disjoint closed subsets of (Y,σ). Since f is a continuous surjection, $f^{-1}(A)$ and $f^{-1}(B)$ are the disjoint closed sets in (X,τ). Since (X,τ) is normal, there exist disjoint open sets U and V such that of (X,τ) such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$. Since f is $s\alpha rps$-closed, by Theorem 3.17, there exist $s\alpha rps$-open sets G and H of (Y,σ) such that $A \subseteq G$, $B \subseteq H$ and $f^{-1}(G) \subseteq U$, $f^{-1}(H) \subseteq V$. Then we have $f^{-1}(G) \cap f^{-1}(H) = \emptyset$ and so $G \cap H = \emptyset$. Since every closed set is αrps-closed, A is αrps-closed. Since G is $s\alpha rps$-open and $A \subseteq G$, by Theorem 3.17 of [19],

$A \subseteq sint G$. Similarly $B \subseteq sint H$. Since $sint G \cap sint H = \emptyset$, $(int\ cl\ int\ sint G) \cap (int\ cl\ int\ sint H) = \emptyset$.

Since $A \subseteq sint G \subseteq int\ cl\ int\ sint G$ and

$B \subseteq sint H \subseteq int\ cl\ int\ sint H$, (Y,σ) is normal.

Theorem 3.19

If a mapping f: $(X,\tau)\rightarrow(Y,\sigma)$ is $s\alpha rps$-closed, then $s\alpha rps$-$cl(f(A)) \subseteq f(clA)$ for every subset A of (X,τ).

Proof

Suppose that f is $s\alpha rps$-closed and $A \subseteq X$. Then clA is closed in (X,τ). Since f is $s\alpha rps$-closed, $f(clA)$ is $s\alpha rps$-closed in (Y,σ).

Therefore $s\alpha rps$-$cl(f(clA)) = f(clA)$.

We have $f(A) \subseteq f(clA)$.

By Lemma 2.7(i), $s\alpha rps$-$cl(f(A)) \subseteq s\alpha rps$-$cl\ (f(clA))$.

That is, $s\alpha rps$-$cl(f(A)) \subseteq f(clA)$.

Theorem 3.20

Let f: $(X,\tau)\rightarrow(Y,\sigma)$ and g: $(Y,\sigma)\rightarrow(Z,\mu)$ be two mappings. Let $h = g\circ f$.

(i) If f is closed and g is $s\alpha rps$-closed, then h is $s\alpha rps$-closed.

(ii) If f is g-closed, g is $s\alpha rps$-closed and (Y,σ) is a $T_{1/2}$ space, then h is $s\alpha rps$-closed.

(iii) If f is regular closed and g is $s\alpha rps$-closed, then h is $s\alpha rps$-closed.

Proof

(i) Let F be closed in (X,τ). Since f is closed, f(F) is closed in (Y,σ). Since g is $s\alpha rps$-closed, g(f(F)) is $s\alpha rps$-closed in (Z,μ). That is h(F) is $s\alpha rps$-closed in (Z,μ). This proves (i).

(ii) Let F be closed in (X,τ). Since f is g-closed, f(F) is g-closed in (Y,σ). Since (Y,σ) is a $T_{1/2}$ space , by Definition 2.10, f(F) is closed in (Y,σ). Since g is $s\alpha rps$-closed, g(f(F)) is $s\alpha rps$-closed in (Z,μ). That is h(F) is $s\alpha rps$-closed in (Z,μ). This proves (ii).

(iii) Let F be closed in (X,τ). Since f is regular closed, f(F) is regular closed in (Y,σ). Since every regular closed set is closed, f(F) is closed in (Y,σ). Since g is $s\alpha rps$-closed, g(f(F)) is $s\alpha rps$-closed in (Z,μ). That is h(F) is $s\alpha rps$-closed in (Z,μ). This proves (iii).

Theorem 3.21

Let f: $(X,\tau)\rightarrow(Y,\sigma)$ and g: $(Y,\sigma)\rightarrow(Z,\mu)$ be two mappings. Let $h = g\circ f$ be $s\alpha rps$-closed.

(i) If f is continuous and surjective, then g is $s\alpha rps$-closed.

(ii) If f is g-continuous and surjective and (X,τ) is a $T_{1/2}$ space, then g is $s\alpha rps$-closed.

(iii) If g is $s\alpha rps$-irresolute and injective, then f is $s\alpha rps$-closed.

(iv) If g is strongly $s\alpha rps$-continuous and injective, then f is $s\alpha rps$-closed.

Proof

(i) Let F be closed in (Y,σ). Since f is continuous, $f^{-1}(F)$ is closed in (X,τ). Since h is $s\alpha rps$-closed, $h(f^{-1}(F))$ is $s\alpha rps$-closed in (Z,μ). That is $(g\circ f)(f^{-1}(F))$ is $s\alpha rps$-closed in (Z,μ). That is g(F) is $s\alpha rps$-closed in (Z,μ). This proves (i).

(ii) Let F be closed in (Y,σ). Since f is g-continuous, $f^{-1}(F)$ is g-closed in (X,τ). Since (X,τ) is a $T_{1/2}$ space, $f^{-1}(F)$ is closed in (X,τ). Since h is $s\alpha rps$-closed, $h(f^{-1}(F))$ is $s\alpha rps$-closed in (Z,μ). That is $(g\circ f)(f^{-1}(F))$ is $s\alpha rps$-closed in (Z,μ). That is g(F) is $s\alpha rps$-closed in (Z,μ). This proves (ii).

(iii) Let F be closed in (X,τ). Since h is $s\alpha rps$-closed, h(F) is $s\alpha rps$-closed in (Z,μ). That is g(f(F)) is $s\alpha rps$-closed in (Z,μ). Since g is $s\alpha rps$-irresolute, $g^{-1}(g(f(F)))$ is $s\alpha rps$-closed in (Y,σ). That is f(F) is $s\alpha rps$-closed in (Y,σ). This proves (iii).

(iv) Let F be closed in (X,τ). Since h is $s\alpha rps$-closed, h(F) is $s\alpha rps$-closed in (Z,μ). That is g(f(F)) is $s\alpha rps$-closed in (Z,μ). Since g is strongly $s\alpha rps$-continuous, $g^{-1}(g(f(F)))$ is closed in (Y,σ). That is f(F) is closed in (Y,σ). Since every closed set is $s\alpha rps$-closed, f(F) is $s\alpha rps$-closed in (Y,σ). This proves (iv).

IV. $S\alpha RPS$-OPEN MAPS IN TOPOLOGICAL SPACES

In this section, we introduce $s\alpha rps$-open maps in topological spaces.

Definition 4.1

A map f: $(X,\tau)\rightarrow(Y,\sigma)$ is called $s\alpha rps$-open if for every open subset U of (X,τ), f(U) is a $s\alpha rps$-open set in (Y,σ).

Proposition 4.2

Every open map is a $s\alpha rps$-open map.

Proof

Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be an open map. Let U be an open subset of (X,τ). Since f is an open map, f(U) is an open set in

(Y,σ). Since every open set is $s\alpha rps$-open, f(U) is $s\alpha rps$-open in (Y,σ). Therefore f is a $s\alpha rps$-open map.

Converse of the above Proposition need not be true as shown in Example 4.4.

Proposition 4.3

(i) Every semi-open map is a $s\alpha rps$-open map.

(ii) Every regular open map is a $s\alpha rps$-open map.

(iii) Every α-open map is a $s\alpha rps$-open map.

Proof

Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be a semi-open (resp. regular open, resp. α-open) map. Let U be an open subset of (X,τ). Since f is a semi-open (resp. regular open, resp. α-open) map, f(U) is a semi-open (resp. regular open, resp. α-open) set in (Y,σ). By Proposition 3.2 of [19], f(U) is $s\alpha rps$-open in (Y,σ). Therefore f is a $s\alpha rps$-open map.

Converse of the above Proposition need not be true as shown in Example 4.4.

Example 4.4

Let X = Y={a,b,c} with topologies $\tau=\{\phi,\{a\},X\}$ and $\sigma=\{\phi,\{a\},\{b,c\},Y\}$ on X and Y respectively. Let the map f: $(X,\tau)\rightarrow(Y,\sigma)$ be defined as f(a) = b, f(b) = c, f(c) = a. Here the open sets in (X,τ) are ϕ, {a} and X. Now f({a}) = {b},which is $s\alpha rps$-open in (Y,σ). Therefore for every open set U in (X,τ), f(U) is $s\alpha rps$-open in (Y,σ). Hence f is $s\alpha rps$-open. But {b} is not open, not semi-open, not regular open, not α-open in (Y,σ). Hence f is not open, not semi-open, not regular open, not α-open.

Proposition 4.5

(i) Every $s\alpha rps$-open map is a gs-open map.

(ii) Every $s\alpha rps$-open map is a gsp-open map.

(iii) Every $s\alpha rps$-open map is a gb-open map.

(iv) Every $s\alpha rps$-open map is a rgb-open map.

Proof

Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be a $s\alpha rps$-open map. Let U be an open subset of (X,τ). Since f is a $s\alpha rps$-open map, f(U) is a $s\alpha rps$-open set in (Y,σ). By Proposition 3.4 of [19], f(U) is gs-open (resp. gsp-open, resp. gb-open, resp. rgb-open) in (Y,σ). Therefore f is a gs-open (resp. gsp-open, resp. gb-open, resp. rgb-open) map.

Converse of the above Proposition need not be true as shown in Example 4.6.

Example 4.6

Let X=Y={a,b,c,d} with topologies $\tau=\{\phi,\{b,d\},X\}$ and $\sigma=\{\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\},Y\}$ on X and Y respectively. Let the map f: $(X,\tau)\rightarrow(Y,\sigma)$ be defined as f(a) = b, f(b) = c, f(c) = d, f(d) = a. Here the open sets in (X,τ) are ϕ,{b,d} and X. Now f({b,d}) = {a,c} is gs-open (resp. gb-open, resp. gsp-open, resp. rgb-open) in (Y,σ). Therefore f is gs-open (resp. gb-open, resp. gsp-open, resp. rgb-open). But {a,c} is not $s\alpha rps$-open in (Y,σ). Hence f is not $s\alpha rps$-open.

The concept $s\alpha rps$-open map is independent from the concepts αg-open map, g-open map, rg-open map, gpr-open map, wg-open map, rwg-open map as shown in the following examples.

Example 4.7

Let X = Y={a,b,c,d} with topologies $\tau=\{\phi,\{a,c\},X\}$ and $\sigma=\{\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\},Y\}$ on X and Y respectively. Let the map f: $(X,\tau)\rightarrow(Y,\sigma)$ be defined as f(a) = b, f(b) = c, f(c) = d, f(d) = a. Here the open sets in (X,τ) are ϕ,{a,c} and X. Now f({a,c}) = {b,d} is $s\alpha rps$-open, but not αg-open in (Y,σ). Hence f is $s\alpha rps$-open, but not αg-open.

Example 4.8

Let X = Y={a,b,c,d} with topologies $\tau=\{\phi,\{b,d\},X\}$ and $\sigma=\{\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\},Y\}$ on X and Y respectively. Let the map f: $(X,\tau)\rightarrow(Y,\sigma)$ be defined as f(a) = b, f(b) = c, f(c) = d, f(d) = a. Here the open sets in (X,τ) are ϕ, {b,d} and X. Now f({b,d}) = {a,c} is αg-open, but not $s\alpha rps$-open in (Y,σ). Hence f is αg-open, but not $s\alpha rps$-open.

Example 4.9

Let X = Y={a,b,c,d} with topologies $\tau=\{\phi,\{a,b,c\},X\}$ and $\sigma=\{\phi,\{a\},\{b\},\{a,b\},\{a,b,c\},\{a,b,d\},Y\}$ on X and Y respectively. Let the map f: $(X,\tau)\rightarrow(Y,\sigma)$ be defined as f(a) = b, f(b) = c, f(c) = d, f(d) = a. Here the open sets in (X,τ) are ϕ,{a,b,c} and X. Now f({a,b,c}) = {b,c,d} is $s\alpha rps$-open, but not rg-open in (Y,σ). Hence f is $s\alpha rps$-open, but not rg-open.

Example 4.10

Let X = Y={a,b,c,d} with topologies $\tau=\{\phi,\{c\},X\}$ and $\sigma=\{\phi,\{a\},\{b\},\{a,b\},\{a,b,c\},\{a,b,d\},Y\}$ on X and Y respectively. Let the map f: $(X,\tau)\rightarrow(Y,\sigma)$ be defined as f(a) = b, f(b) = c, f(c) = d, f(d) = a. Here the open sets in (X,τ) are ϕ,{c} and X. Now f({c}) = {d} is rg-open , but not $s\alpha rps$-open in (Y,σ). Hence f is rg-open , but not $s\alpha rps$-open.

Example 4.11

Let X = Y={a,b,c,d} with topologies $\tau=\{\phi,\{b,c,d\},X\}$ and $\sigma=\{\phi,\{a\},\{a,b\},Y\}$ on X and Y respectively. Let the map f: $(X,\tau)\rightarrow(Y,\sigma)$ be defined as f(a) = b, f(b) = c, f(c) = d, f(d) = a. Here the open sets in (X,τ) are ϕ,{b,c,d} and X. Now f({b,c,d}) = {a,c,d} is $s\alpha rps$-open, but not g-open in (Y,σ). Hence f is $s\alpha rps$-open, but not g-open.

Example 4.12

Let X = Y={a,b,c,d} with topologies τ={ϕ,{a,c},X} and σ={ϕ,{a},{a,b},Y} on X and Y respectively. Let the map f: (X,τ)$\rightarrow$(Y,σ) be defined as f(a) = b, f(b) = c, f(c) = d, f(d) = a. Here the open sets in (X,τ) are ϕ,{a,c} and X. Now f({a,c}) = {b,d} is g-open, but not $s\alpha rps$-open in (Y,σ). Hence f is g-open, but not $s\alpha rps$-open.

Example 4.13

Let X = Y={a,b,c} with topologies τ={ϕ,{a,b},X} and σ={ϕ,{a},{b},{a,b},Y} on X and Y respectively. Let the map f: (X,τ)$\rightarrow$(Y,σ) be defined as f(a) = b, f(b) = c, f(c) = a. Here the open sets in (X,τ) are ϕ,{a,b} and X. Now f({a,b}) = {b,c} is $s\alpha rps$-open, but not gpr-open in (Y,σ). Hence f is $s\alpha rps$-open, but not gpr-open.

Example 4.14

Let X = Y={a,b,c} with topologies τ={ϕ,{a},X} and σ={ϕ,{a},{b},{a,b},Y} on X and Y respectively. Let the map f: (X,τ)$\rightarrow$(Y,σ) be defined as f(a) = c, f(b) = b, f(c) = a. Here the open sets in (X,τ) are ϕ,{a} and X. Now f({a}) = {c} is gpr-open, but not $s\alpha rps$-open in (Y,σ). Hence f is gpr-open, but not $s\alpha rps$-open.

Example 4.15

Let X = Y={a,b,c,d} with topologies τ={ϕ,{a,b,c},X} and σ={ϕ,{a},{b},{a,b},{a,b,c},Y} on X and Y respectively. Let the map f: (X,τ)$\rightarrow$(Y,σ) be defined as f(a) = b, f(b) = c, f(c) = d, f(d) = a. Here the open sets in (X,τ) are ϕ,{a,b,c} and X. Now f({a,b,c}) = {b,c,d} is $s\alpha rps$-open, but not wg-open and not rwg-open in (Y,σ). Hence f is $s\alpha rps$-open, but not wg-open and not rwg-open.

Example 4.16

Let X = Y={a,b,c,d} with topologies τ={ϕ,{a},X} and σ={ϕ,{a},{b},{a,b},{a,b,c},Y} on X and Y respectively. Let the map f: (X,τ)$\rightarrow$(Y,σ) be defined as f(a) = c, f(b) =a, f(c) = d, f(d) = b. Here the open sets in (X,τ) are ϕ,{a} and X. Now f({a}) = {c} is wg-open and rwg-open, but not $s\alpha rps$-open in (Y,σ). Hence f is wg-open and rwg-open, but not $s\alpha rps$-open.

Theorem 4.17

If a mapping f: (X,τ)$\rightarrow$(Y,σ) is $s\alpha rps$-open, then f(*int*A) $\subseteq$ $s\alpha rps$-*int*(f(A)) for every subset A of (X,τ).

Proof

Suppose that f is $s\alpha rps$-open and A $\subseteq$ X. Then *int*A is open in (X,τ). Since f is $s\alpha rps$-open, f(*int*A) is $s\alpha rps$-open in (Y,σ). By Lemma 2.7(vi), $s\alpha rps$-*int* (f(*int*A)) = f(*int*A). We have f(*int*A) $\subseteq$ f(A). By Lemma 2.7(ii), $s\alpha rps$-*int* (f(*int*A)) $\subseteq$ $s\alpha rps$-*int* f(A). That is f(*int*A) $\subseteq$ $s\alpha rps$-*int*(f(A)).

Theorem 4.18

If a map f: (X,τ)$\rightarrow$(Y,σ) is $s\alpha rps$-open, then for each nbhd U of x in (X,τ), there exists a $s\alpha rps$-nbhd W of f(x) in (Y,σ) such that W $\subseteq$ f(U).

Proof

Let f: (X,τ)$\rightarrow$(Y,σ) be $s\alpha rps$-open. Let x $\in$ X and U be an arbitrary nbhd of x in (X,τ). Then there exists an open set G in (X,τ) such that x $\in$ G $\subseteq$ U. Now f(x) $\in$ f(G) $\subseteq$ f(U) and f(G) is a $s\alpha rps$-open set in (Y,σ), as f is a $s\alpha rps$-open map. By Lemma 2.8, f(G) is a $s\alpha rps$-nbhd of each of its points. Taking f(G) = W, W is a $s\alpha rps$-nbhd of f(x) in (Y,σ) such that W $\subseteq$ f(U).

Theorem 4.19

A map f: (X,τ)$\rightarrow$(Y,σ) is $s\alpha rps$-open if and only if for any subset S of (Y,σ) and any closed set F of (X,τ) containing f^{-1}(S), there exists a $s\alpha rps$-closed set K of (Y,σ) containing S such that f^{-1}(K) $\subseteq$ F.

Proof

Suppose f is a $s\alpha rps$-open map. Let S $\subseteq$ Y and F be a closed set of (X,τ) such that f^{-1}(S) $\subseteq$ F. Now X \ F is an open set in (X,τ). Since f is $s\alpha rps$-open, f(X \ F) is $s\alpha rps$-open in (Y,σ). Then K = Y \ f(X \ F) is $s\alpha rps$-closed in (Y,σ). Now f^{-1}(S) $\subseteq$ F implies S $\subseteq$ K and f^{-1}(K) = f^{-1}(Y \ f(X \ F)) $\subseteq$ f^{-1}(Y) \ (X \ F) = F. That is f^{-1}(K) $\subseteq$ F. Conversely, let U be an open set of (X,τ). Then f(U) is a subset of (Y,σ) and so Y \ f(U) is a subset of (Y,σ). Therefore f^{-1}(Y \ f(U)) $\subseteq$ X \ U. By hypothesis, there exists a $s\alpha rps$-closed set K of (Y,σ) such that Y \ f(U) $\subseteq$ K and f^{-1}(K) $\subseteq$ X \ U and so U $\subseteq$ X \ f^{-1}(K). Hence Y \ K $\subseteq$ f(U) $\subseteq$ f(X \ f^{-1}(K)) $\subseteq$ Y \ K which implies f(U) = Y \ K. Since Y \ K is $s\alpha rps$-open in (Y,σ), f(U) is $s\alpha rps$-open in (Y,σ). Therefore f is a $s\alpha rps$-open map.

Theorem 4.20

If a mapping f: (X,τ)$\rightarrow$(Y,σ) is $s\alpha rps$-open, then f^{-1}($s\alpha rps$-*cl*B) $\subseteq$ *cl* f^{-1}(B) for every subset B of (Y,σ).

Proof

Let f: (X,τ)$\rightarrow$(Y,σ) be $s\alpha rps$-open and B be any subset of (Y,σ). Then f^{-1}(B) $\subseteq$ *cl* f^{-1}(B) and *cl* f^{-1}(B) is closed in (X,τ). By Theorem 4.19, there exist a $s\alpha rps$-closed set K of (Y,σ) such that B $\subseteq$ K and f^{-1}(K) $\subseteq$ *cl* f^{-1}(B). Since K is $s\alpha rps$-closed, by Lemma 2.7(v), $s\alpha rps$-*cl*K = K. Since B $\subseteq$ K, by Lemma 2.7(i), $s\alpha rps$-*cl*B $\subseteq$ $s\alpha rps$-*cl*K = K. Therefore f^{-1}($s\alpha rps$-*cl*B) $\subseteq$ f^{-1}(K) $\subseteq$ *cl* f^{-1}(B). Thus f^{-1}($s\alpha rps$-*cl*B) $\subseteq$ *cl* f^{-1}(B).

Theorem 4.21

Let f: (X,τ)$\rightarrow$(Y,σ) and g: (Y,σ) $\rightarrow$(Z,μ) be two mappings. Let h = g$\circ$f.

(i) If f is open and g is $s\alpha rps$-open, then h is $s\alpha rps$-open.

(ii) If f is regular open and g is $s\alpha rps$-open, then h is $s\alpha rps$-open.

Proof

(i) Let F be open in (X, τ). Since f is open, f(F) is open in (Y, σ). Since g is $s\alpha rps$-open, g(f(F)) is $s\alpha rps$-open in (Z, μ). That is h(F) is $s\alpha rps$-open in (Z, μ).

This proves (i).

(ii) Let F be open in (X, τ). Since f is regular open, f(F) is regular open in (Y, σ). Since every regular open set is open, f(F) is open in (Y, σ). Since g is $s\alpha rps$-open, g(f(F)) is $s\alpha rps$-open in (Z, μ). That is h(F) is $s\alpha rps$-open in (Z, μ). This proves (ii).

Theorem 4.22

Let f: (X, τ) $\rightarrow$ (Y, σ) and g: (Y, σ) $\rightarrow$ (Z, μ) be two mappings. Let h = g $\circ$ f be $s\alpha rps$-open.

(i) If f is continuous and surjective, then g is $s\alpha rps$-open.

(ii) If g is $s\alpha rps$-irresolute and injective, then f is $s\alpha rps$-open.

(iii) If g is strongly $s\alpha rps$-continuous and injective, then f is $s\alpha rps$-open.

Proof

(i) Let F be open in (Y, σ). Since f is continuous, $f^{-1}(F)$ is open in (X, τ). Since h is $s\alpha rps$-open, h($f^{-1}(F)$) is $s\alpha rps$-open in (Z, μ). That is (g $\circ$ f) ($f^{-1}(F)$) is $s\alpha rps$-open in (Z, μ). That is g(F) is $s\alpha rps$-open in (Z, μ). This proves (i).

(ii) Let F be open in (X, τ). Since h is $s\alpha rps$-open, h(F) is $s\alpha rps$-open in (Z, μ). That is g(f(F)) is $s\alpha rps$-open in (Z, μ). Since g is $s\alpha rps$-irresolute, g^{-1}(g(f(F))) is $s\alpha rps$-open in (Y, σ). That is f(F) is $s\alpha rps$-open in (Y, σ). This proves (ii).

(iii) Let F be open in (X, τ). Since h is $s\alpha rps$-open, h(F) is $s\alpha rps$-open in (Z, μ). That is g(f(F)) is $s\alpha rps$-open in (Z, μ). Since g is strongly $s\alpha rps$-continuous, g^{-1}(g(f(F))) is open in (Y, σ). That is f(F) is open in (Y, σ). Since every open set is $s\alpha rps$-open, f(F) is $s\alpha rps$-open in (Y, σ). This proves (iii).

Theorem 4.23

Let f: (X, τ) $\rightarrow$ (Y, σ) be a bijection. Then the following are equivalent.

(i) f is $s\alpha rps$-open.

(ii) f is $s\alpha rps$-closed.

(iii) f^{-1} is $s\alpha rps$-continuous.

Proof

(i) $\Rightarrow$ (ii)

Suppose f is $s\alpha rps$-open. Let F be closed in (X, τ). Then X \ F is open in (X, τ). Since f is $s\alpha rps$-open, f(X \ F) is $s\alpha rps$-open in (Y, σ). Since f is a bijection, Y \ f(F) is $s\alpha rps$-open in (Y, σ). Therefore f(F) is $s\alpha rps$-closed in (Y, σ).

(ii) $\Rightarrow$ (iii)

Suppose f is $s\alpha rps$-closed. Let g = f^{-1}. Let U be open in (X, τ). Then X \ U is closed in (X, τ). Since f is $s\alpha rps$-closed, f(X \ U) is $s\alpha rps$-closed in (Y, σ). Since f is a bijection, Y \ f(U) is $s\alpha rps$-closed that implies f(U) is $s\alpha rps$-open in (Y, σ). Since g = f^{-1} and since f and g are bijection, g^{-1}(U) = f(U) so that g^{-1}(U) is $s\alpha rps$-open in (Y, σ). Therefore f^{-1} is $s\alpha rps$-continuous.

(iii) $\Rightarrow$ (i)

Suppose f^{-1} is $s\alpha rps$-continuous. Let U be open in (X, τ). Then X \ U is closed in (X, τ). Since f^{-1} is $s\alpha rps$-continuous, $(f^{-1})^{-1}$(X \ U) = f(X \ U) = Y \ f(U) is $s\alpha rps$-closed in (Y, σ) that implies f(U) is $s\alpha rps$-open in (Y, σ). Therefore f is $s\alpha rps$-open.

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