On Vertex Prime Cordial Labeling On Some Graphs

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Abstract:- **Let G be a (p,q) graph. We define the Vertex Prime Cordial labeling as follows. Let V(G), E(G) denote the vertex set and edge set of G respectively. Consider a bijection $f : E(G) \to \{0, 1, 2, \dots, |E|\}$ such that for each vertex of degree atleast two and the induced function $f^* : V(G) \to \{0, 1\}$ is defined by**

$$f^*(u) = \begin{cases} 1; & \text{if gcd of labels of the edges incident at } u \text{ is } 1 \\ 0; & \text{otherwise} \end{cases}$$

satisfies the condition $|v_f(0) - v_f(1)| \le 1$ where $v_f(i)$ = number of vertices labeled with i where i = 0,1. In this paper we proved the following graphs are Vertex Prime Cordial labeling.

***Keywords*: Pyramid graph, Duplicating all the vertices of path, Closed Helm.**

Introduction:

A graph labeling (or) valuation of a graph G is an assignment of labels to the vertices of G that induces for each xy a labels depending on the vertex labels f(x) and f(y). For all terminology and notations we use [5]. In 1987 Cahit [1] introduced a variation of both graceful and harmonious labeling and called such labeling as cordial labeling. In 2005 Sundaram, Ponraj and Somasundarm [6] have introduced the notion of prime cordial labeling. In 2006 Sundaram, and Somasundarm introduced the class of product cordial labeling and total product cordial labeling and studied in detail.

Definition:1.1

A binary vertex labeling f of a graph G is called a **cordial labeling** if $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$. A graph G is called cordial graph if it admits cordial labeling.

Definition :1.2

A **prime cordial labeling** of a graph G with vertex set V(G) is a bijection $f : V(G) \to \{1, 2, 3, \dots, |V(G)|\}$ and the induced function $f^* : E(G) \to \{0, 1\}$ is defined by

$$f^*(e = uv) = \begin{cases} 1; & \text{if gcd } (f(u), f(v)) = 1 \\ 0; & \text{otherwise} \end{cases}$$

satisfies the condition $|e_f(0) - e_f(1)| \le 1$. A graph which admits prime cordial labeling is called a prime cordial graph.

In this paper we define the new concept **Vertex Prime Cordial Labeling** of graphs as follows.

Definition:1.3

A **vertex prime cordial labeling** of a graph G with edge set E(G) is a bijection $f : E(G) \to \{0, 1, 2, \dots, |E|\}$ such that for each vertex of degree atleast two and the induced function $f^* : V(G) \to \{0, 1\}$ defined by

$$f^*(u) = \begin{cases} 1; & \text{if gcd of labels of the edges incident at u is 1} \\ 0; & \text{otherwise} \end{cases}$$

satisfies the condition $|v_f(0) - v_f(1)| \le 1$ where $v_f(i)$ = number of vertices labelled with i where i = 0, 1. A graph which admits vertex prime cordial labeling is called a vertex prime cordial graph.

Theorem:1.1

Cycle graph C_n is a vertex prime cordial graph for all $n \ge 3$.

Proof

Let the vertex set of C_n be $V(C_n) = \{u_i / 1 \le i \le n\}$. And the edge set of C_n be $E(G) = \{(u_i u_{i+1}) / 1 \le i \le n-1\} \cup \{(u_1 u_n)\}$. It has n vertices and n edges.

Define $f : E(C_n) \to \{0, 1, 2, \dots, |E|\}$ as follows

Case (i) Suppose n is even

$$f(e_i) = f(u_i u_{i+1}) = 2i - 1, \quad 1 \le i \le \frac{n}{2}$$

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$f(e_i) = f(u_i u_{i+1}) = 2\left(i - \left(\frac{n}{2}+1\right)\right), \frac{n}{2}+1 \le i \le n - 1$

$f(e_n) = f(u_1 u_n) = n - 2$

Thus all the edge values are distinct.
Now the corresponding vertex labels are

$f^*(u_i) = \gcd(f(e_{i-1}), f(e_i)), 1 \le i \le \frac{n}{2}$

$= \gcd(2i - 3, 2i - 1)$ where i is odd
$= \gcd(k, k + 2)$ where k is odd

$f^*(u_i) = \gcd(f(e_{i-1}), f(e_i)), \frac{n}{2} + 1 \le i \le n -1$

$= \gcd\{2\left(i - 1 - \left(\frac{n}{2}+1\right)\right), \left(i - \left(\frac{n}{2}+1\right)\right)\}$

where i is even
$= \gcd(k_1, k_1 + 2)$ where k_1 is even
$= 0$

Thus $f^*(u_i) = 1, 1 \le i \le \frac{n}{2}$ and

$f^*(u_i) = 0, \frac{n}{2} + 1 \le i \le n$

Then $v_f(1) = v_f(0) = \frac{n}{2}$

Case (ii) Suppose n is odd

$f(e_i) = f(u_i u_{i+1}) = 2i - 1, 1 \le i \le \left\lceil \frac{n}{2} \right\rceil$

$f(e_i) = f(u_i u_{i+1}) = 2\left(i - \left(\left\lceil \frac{n}{2} \right\rceil + 1\right)\right), \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n -1,$

$f(e_n) = f(u_1 u_n) = n - 3$

Thus all the edge values are distinct.
Now the corresponding vertex labels are

$f^*(u_i) = \gcd(f(e_{i-1}), f(e_i)), 1 \le i \le \left\lceil \frac{n}{2} \right\rceil$

$= \gcd(2i - 3, 2i - 1)$ where i is odd
$= \gcd(k, k + 2)$ where k is odd

$f^*(u_i) = \gcd(f(e_{i-1}), f(e_i)), \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n -1$

$= \gcd\{2\left(i - 1 - \left(\left\lceil \frac{n}{2} \right\rceil + 1\right)\right), 2\left(i - \left(\left\lceil \frac{n}{2} \right\rceil + 1\right)\right)\}$ where i is even

$f^*(u_i) = \gcd(k_1, k_1 + 2)$ where k_1 is even
$= 0$

Thus $f^*(u_i) = 1, 1 \le i \le \left\lceil \frac{n}{2} \right\rceil$ and

$f^*(u_i) = 0, \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n$

Then $v_f(1) = \left\lceil \frac{n}{2} \right\rceil$ and $v_f(0) = \left\lfloor \frac{n}{2} \right\rfloor$

From the above results we have $| v_f(1) - v_f(0) | \le 1$.
Hence C_n is vertex prime cordial graph.

***Illustration*:1**

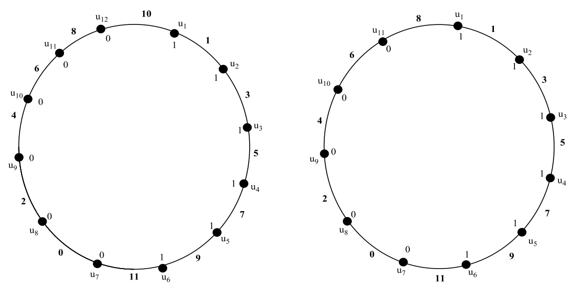

Fig:1 C_{12} is vertex prime cordial graph

C_{11} is vertex prime cordial graph.

Theorem:1.2

Path graph P_n is a vertex prime cordial graph for all n.

Definition:1.4 *(Pyramid Graph)*

The graph obtained by arranging vertices into a finite number of rows with i vertices in the i^{th} row and in every row the j^{th} vertex and $j +1^{st}$ vertex of the next row is called the pyramid. We denote the pyramid with n rows by Py_n.

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Theorem:1.3

All pyramids are vertex prime cordial graph.

Proof

Let a_{11} be the unique vertex of the Py_n. Let a_{21} a_{22} be the vertices in the first level with 2 edges. Let a_{31}, a_{32}, a_{33} be the vertices in the second level. Hence a_{3j}, j = 1, 2, 3 be the vertices in the third level with n = 3 and with 6 edges. proceeding like this, a_{n1}, a_{n2}, a_{n3} , . . a_{nn} be the vertices in the n^{th} level and the corresponding edges will be (n – 1)n.

Now the graph Py_n has $\frac{n(n+1)}{2}$ vertices and n(n – 1) edges.

Define $f : E(Py_n) \rightarrow \{0, 1, 2, . . . |E|\}$ as follows

$f(a_{11}a_{21}) = 1$ $\quad$ $f(a_{31}a_{41}) = 7$

$f(a_{11}a_{22}) = 0$ $\quad$ $f(a_{31}a_{42}) = 9$

$f(a_{21}a_{31}) = 3$ $\quad$ $f(a_{33}a_{44}) = 6$

$f(a_{22}a_{33}) = 2$ $\quad$ $f(a_{32}a_{42}) = 11$

$f(a_{21}a_{32}) = 5$ $\quad$ $f(a_{32}a_{43}) = 10$

$f(a_{22}\, a_{32}) = 4$ $\quad$ $f(a_{33}a_{43}) = 8$

Proceeding like this In general

$$f(a_{ij}a_{(i+1)j}) = 2m + 1 \text{ , } m = 0, 1, 3, 5, 6, 8 \dots \frac{n}{2},$$

$$1 \le i \le n, 1 \le j \le \left\lceil \frac{n}{2} \right\rceil$$

$$f(a_{ij}a_{(i+1)(j+1)}) = 2m+1, m = 2, 4, 7, \dots \frac{n}{2}, 1 \le i \le n,$$

$$1 \le j \le \left\lceil \frac{n}{2} \right\rceil$$

$$f(a_{ij}a_{(i+1)(j)}) = 2m, m = 2, 4, 7, 9 \dots \frac{n}{2}, 1 \le i \le n-1,$$

$$2 \le j \le \left\lfloor \frac{n}{2} \right\rfloor$$

$$f(a_{ij}a_{(i+1)(j+2)}) = 2m \text{ , } m = 0, 1, 3, 5, 6, 8, \dots \frac{n}{2},$$

$$1 \le i \le n-1, 2 \le j \le \left\lfloor \frac{n}{2} \right\rfloor.$$

Then the vertex labels are

$$f(a_{ij}) = \begin{cases} 1, & \text{if } 1 \le j \le \left\lceil \frac{n}{2} \right\rceil, \\ 0, & \text{if } \left\lfloor \frac{n}{2} \right\rfloor \le j \le n \end{cases}$$

$$1 \le i \le n$$

Case (i) Suppose n is even

$$v_f(1) - v_f(0) = \frac{n}{2}.$$

Case (ii) Suppose n is odd

$$v_f(1) = \left\lceil \frac{n}{2} \right\rceil \text{ and } v_f(0) = \left\lfloor \frac{n}{2} \right\rfloor.$$

From above result we get $| v_f(1) - v_f(0) | \le 1$, which satisfy the condition of vertex prime cordial graph.

Hence Py_n is vertex prime cordial graph.

We now prove in particular for the vertices $\mathbf{a_{32}}$ and $\mathbf{a_{43}}$ as follows

$f(a_{32}) = \gcd\{ f(a_{21}a_{32}), f(a_{22}a_{32}), f(a_{32}a_{42}), f(a_{32}a_{43})\}$

$= \gcd\{3,4,11,10\}$

$= 1$

$f(a_{43}) = \gcd\{ f(a_{32}a_{43}), f(a_{33}a_{43}), f(a_{43}a_{53}), f(a_{43}a_{54})\}$

$= \gcd\{10,8,18,16\}$

$= 0$ and so on.

Illustration:2

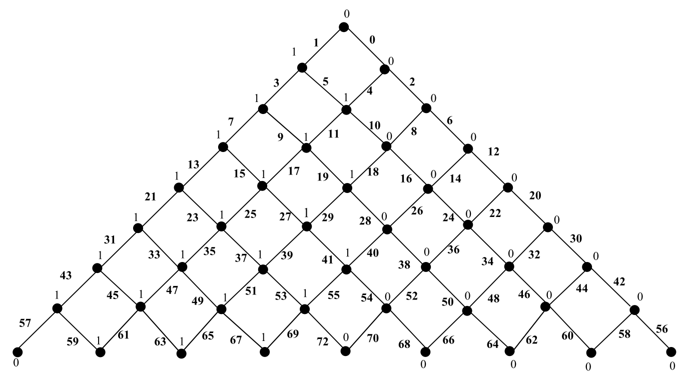

Fig: 2 Py_9 is vertex prime cordial graph.

Theorem:1.4

The graph obtained by duplicating all the vertices by edges in path P_n is vertex prime cordial graph.

Proof

Let $v_1, v_2, \dots , v_n$ be the vertices and $e_1, e_2, \dots , e_{n-1}$ be edges of path P_n.

Let the graph obtained by duplicating all the vertices by edges in path P_n is G. Let the edge so added corresponding to vertex v_n has end vertices v_n' and v_n''.

Let the vertex set of $V(G) = \{v_i / 1 \le i \le n\}$

$\cup \{ v_i' / 1 \le i \le n \} \cup \{ v_i'' / 1 \le i \le n \}$

and the edge set of be $E(G) = \{ v_i v_{i+1} / 1 \le i \le n-1\}$

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$\cup\{v_i' v_i'' \,/\, 1 \le i \le n\} \cup \{v_i v_i' \,/\, 1 \le i \le n\}$
$\cup\{v_i v_i'' \,/\, 1 \le i \le n\}$.

Note that the graph obtained by duplicating all the vertices by edges in path P_n has 3n vertices and 4n - 1 edges.

Define $f : E(G) \to \{0, 1, 2, \ldots, |E|\}$ as follows

$$f(v_i v_i') = 6(i-1)+1, \quad 1 \le i \le \left\lceil \frac{n}{2} \right\rceil$$

$$f(v_i v_i'') = 6(i-1)+5, \quad 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$$

$$f(v_i' v_i'') = 6(i-1)+3, \quad 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$$

$$f(v_i v_i') = 6\left(i - \left\lfloor \frac{n}{2} \right\rfloor\right), \quad \left\lfloor \frac{n}{2} \right\rfloor \le i \le n$$

$$f(v_i v_i'') = 6\left(i - \left\lceil \frac{n}{2} \right\rceil\right) + 4, \quad \left\lceil \frac{n}{2} \right\rceil \le i \le n$$

$$f(v_i' v_i'') = 6\left(i - \left\lceil \frac{n}{2} \right\rceil\right) + 2, \quad \left\lceil \frac{n}{2} \right\rceil \le i \le n$$

$$f(v_i v_{i+1}) = 3n + 2(i-1), \quad 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor, \text{ n is odd}$$

$$f(v_i v_{i+1}) = 3n + 1 + 2(i-1), \quad 1 \le i \le \frac{n}{2} - 1, \text{ n is even}$$

$$f(v_i v_{i+1}) = 3n+1+2\left(i - \left(\left\lfloor \frac{n}{2} \right\rfloor + 1\right)\right), \quad \left\lfloor \frac{n}{2} \right\rfloor + 1 \le i \le n-1, \text{ n is odd}$$

$$f(v_i v_{i+1}) = 3n + 2\left(i - \frac{n}{2}\right), \quad \frac{n}{2} \le i \le n-1, \text{ n is even}$$

Then the vertex labels are as follows

Case (i) When n is odd

$$f^*(v_i) = 1, \quad 1 \le i \le \left\lceil \frac{n}{2} \right\rceil$$

$$f^*(v_i) = 0, \quad \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n$$

$$f^*(v_i') = 1, \quad 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$$

$$f^*(v_i') = 0, \quad \left\lfloor \frac{n}{2} \right\rfloor + 1 \le i \le n$$

$$f^*(v_i'') = 1, \quad 1 \le i \le \left\lceil \frac{n}{2} \right\rceil$$

$$f^*(v_i'') = 0, \quad \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n.$$

In view of the above labeling pattern we have

$$v_f(1) = \left\lceil \frac{n}{2} \right\rceil \text{ and } v_f(0) = \left\lfloor \frac{n}{2} \right\rfloor.$$

Case (ii) When n is even

$$f^*(v_i) = 1, \quad 1 \le i \le \frac{n}{2}$$

$$f^*(v_i) = 0, \quad \frac{n}{2} + 1 \le i \le n$$

$$f^*(v_i') = 1, \quad 1 \le i \le \frac{n}{2}$$

$$f^*(v_i') = 0, \quad \frac{n}{2} + 1 \le i \le n$$

$$f^*(v_i'') = 1, \quad 1 \le i \le \frac{n}{2}$$

$$f^*(v_i'') = 0, \quad \frac{n}{2} + 1 \le i \le n$$

In view of the above labeling pattern
we have $v_f(1) = v_f(0) = \frac{n}{2}$.

Then from the case 1 and case 2
we have $|v_f(1) - v_f(0)| \le 1$.

Hence duplicating all the vertices by edges in path P_n is vertex prime cordial graph.

***Illustration*:3**

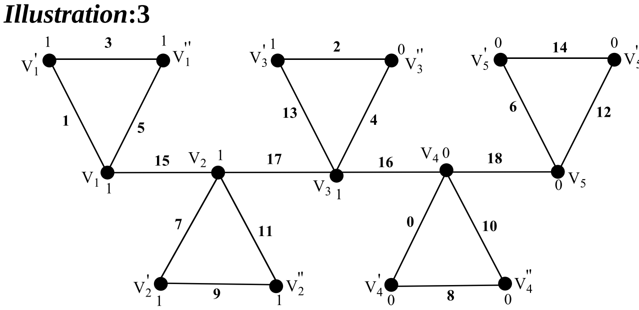

Fig: 3 The graph obtained by duplicating all the vertices by edges in path P_5 is vertex prime cordial graph

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Illustration:4

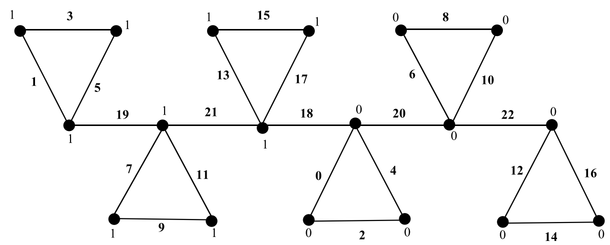

Fig: 4 The graph obtained by duplicating all the vertices by edges in path P_6 is vertex prime cordial graph.

***Definition*:1.5**

The closed helm CH_n is the graph obtained from a helm H_n by joining each pendent vertex to form a cycle.

***Theorem* : 1.5**

Closed hem CH_n is a vertex prime cordial graph.

Proof

Let v be the apex vertex $v_1, v_2, \ldots, v_n$ be the vertices of inner cycle and $u_1, u_2, \ldots, u_n$ be the vertices of outer cyle CH_n.

Let the vertex set be $V(CH_n) = \{ v_i / 1 \le i \le n \} \cup \{ u_i / 1 \le i \le n \} \cup \{ v \}$

and the edge set be $E(CH_n) = \{ vv_i / 1 \le i \le n \} \cup \{v_i v_{i+1} / 1 \le i \le n - 1\} \cup \{v_1 v_n\} \cup \{v_i u_i / 1 \le i \le n \} \cup \{u_i u_{i+1} / 1 \le i \le n -1 \} \cup \{u_1 u_n\}$.

Note that the graph CH_n has 2n + 1 vertices and 4n edges.

Now define $f : E(G) \to \{0, 1, 2, \ldots, |E|\}$ as follows

Case (i) If n is odd

$$f(vv_i) = 2i - 1, \quad 1 \le i \le \left\lceil \frac{n}{2} \right\rceil$$

$$f(vv_i) = 2\left(i - \left(\left\lceil \frac{n}{2} \right\rceil + 1\right)\right), \quad 1 \le i \le \left\lceil \frac{n}{2} \right\rceil + 1$$

$$f(v_i v_{i+1}) = n + 2i, \quad 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$$

$$f(v_i v_{i+1}) = n - 1 + 2\left(i - \left(\left\lfloor \frac{n}{2} \right\rfloor + 1\right)\right), \quad \left\lfloor \frac{n}{2} \right\rfloor + 1 \le i \le n$$

$$f(v_i u_i) = 2n + 1 + 2(i - 1), \quad 1 \le i \le \left\lceil \frac{n}{2} \right\rceil$$

$$f(v_i u_i) = 2n + 2\left(i - \left(\left\lceil \frac{n}{2} \right\rceil + 1\right)\right), \quad \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n$$

$$f(u_i u_{i+1}) = 3n + 2 + 2(i - 1), \quad 1 \le i \le \left\lceil \frac{n}{2} \right\rceil$$

$$f(u_i u_{i+1}) = 3n - 1 + 2\left(i - \left(\left\lceil \frac{n}{2} \right\rceil + 1\right)\right), \quad \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n.$$

Then the vertex labels corresponding to the edges are as follows.

$$f^*(v) = 0$$

$$f^*(v_i) = 1, \quad 1 \le i \le \left\lceil \frac{n}{2} \right\rceil$$

$$f^*(v_i) = 0, \quad \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n$$

$$f^*(u_i) = 1, \quad 1 \le i \le \left\lceil \frac{n}{2} \right\rceil$$

$$f^*(u_i) = 0, \quad \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n.$$

In view of the above labeling pattern we have

$$v_f(1) = \left\lceil \frac{n}{2} \right\rceil, \; v_f(0) = \left\lfloor \frac{n}{2} \right\rfloor.$$

Case (ii) If n is even

$$f(vv_i) = 2i - 1, \quad 1 \le i \le \frac{n}{2}$$

$$f(vv_i) = 2\left(i - \left(\frac{n}{2} + 1\right)\right), \quad \frac{n}{2} + 1 \le i \le n$$

$$f(v_i v_{i+1}) = n + 1 + (i - 1), \quad 1 \le i \le \frac{n}{2}$$

$$f(v_i v_{i+1}) = n + 2\left(i - \left(\frac{n}{2} + 1\right)\right), \quad \frac{n}{2} + 1 \le i \le n - 1$$

$$f(v_n v_1) = 2n - 2$$

$$f(v_i u_i) = 2n - 1 + 2(i - 1), \quad 1 \le i \le \frac{n}{2}$$

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$$f(v_i u_i) = 2n + 2\left(i - \left(\frac{n}{2} + 1 \right) \right), \quad \frac{n}{2} + 1 \le i \le n$$

$$f(u_i u_{i+1}) = 3n - 1 + 2(i - 1), \quad 1 \le i \le \frac{n}{2}$$

$$f(u_i u_{i+1}) = 3n + 2\left(i - \left(\frac{n}{2} + 1 \right) \right), \quad \frac{n}{2} + 1 \le i \le n$$

$$f(u_i u_n) = 4n - 2$$

Then the vertex labels are as follows

$$f^*(v) = 0$$

$$f^*(v_i) = 1, \qquad 1 \le i \le \frac{n}{2}$$

$$f^*(v_i) = 0, \qquad \frac{n}{2} + 1 \le i \le n$$

$$f^*(u_i) = 1, \qquad 1 \le i \le \frac{n}{2} + 1$$

$$f^*(u_i) = 0, \quad \frac{n}{2} + 2 \le i \le n.$$

In view of the above labeling pattern

we have $v_f(1) = \left\lceil \frac{n}{2} \right\rceil$ $v_f(0) = \left\lfloor \frac{n}{2} \right\rfloor$.

Thus from case 1 and case 2 we have

$| v_f(1) - v_f(0) | \le 1$, which satisfy the condition of vertex prime cordial graph. Hence CH_n is a vertex prime cordial graph.

Illustration:6

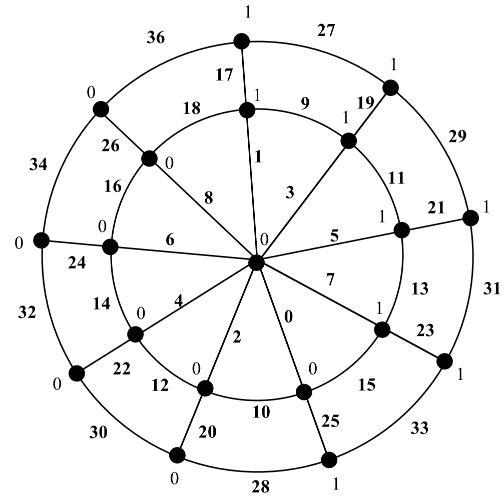

Fig: 6 CH_9 is a vertex prime cordial graph.

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