On Vertex Polynomial of Comb and Crown

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Abstract **- Let $G = (V, E)$ be a graph. The vertex polynomial of the graph $G = (V, E)$ is defined as $V(G, x) = \sum_{k=0}^{\Delta(G)} v_k x^k$, where $\Delta(G) = \max\{d(v)/v\epsilon V\}$ and v_k is the number of vertices of degree k. In this paper we seek to find the vertex polynomial of Comb, Crown and some other Graphs.**

Keywords**- Comb, Crown, Vertex Polynomial, Union, Sum.**

1. INTRODUCTION

In a graph $G = (V, E)$, we mean a finite undirected, non-trivial graph without loops and multiple edges. The vertex set is denoted by V and the edge set by E. For $v\epsilon V$, $d(v)$ is the number of edges incident with v, the maximum degree of G is defined as $\Delta(G) = \max\{d(v)/v\epsilon V\}$. For terms not defined here, we refer to Frank Harary[6]. The graph $G = (V, E)$ is simply denoted by G. Let G_1 and G_2 be two graphs, the union $G_1 \cup G_2$ is defined to be (V, E) where $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$, the sum $G_1 + G_2$ is defined as $G_1 \cup G_2$ together with all the lines joining points of V_1 to V_2. The graph obtained by joining a single pendent edge to each vertex of a path is called Comb. Any cycle with pendant edge attached to each vertex is called Crown.

2. VERTEX POLYNOMIAL OF COMB, THEIR UNION AND THEIR SUM.

Definition:2.1

The graph obtained by joining a single pendent edge to each vertex of a path is called Comb.

Theorem: 2.2

Let G be a Comb with order $2n, (n \geqslant 2)$. The vertex polynomial of G is

$$V(G, x) = (n-2)x^3 + 2x^2 + nx, n \geqslant 2.$$

Proof:

Let G be a Comb with order $2n, (n \geqslant 2)$. By Comb definition, n vertices are pendent vertices, among remaining n vertices $n - 2$ have degree 3 and 2 vertices have degree 2.

Example: 2.3

Take $n = 3$ in the above theorem. We have the graph

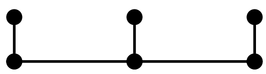

Figure: 1

Here, $V(G, x) = x^3 + 2x^2 + 3x$.

Theorem: 2.4

Let G be a Comb with order $2n, (n \geqslant 2)$. The vertex polynomial of $\zeta = G \cup G \cup \ldots \cup G$ $(m\ times)$ is

$$V(G, x) = m(n-2)x^3 + 2mx^2 + mnx, n \geqslant 2.$$

Proof:

Let G be a Comb with order $2n, (n \geqslant 2)$. Consider m copies of G. Since, G has order $2n, (n \geqslant 2)$ we have m copies of G has order $2mn, (n \geqslant 2)$. In m copies of G, mn vertices are pendent vertices, $2m$ vertices having degree 2, $m(n-2)$ vertices have degree 3 gives the required result.

Theorem: 2.5

Let G be a Comb with order $2n, (n \geqslant 2)$. The vertex polynomial of mG is

$$V(G, x) = m(n-2)x^{3+2n(m-1)} + 2mx^{2+2n(m-1)} + mnx^{2n(m-1)},\ n \geqslant 2.$$

Proof:

Using definition of sum of graphs, degree of each vertex in ζ is increased by $2n(m-1)$ in mG gives the required result.

3. VERTEX POLYNOMIAL OF CROWN ($C_N \odot K_1$), THEIR UNION AND THEIR SUM.

Definition: 3.1

Any cycle with pendant edge attached to each vertex is called Crown.

Theorem: 3.2

Let G be a Crown with order $2n$. Then the vertex polynomial of G is given by $V(G, x) = nx^3 + nx, n \geqslant 3$.

Proof:

Let G be a Crown with order $2n, (n \geqslant 3)$. By Crown definition, n vertices are pendant vertices; remaining n vertices have degree 3 gives the required result.

Example: 3.3

Take $n = 3$ in the above theorem. We have the graph,

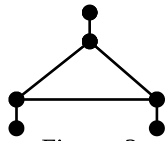

Figure: 2

Here, $V(G, x) = 3x^3 + 3x$.

Theorem: 3.4

Let G be a Crown with order $2n, (n \geqslant 3)$. The vertex polynomial of $\zeta = G \cup G \cup \ldots \cup G$ $(m\ times)$ is

$$V(G, x) = mnx^3 + mnx,\ n \geqslant 3.$$

Proof:

Let G be a Crown with order $2n, (n \geqslant 3)$. Consider m copies of G. Since, G has order $2n, (n \geqslant 3)$ we have m copies of G has order $2mn, (n \geqslant 3)$. In m copies of G, mn vertices are pendent vertices, mn vertices having degree 3 gives the required result.

Theorem: 3.5

Let G be a Crown with order $2n, (n \geqslant 3)$. The vertex polynomial of mG is given by

$$V(G,x) = mnx^{3+2n(m-1)} + mnx^{1+2n(m-1)},\ n \geqslant 3.$$

Proof:

Using definition of sum of graphs, degree of each vertex in $\zeta = G \cup G \cup \dots \cup G\ (m\ times)$ is increased by $2n(m-1)$ in mG gives the required result.

4. Vertex Polynomial of $P_N \odot \overline{K}_M, (N \geqslant 2)$, their union and their sum

Theorem: 4.1

Let G be $P_n \odot \overline{K}_m$. Then the vertex polynomial of G is given by

$$V(G,x) = (n-2)x^{m+2} + 2x^{m+1} + nmx, n \geqslant 2.$$

Proof:

Let G be $P_n \odot \overline{K}_m$. We can observe that, $n-2$ vertices have degree $m+2$, 2 vertices have degree $m+1$ and nm vertices have degree 1 shows the required result.

Example: 4.2

Consider the Graph $P_6 \odot \overline{K}_3$. Then the corresponding graph is illustrated as follows;

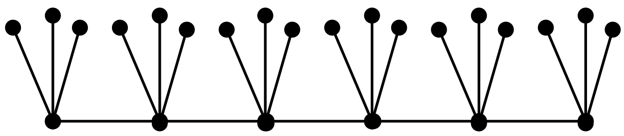

Figure: 3

Here, $V(G,x) = 4x^5 + 2x^4 + 18x$.

Results: 4.3

(i) Let G be $P_n \odot \overline{K}_m$. Then the vertex polynomial of $\zeta = G \cup G \cup \dots \cup G\ (k\ times)$ is given by $V(G,x) = k(n-2)x^{m+2} + 2kx^{m+1} + knmx, n \geqslant 2$.

(ii) Let G be $P_n \odot \overline{K}_m$. Then the vertex polynomial of kG is given by

$$V(G,x) = k(n-2)x^{(m+2)+n(k-1)(m+1)} + 2kx^{(m+1)+n(k-1)(m+1)} + knmx^{n(k-1)(m+1)}, n \geqslant 2.$$

5. Vertex Polynomial of $C_N \odot \overline{K}_M, (N \geqslant 3)$, their union and their sum

Theorem: 5.1

Let G be $C_n \odot \overline{K}_m, (n \geqslant 3)$. Then the vertex polynomial of G is given by

$$V(G,x) = nx^{m+2} + nmx,\ n \geqslant 3.$$

Proof:

Let G be $C_n \odot \overline{K}_m$. We can observe that, n vertices have degree $m+2$ and nm vertices have degree 1 shows the required result.

Example: 5.2

Consider the Graph $C_4 \odot \overline{K}_2$. Then the corresponding graph is depicted as follows;

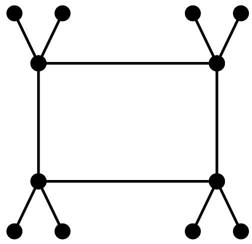

Figure: 4

Here, $V(G,x) = 4x^4 + 8x$.

Results: 5.3

(i) Let G be $C_n \odot \overline{K}_m, (n \geqslant 3)$. Then the vertex polynomial of $\zeta = G \cup G \cup \dots \cup G\ (k\ times)$ is given by $V(G,x) = knx^{m+2} + knmx$.

(ii) Let G be $C_n \odot \overline{K}_m, (n \geqslant 3)$. Then the vertex polynomial of kG is given by $V(G,x) = knx^{(m+2)+n(k-1)(1+m)} + knmx^{n(k-1)(1+m)}$.

6. Vertex Polynomial of $L_N \odot \overline{K}_M, (N \geqslant 2)$, their union and their sum

Theorem: 6.1

Let G be $L_n \odot \overline{K}_m, (n \geqslant 2)$. Then the vertex polynomial of G is given by

$$V(G,x) = (2n-4)x^{m+3} + 4x^{m+2} + 2nmx.$$

Proof:

Let G be $L_n \odot \overline{K}_m, (n \geqslant 2)$.. We can observe that, $2n-4$ vertices have degree $m+3$, 4 vertices have degree $m+2$ and $2nm$ vertices have degree 1 gives the required result.

Example: 6.2

Consider the Graph $L_4 \odot \overline{K}_3$. Then the corresponding graph is depicted as follows;

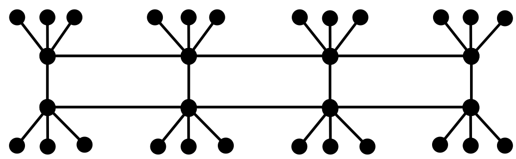

Figure: 5

Here, $V(G,x) = 4x^6 + 4x^5 + 24x$.

Results: 6.3

(i) Let G be $L_n \odot \overline{K}_m, (n \geqslant 2)$. Then the vertex polynomial of $\zeta = G \cup G \cup \dots \cup G\ (k\ times)$ is given by $V(G,x) = k(2n-4)x^{m+3} + 4kx^{m+2} + 2knmx$.

(ii) Let G be $P_n \odot \overline{K}_m, (n \geqslant 2)$. Then the vertex polynomial of kG is given by $V(G,x)$

$$= k(2n-4)x^{(m+3)+2n(1+m)(k-1)} + 4kx^{(m+2)+2n(1+m)(k-1)} + 2knmx^{2n(1+m)(k-1)}.$$

7. VERTEX POLYNOMIAL OF $C_N \odot K_2, (N \geqslant 3)$, THEIR UNION AND THEIR SUM.

Theorem: 7.1

Let G be $C_n \odot K_2, (n \geqslant 3)$. Then the vertex polynomial of G is given by
$V(G,x) = nx^4 + 2nx^2,\ n \geqslant 3.$

Proof:

Let G be $C_n \odot K_2, (n \geqslant 3)$. We can observe that, n vertices have degree 4 and $2n$ vertices have degree 2 shows the required result.

Example: 7.2

Consider the Graph $C_4 \odot K_2$. Then the corresponding graph is depicted as follows;

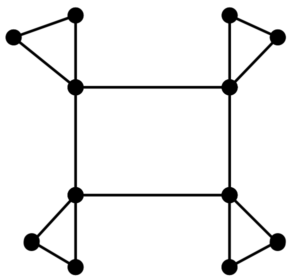

Figure: 6

Here, $V(G,x) = 4x^4 + 8x^2$.

Results: 7.3

(i) Let G be $C_n \odot K_2$, $(n \geqslant 3)$. Then the vertex polynomial of $\zeta = G \cup G \cup \ldots \cup G\ (k\ times)$ is given by $V(G,x) = knx^4 + 2knx^2$.

(ii) Let G be $C_n \odot K_2$, $(n \geqslant 3)$. Then the vertex polynomial of kG is given by
$$V(G,x) = knx^{4+3n(k-1)} + 2knx^{2+3n(k-1)}.$$

8. VERTEX POLYNOMIAL OF $P_N \odot K_2, (N \geqslant 2)$, THEIR UNION AND THEIR SUM.

Theorem: 8.1

Let G be $P_n \odot K_2, (n \geqslant 2)$. Then the vertex polynomial of G is given by
$V(G,x) = (n-2)x^4 + 2x^3 + 2nx^2,\ n \geqslant 2.$

Proof:

Let G be $P_n \odot K_2, (n \geqslant 2)$. We can observe that, $n-2$ vertices have degree 4, 2 vertices have degree 3 and $2n$ vertices have degree 2 shows the required result.

Example: 8.2

Consider the Graph $P_6 \odot K_2$. Then the corresponding graph is illustrated as follows;

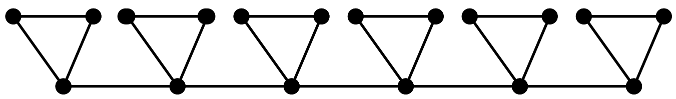

Figure: 7

Here, $V(G,x) = 4x^4 + 2x^3 + 12x^2$.

Results: 8.3

(i) Let G be $P_n \odot K_2, (n \geqslant 2)$. Then the vertex polynomial of $\zeta = G \cup G \cup \ldots \cup G\ (k\ times)$ is given by $V(G,x) = k(n-2)x^4 + 2kx^3 + 2knx^2$.

(ii) Let G be $P_n \odot K_2, (n \geqslant 2)$. Then the vertex polynomial of kG is given by $V(G,x) = k(n-2)x^{4+3n(k-1)} + 2kx^{3+3n(k-1)} + 2knx^{2+3n(k-1)}$.

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