Diophantine 3-Tuple in A.P with property D(α^2)

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Abstract— **In this communication, we exhibit a triple in Arithmetic Progression such that the product of any two added with a square is a perfect square. This result is presented as a theorem.**

Keywords— **Diophantine triple, A.P, quadruple.**

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I. Introduction

The Problem of constructing the sets with property that product of any two of its distinct elements is one less than a square has a very long history and such sets were studied by Diophantus [1]. A set of positive integers $\{a_1, a_2 ......a_m\}$ is said to have the property D (n), $n \in Z - \{0\}$, if $a_i a_j + n$, a perfect square for all $1 \le i < j \le m$ and such a set is called a Diophantine m-tuples with property D(n). Many mathematicians considered the Construction of different formulations of Diophantine triples with property D(n) for any arbitrary integer n and also, for any linear polynomials in n. In this context, one may refer [2-19] for an extensive review of various problem on Diophantine triples. In [20-22], special mention is provided because it differs from the earlier one. This paper aims at constructing an interesting family of 3-tuples different from the earlier one. This interesting triple is constructed where the product of any two members of the triple in A.P added with a square is a perfect square.

II. Theorem

Let (a, b, c) be a triple in A.P such that $a = 3(k+1)^2 r^2 + s^2 - 4(k+1)rs$,

$b = 3(k+1)^2 r^2 + s^2$, $c = 3(k+1)^2 r^2 + s^2 + 4(k+1)rs$

Then, the product of any two added with $D\left((2(k+1)rs)^2\right)$ is a perfect square.

Proof:

$ab + D\left((2(k+1)rs)^2\right)$

$= \left(3(k+1)^2 r^2 + s^2 - 4(k+1)rs\right)\left(3(k+1)^2 r^2 + s^2\right) + (2(k+1)rs)^2$

$= \left(3(k+1)^2 r^2 + s^2 - 2(k+1)rs\right)^2 = \alpha^2$ = Perfect square.

$bc + D\left((2(k+1)rs)^2\right)$

$= \left(3(k+1)^2 r^2 + s^2\right)\left(3(k+1)^2 r^2 + s^2 + 4(k+1)rs\right) + (2(k+1)rs)^2$

$= \left(3(k+1)^2 r^2 + s^2 + 2(k+1)rs\right)^2 = \beta^2$ = Perfect square.

$ac + D\left((2(k+1)rs)^2\right)$

$= \left(3(k+1)^2 r^2 + s^2 - 4(k+1)rs\right)\left(3(k+1)^2 r^2 + s^2 + 4(k+1)rs\right)$

$+ (2(k+1)rs)^2$

$= \left(3(k+1)^2 r^2 - s^2\right)^2 = \gamma^2$ = Perfect square.

Hence, (a, b, c) is a Diophantine 3-tuple in A.P with property $D\left((2(k+1)rs)^2\right)$.

REMARK:

When s=1, the above triples is extended to quadruple with Property $D\left((2(k+1)r)^2\right)$ which is illustrated below:

When s=1, we have

$a = 3(k+1)^2 r^2 - 4(k+1)r + 1,\quad b = 3(k+1)^2 r^2 + 1,$

$c = 3(k+1)^2 r^2 + 4(k+1)r + 1$ which is such that the product of any two added with is a perfect square. If d is the fourth tuple with property $D\left((2(k+1)r)^2\right)$, then we have from Euler's formula

$$d = a + b + c + \frac{1}{2}[abc + \alpha\beta\gamma]$$

ie, $\quad d = 27(k+1)^4 r^4 - 3(k+1)^2 r^2$

Now,

$ad + D\left((2(k+1)r)^2\right)$

$= \left(3(k+1)^2 r^2 - 4(k+1)r + 1\right)\left(27(k+1)^4 r^4 - 3(k+1)^2 r^2\right) + (2(k+1)r)^2$

$= (k+1)^2 r^2 \left(9(k+1)^2 r^2 - 6(k+1)r - 1\right)^2$ = Perfect square.

$cd + D\left((2(k+1)r)^2\right)$
$= \left(3(k+1)^2 r^2 + 4(k+1)r + 1\right)\left(27(k+1)^4 r^4 - 3(k+1)^2 r^2\right)$
$+ (2(k+1)r)^2$

$= (k+1)^2 r^2 \left(9(k+1)^2 r^2 + 6(k+1)r - 1\right)^2$ = Perfect square.
$bd + D\left((2(k+1)r)^2\right)$
$= \left(3(k+1)^2 r^2 + 1\right)\left(27(k+1)^4 r^4 - 3(k+1)^2 r^2\right) + (2(k+1)r)^2$
$= (k+1)^2 r^2 \left(9(k+1)^2 r^2 + 1\right)^2$ = Perfect square.

Thus, the quadruples (a, b, c, d) is a Diophantine quadruple with property $D\left((2(k+1)r)^2\right)$

It is to be noted that the triple (a, b, c) is only in A.P.

III. CONCLUSION

In this paper, we have presented a triple in Arithmetic Progression such that the product of any two added with a square is a perfect square and extended it to a quadruple (a, b, c, d) whereas (a, b, c) is in A.P . To conclude, one may attempt to find Diophantine quadruple (a, b, c, d) is in A.P with suitable property.

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