On non-homogeneous octic equation with four unknowns $x^2 = y^3 + z^5 w^3$

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Abstract— **The non-homogeneous octic equation with four unknowns represented by the diophantine equation $x^2 = y^3 + z^5 w^3$ is analyzed for its patterns of non-zero distinct integral solutions and six different patterns of integral solutions are illustrated.**

Keywords—**Octic non-homogeneous equation, octic equation with four unknowns, integer solutions.**

Introduction

The theory of diophantine equations offers a rich variety of fascinating problems. In particular, homogeneous and non homogeneous equations of higher degree have aroused the interest of numerous Mathematicians since antiquity [1-3].Particularly in [4, 5] special equations of sixth degree with four and five unknowns are studied. In [6-9] heptic equations with three, five and six unknowns are analysed. In [10-11] equations of eighth degree with five and six unknowns are analysed. This paper concerns with the problem of determining non-trivial integral solution of the non-homogeneous equation of eighth degree with four unknowns

I. Method of Analysis

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The eighth degree equation with four unknowns to be solved is

$$x^2 = y^3 + z^5 w^3 \qquad (1)$$

The substitution of the transformations

$$\left.\begin{aligned} x &= t(n+w)^8 \\ y &= n(n+w)^5 \\ z &= (n+w)^3 \end{aligned}\right\} \qquad (2)$$

in (1) gives

$$t^2 = n^2 - nw + w^2 \qquad (3)$$

Again introducing the linear transformations

$$n = u \pm v \quad \text{and} \quad w = u \mp v \qquad (4)$$

in (3), it leads to

$$t^2 = u^2 + 3v^2 \qquad (5)$$

The above equation is solved through different methods and using (4)& (2), different patterns of integer solutions to (1) are obtained.

Pattern 1:

It is well-known that (5) is satisfied by

$$t = 3r^2 + s^2, v = 2rs, u = 3r^2 - s^2$$

Substituting the above values of u, v and t in (4) and (2) , the corresponding values of x, y, z, w are given by

$$x = (3r^2 + s^2)(6r^2 - 2s^2)^8$$

$$y = (3r^2 - s^2 \pm 2rs)(6r^2 - 2s^2)^5$$

$$z = (6r^2 - 2s^2)^3$$

$$w = 3r^2 - s^2 \mp 2rs$$

Pattern 2:

Express (5) as the system of double equations as

$$\left.\begin{aligned} t + u &= v^2 \\ t - u &= 3 \end{aligned}\right\} \qquad (6)$$

Solving the above system of equations (6), the corresponding values of u, v and v are given by

$$u = 2k^2 + 2k - 1$$

$$v = 2k + 1$$

$$t = 2k^2 + 2k + 2$$

Substituting these values of u, v and v in (4) and (2), the corresponding integer solutions to (1) are obtained as

$$x = (2k^2 + 2k + 2)(4k^2 + 4k - 2)^8$$

$$y = (2k^2 - 2)(4k^2 + 4k - 2)^5 (or) (2k^2 + 4k)(4k^2 + 4k - 2)^5$$

$$z = (4k^2 + 4k - 2)^3$$

$$w = (2k^2 + 4k)(or)(2k^2 - 2)$$

Pattern 3:

Writing (5) as the system of double equations as

$$\left.\begin{aligned} t + u &= 3v^2 \\ t - u &= 1 \end{aligned}\right\} \qquad (7)$$

Repeating the process as in Pattern 2, the values x, y, z, and w satisfying (1) are given

$$x = (6k^2 + 6k + 2)(12k^2 + 12k + 2)^8$$
$$y = (6k^2 + 8k + 2)(12k^2 + 12k + 2)^5 (or)(6k^2 + 4k)(12k^2 + 12k + 2)^5$$
$$z = (12k^2 + 12k + 2)^3$$
$$w = (6k^2 + 8k + 2)(or)(6k^2 + 4k)$$

Pattern 4 :

Consider (3) as

$$t^2 - w^2 = n^2 - nw \qquad (8)$$

Write (8) in the form of ratio as

$$\frac{t+w}{n} = \frac{n-w}{t-w} = \frac{\alpha}{\beta}, \beta \neq 0$$

which is equivalent to the following two equations

$$t\beta + w\beta - n\alpha = 0$$
$$-t\alpha + w(\alpha - \beta) + n\alpha = 0$$

On employing the method of cross multiplication, we get

$$t = \beta^2 - \alpha^2 + \alpha\beta$$
$$w = \alpha^2 - \beta^2$$
$$n = 2\alpha\beta - \beta^2$$

Substituting these values of t, w and n in (2),the non-zero distinct integral values of $x, y, z, and\ w$ are given by

$$x = (\beta^2 - \alpha^2 + \alpha\beta)(\alpha^2 - 2\beta^2 + 2\alpha\beta)^8$$
$$y = (2\alpha\beta - \beta^2)(\alpha^2 - 2\beta^2 + 2\alpha\beta)^5$$
$$z = (\alpha^2 - 2\beta^2 + 2\alpha\beta)^3$$
$$w = (\alpha^2 - \beta^2)$$

Note:

Equation (8) can also be expressed in the form of ratio as

$$\frac{t+w}{n-w} = \frac{n}{t-w} = \frac{\alpha}{\beta}, \beta \neq 0$$

Repeating the analysis as above, the corresponding integer solutions to (1)are presented below

$$x = -(\beta^2 + \alpha^2 + \alpha\beta)(\beta^2 - 2\alpha^2 - 2\alpha\beta)^8$$
$$y = -(2\alpha\beta + \alpha^2)(\beta^2 - 2\alpha^2 - 2\alpha\beta)^5$$
$$z = (\beta^2 - 2\alpha^2 - 2\alpha\beta)^3$$
$$w = (\beta^2 - \alpha^2)$$

Pattern 5:

Consider (3) as

$$t^2 - n^2 = w^2 - nw \qquad (9)$$

Write (9) in the form of ratio as

$$\frac{t-n}{w} = \frac{w-n}{t+n} = \frac{\alpha}{\beta}, \beta \neq 0$$

which is equivalent to the following two equations

$$t\beta - n\beta - w\alpha = 0$$
$$-t\alpha - n(\alpha + \beta) + w\beta = 0$$

On employing the method of cross multiplication, we get

$$t = -(\beta^2 - \alpha^2 - \alpha\beta)$$
$$w = (\alpha^2 - \beta^2$$
$$n = -2\alpha\beta - \beta^2$$

Substituting these values of t, w and n in (2), the non-zero distinct integral values of $x, y, z, and\ w$ are given by

$$x = -(\alpha^2 + \beta^2 + \alpha\beta)(\alpha^2 - 2\beta^2 - 2\alpha\beta)^8$$
$$y = (\alpha^2 - \beta^2)(\alpha^2 - 2\beta^2 - 2\alpha\beta)^5$$
$$z = (\alpha^2 - 2\beta^2 - 2\alpha\beta)^3$$
$$w = -2\alpha\beta - \beta^2$$

Note:

Equation (8) can also be expressed in the form of ratio as

$$\frac{t-n}{w-n} = \frac{w}{t+n} = \frac{\alpha}{\beta}, \beta \neq 0$$

Repeating the analysis as above the corresponding integer solutions to (1) are given below

$$x = (\alpha\beta - \beta^2 - \alpha^2)(2\alpha^2 - \beta^2 - 2\alpha\beta)^8$$
$$y = (\alpha^2 - \beta^2)(2\alpha^2 - \beta^2 - 2\alpha\beta)^5$$
$$z = (2\alpha^2 - \beta^2 - 2\alpha\beta)^3$$
$$w = (\alpha^2 - 2\alpha\beta)$$

Pattern 6:

Write (5) as

$$t^2 - 3v^2 = u^2 * 1 \qquad (10)$$

Take $u = a^2 - 3b^2$ (11)

and write 1 as

$$1 = (2 + \sqrt{3})(2 - \sqrt{3}) \qquad (12)$$

Substituting (11), (12) in (10) and applying the method of factorization, define

$$(t + \sqrt{3}v) = (a + \sqrt{3}b)^2 (2 + \sqrt{3})$$

Equating the rational and irrational parts in the above equation , we have

$$t = (2a^2 + 3b^2 + 4ab)$$
$$v = (a^2 + 3b^2 + 4ab)$$

Substituting these values of $t, v\ and\ u$ in (4) and (2), the corresponding integer solutions to (1) are given by

$$x = (2a^2 + 6b^2 + 6ab)(2a^2 - 6b^2)^8$$
$$y = (2a^2 + 4ab)(2a^2 - 6b^2)^5 (or)(-6b^2 - 4ab)(2a^2 - 6b^2)^5$$
$$z = (2a^2 - 6b^2)^3$$
$$w = (-6b^2 - 4ab)(or)(2a^2 + 4ab)$$

Note:

1 can also be written as

$$1=(7+4\sqrt{3})(7-4\sqrt{3})$$

For this choice, the integer solutions to (1) are found to be

$x=(7a^2+21b^2+24ab)(2a^2-6b^2)^8$

$y=(5a^2+9b^2+14ab)(2a^2-6b^2)^5 (or)$

$(-3a^2-15b^2-14ab)(2a^2-6b^2)^5$

$z=(2a^2-6b^2)^3$

$w=(-3a^2-15b^2-14ab)(or)(5a^2+9b^2+4ab)$

Conclusion

In this paper, we have obtained different patterns of solutions to Octic equation with four unknowns. As Diophantine equations are rich in variety one may search for other choices of octic equations with multi variables for obtaining there integer solutions with suitable properties.

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